On the Characteristics of a Bolted Joint with Tap Bolts*

(the case where a clamped part is a cover of a pressure vessel)

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This paper discusses on a distribution of contact stresses when the cover of a pressure vessel is fastened with tap bolts to a vessel. In the analysis, by replacing the cover and the vessel by a finite hollow cylinder the distribution of contact stresses is analyzed by three dimensional theory of elasticity. In addition, the force ratio, which is the ratio of an increment of a bolt axial force to a load when a pressure is applied, is analyzed by using three dimensional theory of elasticity and theory of beam on an elastic foundation. Moreover, the maximum stress caused in bolts when a pressure is applied, is also analyzed. Then, a method to estimate the pressures causing a separation of the interface and a leakage of fluid from the joint is proposed. For verification, experiments are carried out. Analytical results are in a fairly good agreement with experimental ones.

Key words : Tap Bolt, Contact Stress, Force Ratio, Pressure Vessel, Structural Analysis, Fixing Element

1. Introduction

In a reasonable design of bolted joints, it is important to take account of the distribution of contact stresses and the force ratio (the ratio of an increment of a bolt axial force to a load). Up to now, many works have been carried out on the distributions of contact stresses and the force ratios, but they⁽¹⁾⁻⁽¹⁰⁾ deal with bolted joints in which bolts and nuts are used and there are few works in which tap bolts are used except some experimental works⁽¹¹⁾⁻⁽¹³⁾. For this reason, in the series of our works with respect to the characteristics of a bolted joint with tap bolts, distributions of contact stresses and the force ratios are studied on the case⁽¹⁴⁾ where a hollow cylinder is fastened to a base with a tap bolt and they are also studied on the cases of a T-flange connection⁽¹⁵⁾ and a circular flanged connection⁽¹⁶⁾ as examples of bolted connections subjected to an eccentric load. It is shown that these results are in a fairly good agreement with experimental ones.

In the present paper, on the case where the cover of a pressure vessel is fastened to the vessel with multiple tap bolts, which is an important example of practical use, the distributions of contact stresses and the force ratios

are studied by the methods⁽¹⁴⁾⁻⁽¹⁶⁾ adopted in the works mentioned above. The purpose of the present paper is a contribution to the reasonable design of bolted joints with tap bolts.

By replacing the cover of a pressure vessel and the base by a finite hollow cylinder, the distribution of the contact stresses is analyzed using three dimensional theory of elasticity (three dimensional model). The distribution of contact stresses near a bolt hole is analyzed by the method in reference (14), namely the contact stress between two hollow cylinders is analyzed as a two body contact problem using three dimensional theory of elasticity. Then, the force ratio when a pressure acts is discussed using three dimensional theory of elasticity. In order to estimate the force ratio more simply, theory of beam on an elastic foundation is used (beam model). Moreover, with an application of a pressure, a bending moment is produced in bolts and the stress due to this moment is added to the stress due to an increment of a bolt axial force. The stress due to this moment is also analyzed with the three dimensional model. Experiments are performed to verify the analyses and analytical results are compared with experimental ones.

2. Theoretical Analysis

2.1 Model for analysis

Figure 1 shows a bolted connection in which the cover of a pressure vessel is fastened by N sets of tap bolts to the vessel with an initial tightening force F , and on which a pressure P is applied. In the first place, in order to analyze the distribution of contact stresses in the

