Synthesis of Leg-Mechanisms of Biped Walking Machines*

(Part II, Synthesis of Foot-Driving Mechanism)

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The paper describes a kinematic synthesis of a planar link mechanism with only revolute pairs to drive the foot which is attached to the ankle-path-generator obtained in the 1st paper.

The ankle-joint angle and foot dimension necessary to realize a required gait have been determined in the case where the ankle-path-generator is pre-determined. Then, a mechanism which is composed of a six-bar mechanism fixed on the thigh and a four-bar mechanism on the shank has been proposed and synthesized to drive the foot.

A walking machine has been manufactured by combining the synthesized ankle-path-generator and the foot-driving mechanism with a mechanism for compensation of moments due to the gravitational and inertial forces. The machine has been experimentally confirmed to walk satisfying the given conditions.

Key Words: Mechanism, Biped Walking Machine, Linkage, Kinematic Synthesis

1. Introduction

In order to enable a synthesis of leg-mechanisms for biped walking machines, it has been proposed to divide the mechanisms into two sub-mechanisms in the 1st part of the paper⁽¹⁾.

Comparing the leg-mechanisms to human legs, their main parts are composed of six links connected mutually by revolute pairs as shown in Fig.1, and the joints J_H, J_K and J_A are called respectively the hip, knee and ankle joints. One of the sub-mechanisms called the ankle-path-generator, which generates a required trajectory of the ankle joint J_A relative to the upper body, has been already synthesized. And the angular displacements $\varphi_H(\theta)$ and $\varphi_K(\theta)$ of the links $J_H J_K$ and $J_K J_A$ relative to the upper body and the link $J_H J_K$ respectively have been determined as functions of the crank angle θ of the mechanism.

In the present paper, the other sub-mechanism called the foot-driving mechanism, which generates a required angular displacement $\varphi_A(\theta)$ of a foot PQ about the ankle joint J_A , will be synthesized in the case where a biped machine walks straight on flat grounds.

2. Type of Foot-Driving Mechanism

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As foot-driving mechanisms driven synchronously with the ankle-path-generator, three types of mechanisms may be considered as shown in Fig.2. In the figure, the links to which arrows are attached are driving links.

Fig.1 Basic construction of leg-mechanism

Fig.2 Types of foot-driving mechanism

The type (a) mechanism is composed of two parallel-crank mechanisms to keep the upper body parallel to the walking plane. In the case where the vertical displacement between the hip joint J_H and the ankle joint J_A varies as is observed in the ankle-path-generator obtained in the former paper, the double-supporting phase may exist only instantaneously. Therefore the leg-mechanisms have to be driven at a low speed while the supporting leg is changed.

The type (b) is an example of leg-mechanisms driven by only one actuator. Leg-mechanisms of this type may be composed of a few links as shown in the figure, where two links are added to the ankle-path-generator. However, it is generally difficult to remove the slip between the foot and the walking plane during the double-supporting phase.

The type (c) is an example of leg-mechanisms whose ankle-path-generator and foot-driving mechanism are driven independently by two actuators. In the case where a biped machine walks with a constant stride, the type (b) mechanism may be adopted. However, when it is required to change the stride arbitrarily, the type (c) mechanism should be adopted. Thus, the type (c) mechanism is studied in this paper.

3. Angular Displacement of Foot

In Fig.1, the points P and Q denote respectively the toe and heel of a foot and the point H denotes the point of intersection of the segment PQ and its perpendicular $J_A H$ dropped from the ankle joint J_A . The dimensions of the foot are $\overline{HP} = l_1$, $\overline{HQ} = l_2$ and $\overline{J_A H} = l_3$. And the angle $\varphi(\theta)$ between the foot and the walking plane is measured counterclockwise.

Now, two same ankle-path-generators are connected with each other by making their hip joints J_H coincide mutually, and their cranks are driven synchronously with a phase angle of π . Denoting by θ the crank angle of the ankle-path-generator whose foot is in contact with the walking plane, the angle $\beta(\theta)$ between