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state of the initial tightening, a model for analysis shown in Fig.2 is assumed. Namely, the cover and the vessel are replaced with a hollow cylinder of which the inner diameter is $2a$, the outer diameter is $2b$ and the height is (h_1+h_2) , where h_1 is the thickness of the cover and h_2 is the height of the vessel. In this hollow cylinder the plane of $z=h-h_1$ is assumed to be the interface between the cover and the vessel. In addition, Young's modulus and Poisson's ratio of the cover and the vessel are denoted by E and ν , respectively. It is assumed that the tightening force $F_t \times N$ acts on the bearing surface of bolts ($d-c \leq r \leq d+c, z=h$) annularly as a uniform pressure P_0 and that the shearing force acts on the lower surface ($d-e \leq r \leq d+e, z=-h$) annularly as a uniform pressure P_1 as shown in Fig.2 (a). Figure 2 (b) shows a model for analysis when a pressure P is applied. When the pressure P is applied, a load $W (=P\pi a^2)$ is caused in the axial direction. The load in the axial direction is assumed to act on the upper and the lower surfaces ($a \leq r \leq b$) as a uniform pressure P_2 annularly. The analysis of the bolted connection when a pressure is applied is done by superposing the case of Fig.2 (a) and (b). In order to take account of an effect of the tightening position of the screw thread, a model ⁽¹⁴⁾ for analysis shown in Fig.3 is assumed to analyze the distribution of contact stresses near a bolt hole. Namely, in this model it is assumed that a stress Q_0 acts on the bearing surface ($a_1 \leq r \leq c$) of the finite cylinder [I] and that a uniform shearing force Q_1 acts on the inner circumference ($r=a_1, h'_2 \leq z \leq h'_1$) of the finite cylinder [II].

Fig.1 Model of pressure vessel when internal pressure is applied (the case where gaskets are not used)

2.1.1 Distribution of contact stresses

In the model for analysis shown in Fig.2 (a) and (b), the distribution of contact stresses is analyzed using the theory of a finite hollow cylinder ⁽¹⁷⁾. Using the Fourier and the Bessel expansions the boundary conditions are expressed as follows :

For the case of Fig.2 (a)

$$r=a: \sigma_r = \tau_{rz} = 0,$$

$$r=b: \sigma_r = \tau_{rz} = 0$$

$$z=h: \sigma_z = P_0 \left[a_0 + \sum_{s=1}^{\infty} a_s C_0(\gamma_s r) \right], \quad \tau_{rz} = 0$$

$$z=-h: \sigma_z = P_1 \left[b_0 + \sum_{s=1}^{\infty} b_s C_0(\gamma_s r) \right], \quad \tau_{rz} = 0$$

$$a_0 = \frac{d_2^2 - d_1^2}{b^2 - a^2},$$

$$a_s = \frac{2 \{ d_2 C_1(\gamma_s d_2) - d_1 C_1(\gamma_s d_1) \}}{\gamma_s \{ b^2 C_0^2(\gamma_s b) - a^2 C_0^2(\gamma_s a) \}}$$

$$b_0 = \frac{r_2^2 - r_1^2}{b^2 - a^2},$$

$$b_s = \frac{2 \{ r_2 C_1(\gamma_s r_2) - r_1 C_1(\gamma_s r_1) \}}{\gamma_s \{ b^2 C_0^2(\gamma_s b) - a^2 C_0^2(\gamma_s a) \}}$$

(a) the case of initial tightening

(b) the case when a load due to internal pressure acts

Fig.2 Analysis by three dimensional model

Fig.3 Model for analysis near a bolt hole

$$P_0 = \frac{F_f \times N}{\pi(d_1^2 - d^2)}, \quad P_1 = \frac{F_f \times N}{\pi(r_1^2 - r^2)}$$

$$d_1 = d - c, \quad d_2 = d + c, \quad r_1 = d - e, \quad r_2 = d + e$$

$$\dots\dots\dots(1)$$

For the case of Fig.2 (b)

$$r=a: \quad \sigma_r = -P \left[g_0 + \sum_{n=1}^{\infty} \left\{ g_n \cos \frac{n\pi}{h} z + g'_n \sin \frac{n\pi}{h} z \right\} \right], \quad \tau_{rz} = 0$$

$$r=b: \quad \sigma_r = \tau_{rz} = 0$$

$$z = \pm h: \quad \sigma_z = P_2 \left[e_0 + \sum_{s=1}^{\infty} e_s c_0(\gamma_s r) \right], \quad \tau_{rz} = 0$$

$$g_0 = \frac{h_c + h}{h}, \quad g_n = \frac{1}{n\pi} \sin \frac{n\pi h_c}{h}$$

$$g'_n = \frac{-1}{n\pi} \left\{ \cos \frac{n\pi h_c}{h} + (-1)^{n+1} \right\}$$

$$e_s = \frac{f^2 - a^2}{b^2 - a^2}$$

$$e_s = \frac{2fc_1(\gamma_s f)}{\gamma_s \{ b^2 c_0^2(\gamma_s b) - a^2 c_0^2(\gamma_s a) \}}$$

$$P_2 = W / \{ \pi(f^2 - a^2) \}$$

$$W = P\pi a^2, \quad h_c = h - h_1$$

$\dots\dots\dots(2)$

where $c_\mu(\gamma_s r)$ is $c_\mu(\gamma_s r) = J_\mu(\gamma_s r) - J_1(\gamma_s b) Y_\mu(\gamma_s r) / Y_1(\gamma_s b)$ and $J_\mu(\gamma_s r)$, $Y_\mu(\gamma_s r)$ are the first and second kind of Bessel functions in order μ respectively. γ_s is λ_s/a ($s=1,2,3,\dots$) and λ_s is the s -th root satisfying $c_1(\lambda_s)=0$. The distribution of contact stresses in the model for analysis shown in Fig.3 is analyzed by the method shown in the reference (14).

(c) determination of $2b_1$ and $2b_2$

Fig.4 Definition of spring constants K_t and K_c

2.2 Analysis of force ratio

When a pressure P is applied on the connection of the pressure vessel which is fastened by N sets of tap bolts with an initial tightening force F_i as shown in Fig.1, a load W is caused in the axial direction. So an increment F_t of a bolt axial force is produced in bolts. The ratio of F_t to W/N (which is a load per a bolt), that is the force ratio, is given by Eq.(3) using the condition that the elongation between the bearing surface of a clamped part and the middle position of the screw thread of the base equals to that of the bolt⁽¹⁴⁾⁻⁽¹⁶⁾.

$$\phi = \frac{F_t}{W/N} = \frac{K_t}{K_t + K_c} \left(\frac{K'_c}{K_{pt}} \right) \dots\dots\dots(3)$$

where K_t is the spring constant of a bolt, K_c and K'_c are the compressive spring constants of a clamped part and K_{pt} is the tensile one of a clamped part. The value of $K_t/(K_t + K_c)$ conforms to Thum's formula⁽¹⁸⁾. It is thought that the value K'_c/K_{pt} conforms to Junker's correction factor⁽¹⁹⁾ which indicates the relation of the distance between a position of the load application and a bolt axis. The spring constants K_t and K_c in Eq.(3) are defined as undermentioned. The spring constant K_t for a bolt is defined by the equation $K_t = F_b/(\epsilon'_b + \epsilon''_b)$ using the displacement $\epsilon'_b + \epsilon''_b$ when the force $F_b(=F_t + F_c)$ acts as shown in Fig.4(a) and K_t is obtained from the reference (20). In order to obtain the compressive spring constant K_c near a bolt hole an equivalent hollow cylinder shown in Fig.4 (b) is assumed. The diameter $2b_1$ of the upper finite hollow cylinder is defined as shown in Fig.4 (c). Namely, denote by $2b'$ the diameter of the circle in which one circle with its center at the center of a bolt hole comes in contact with an adjacent circle with its center at the center of an adjacent bolt. Denote by $2b''$ the diameter of the circle which comes in contact with the inner circumference of the pressure vessel. Denote by $2b'''$ the diameter of the circle which comes in contact with the outer circumference of the pressure vessel. Comparing the values of $2b'$, $2b''$ and $2b'''$, the smallest one is taken as the diameter $2b_1$ of the upper hollow cylinder. The diameter $2b_2$ of the lower hollow cylinder is assumed to be the diameter of the circle which comes in contact with the inner and the outer circumferences simultaneously.

Fig.5 Analysis of correction factor K'_c/K_{pt} by beam model

$2a_1$ is the diameter of the bolt hole and $2a_2$ is the effective diameter of the thread. The value of K_c is obtained by the equation $K_c = F_b / (\epsilon'_1 + \epsilon''_1)$ using the displacements shown in Fig.2 (b). Next, the three dimensional model shown in Fig.2 is used to obtain the value of K'_c/K_{pt} . The value of K'_c is obtained by the equation $K'_c = F_b / (\epsilon'_1 + \epsilon''_1)$ using the displacement $(\epsilon'_1 + \epsilon''_1)$ between the bearing surface and the middle position of the screw thread when the tightening force F_t acts as shown in Fig.2 (a). When a load W is applied to such extent that the interface is not separated, the value of K_{pt} is defined by the equation $K_{pt} = (W/N) / (\epsilon'_2 + \epsilon''_2)$ using the displacement $(\epsilon'_2 + \epsilon''_2)$ between the bearing surface and the middle position of the screw thread as shown in Fig.2 (b).

For simplification, the cover and the vessel are replaced with a beam on an elastic foundation as shown in Fig.5. The value of K_{pt} is defined by the equation $K_{pt} = (W/N) / \delta_2$ using the displacement δ_2 at the center of a bolt as shown in Fig.5 (a). The value of K'_c is defined by the equation $K'_c = F_t / \delta_1$ using the displacement δ_1 at the center of a bolt as shown in Fig.5 (b). The values of δ_1 and δ_2 are obtained from theory of beam on an elastic foundation⁽²¹⁾. The correction factor K'_c/K_{pt} is expressed as follows:

$$\frac{K'_c}{K_{pt}} = 2e^{-\beta c_1} \cos \beta c_1 / (1 + 2e^{-2\beta c_1} \cos^2 \beta c_1) \quad \dots\dots\dots (4)$$

where $\beta = \sqrt{k/4EI_x}$, E is Young's Modulus of the beam, I_x is the moment of inertia of area of the beam and k is the modulus of the foundation. k is obtained by the equation $k = 2K_c W_b / \{\pi(c^2 - a^2)\}$. W_b is the width of the beam. W_b is determined to be equal to the width $2c$ of bearing surface of a bolt.

2.3 Estimation of pressures causing the separation of interface and leakage of fluid

With an increase of the pressure, the separation of the interface is caused. In order to estimate a pressure causing a separation of the interface, by analyzing the case of Fig.2 (a) and (b) and by superposing the stresses at the interface, the value of $W (=P\pi a^2)$ for $F_b (=F_t + F_i)$ is determined such that stress at $r=a$, $z=h-h_1$ becomes zero. Thus, the pressure P causing the separation of the interface is determined. As the pressure P increases further, the separation of the interface progresses to a position under the bearing surface ($r=R_{c\max}$) as shown in Fig.6 (a) and then, a leakage of fluid is caused. In practical use, it is necessary to know a pressure P which does not cause a leakage of internal fluid. The pressure under which the separation progresses to the position $r=R_c$ is determined as such value of $W (=P\pi R_c^2)$ that the stress at $r=R_c$, $z=h-h_1$ becomes zero by analyzing the case of Fig.6 (b). Further, the pressure P causing the leakage of fluid is determined as such value of $W (=P\pi R_{c\max}^2)$ that the stress at $r=R_{c\max}$, $z=h-h_1$ becomes zero. In the beam model, the pressure P causing the separation of the interface is determined as such value of W that the displacement at the inner circumference of the beam becomes zero when the beam is fastened with the initial tightening force F_t and the load W/N acts as shown in Fig.5 (b). For the case when the separation progresses further as shown in Fig.7 (a), the cover is replaced with a beam on an elastic foundation, in which the moment M_0 and the shearing force V acts at the end of the beam as shown in Fig.7 (b).

Fig.6 Analysis when interface is separated (three dimensional model)

Fig.7 Analysis when interface is separated (beam model)

of the stress produced in a bolt is expressed as follows:

$$\sigma_{t \max} = \left(\frac{\phi}{A_s} + \frac{|\psi + \eta|}{z} \right) \left(\frac{W}{N} \right) \dots \dots \dots (6)$$

where, A_s : cross sectional area of a bolt
 z : modulus of section of a bolt
 In Eq.(6), when the value of $\psi + \eta$ is positive, a stress $\sigma_{t \max}$ develops at the side of the load application of a bolt and when the value of $\psi + \eta$ is negative, it develops at the opposite side of a bolt. Next, γ_p and γ_c are obtained from the beam model shown in Fig.9. In the calculation of γ_p eliminating θ_c in Fig.5 (b) from $(\theta_p - \theta_c)$ in Fig.9 (a), γ_p is obtained by the equation $\gamma_p = (W/N)/\theta_p$. The value of γ_c is obtained by the equation $\gamma_c = M_s/\theta_c$ as shown in Fig.9 (b). The value of $\sigma_{t \max}$ is obtained by Eqs.(5) and (6). In the beam model the displacements at the either end of bolt are not taken into consideration. In the case when the interface is separated, similar analyses are carried out.

3. Experimental Method

Figure 10 shows dimensions of the cover of a pressure vessel, the vessel and the bolt used in experiments. Three kinds of the covers of vessels, in which the pitch circle diameter $2d$ is varied as 100, 115 and 130mm, are prepared. In order to change the tightening position of the screw thread as shown in Fig.11, bolts with different lengths of the shank are prepared. The cover of the vessel is fastened to the vessel with eight tap bolts. Figure 12 is the experimental setup to measure the force ratio of the pressure vessel. A pressure is produced in the pressure vessel by an electric hydraulic pump. The pressure is picked up by a pressure transducer and the bolt axial force by strain gages glued to the shanks of bolts. Both signals are recorded by an X-Y recorder through dynamic strain amplifiers.

4. Comparisons of Analytical Results with Experimental ones

4.1 Distribution of contact stresses

Numerical computations are carried out under the condition that dimensions of the finite cylinder shown in Fig.2 (a) are $a=34\text{mm}$, $b=85\text{mm}$, $h_1=26\text{mm}$, $h_2=55\text{mm}$ and $h=40.5\text{mm}$, Poisson's ratio is 0.3, Young's modulus is 206GPa, the width $2c$ of the bearing surface is 17.6mm, $2e$ is 12mm and the pitch circle diameter $2d$ is varied as $2d=100, 115$ and 130mm . The face at $z=h-h_1=14.5\text{mm}$ is assumed to be the interface. In numerical analyses, 15 terms of the series are taken. Analytical results on the case of Fig.2 (a) are shown in Fig.13 (a), indicating the influence of $2d$ on the distribution of contact stresses. The ordinate is the ratio σ_z/P_0 , where P_0 is the stress at the bearing surface. With an increase of $2d$, the

stress at $r=a$ on the interface decreases. Analytical results on the case of Fig.2 (b) are shown in Fig.13 (b). The stress at $r=a$ on the interface increases when a load W is applied. The stress decreases with an increase of r . According to the result of Fig.13 (a) and (b), with an increase of $2d$ the load causing the separation of the interface becomes small. Figure 13 (c) shows a distribution of contact stresses near a bolt hole. The ordinate is the ratio σ_z/P_M , where P_M is the mean stress at the interface. From these analyses it is noticed that when the tightening position of the thread is lower the stress distribution becomes closer to average.

4.2 Force ratio

In order to obtain the force ratio the spring constants are calculated. As an example, calculations are done under the condition that the pitch circle diameter $2d$ is 100mm and the number N of bolts is 8. Moreover, in order to examine the influence of the tightening position of the screw thread on the force ratio, calculations are also done for the upper and the lower tightening positions. In order to obtain the spring constant K_c , the diameter of the equivalent hollow cylinder is determined as $2b_1=16\text{mm}$ and $2b_2=51\text{mm}$ by the method shown in Fig.4 (c). For the upper tightening position, the value of the compressive spring constant K_c of a clamped part is obtained as $2646\text{N}/\mu\text{m}$ by analyzing the model shown in Fig.4 (b) and the value of K_t for a bolt is $389\text{N}/\mu\text{m}$. The force ratio is calculated using the values of spring constants mentioned above and as a result, the force ratio ϕ is obtained as 0.094 in the three dimensional model and 0.021 in the beam model. Similarly, for the lower tightening position the force ratio is 0.101 in the three dimensional model and 0.030 in the beam model. Next, calculations are carried out on the case where

Fig.11 Tightening position of screw thread

Fig.12 Photograph of experimental setup

2d is 115mm. For the upper tightening position the force ratio is 0.051 in the three dimensional model and the interface has already separated in the beam model. For the lower tightening position the force ratio is 0.062 in the three dimensional model and the interface has already separated in the beam model. Figure 14 shows a comparison between numerical and experimental results with respect to the force ratio. The abscissa is the pressure P and the ordinate is the bolt axial force $F_t + F_b$. The initial tightening force F_t is 20kN. With an increase of the pressure, an increment of the bolt axial force F_t is caused linearly. When the pressure increases further, F_t comes to plot a curved line since a separation of the interface is caused. The force ratio means the relation (Eq.(3)) between P and F_t on the linear line. The force ratios obtained by the experiments, in the case of 2d=100mm, are 0.017 for the upper tightening position and 0.028 for the lower tightening position. In the

case of 2d=115mm, an increment of the bolt axial force is very small. Analytical results are in a fairly good agreement with experimental ones, but analytical results obtained by the three dimensional model are a little larger than experimental ones. Figure 15 shows comparisons between numerical and experimental results with respect to the maximum stress taking account of a bending moment. Although the influence of bending moment is small, analytical results are in a fairly good agreement with experimental ones.

4.3 Estimation of pressures causing separation of interface and leakage of fluid

Pressures causing a separation of the interface and a leakage of fluid are calculated using the method given in section 2.3. In the three dimensional model, the pressures causing them are 16.3 and 19.3 MPa in the case of 2d=100mm and 7.7 and 13.4MPa in the case of 2d=115mm. In the

Fig.13 Results of analyses with respect to distribution of contact stresses

Fig.14 Comparisons between numerical results and experimental ones with respect to the force ratio

three dimensional model the influence of the tightening position of the thread is not considered. The case of the lower tightening position is closer to practical conditions. In the beam model, the pressures causing them are 5.6 and 17.3 MPa respectively in the case of $2d=100\text{mm}$ and they are 9.6 and 18.7 MPa for the lower position. In the case of $2d=115\text{mm}$, the interface has already separated in the state of the initial tightening. The pressure causing the leakage of fluid is 11.0 MPa for the upper tightening position and it is 12.0 MPa for the lower tightening position. By experiments the values of the pressure causing the leakage of fluid are 22.4 MPa in Fig.13 (a), 23.8 MPa in (b), 12.5 MPa in (c) and 15.2 MPa in (d). The values of the pressure causing the leakage of fluid, obtained by the three dimensional model and the beam model, are in a fairly good agreement with experimental ones. As a whole, it is effective for the prevention of leakage to minimize the pitch circle diameter $2d$ and fasten the bolts in the lower tightening position. Using the beam model it is possible to estimate the pressure causing the leakage of fluid.

5. Conclusions

In the present paper, discussions are made on the characteristics of a pressure vessel with tap bolts, which is an important example of practical use. The following results are obtained:

- (1) Replacing a connection, in which the cover of a pressure vessel is fastened to the vessel with tap bolts, with a finite hollow cylinder, a method for analysis of the force ratio is demonstrated using three dimensional theory of elasticity. Methods to calculate the stress taking bending moments caused in bolts into consideration and to estimate a pressure causing a separation of the interface are shown.
- (2) For simplification, replacing the cover of a pressure vessel with a beam on an elastic foundation, methods for analysis of the force ratio, the stress taking a bending moment caused in bolts into consideration and pressures causing a separation of the interface and a leakage of fluid, are demonstrated using the beam theory.
- (3) Experiments are performed to verify the results obtained by (1) and (2). Analytical results are in a fairly good agreement with experimental ones.
- (4) In the case of a pressure vessel with tap bolts, the force ratio is considerably small. In order to prevent the leakage it is effective to minimize the pitch circle diameter and to fasten the bolts in the lower tightening position of the thread.

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(the case of $2d=115\text{mm}$ and the lower tightening position)

Fig15 Comparisons taking account of bending moment

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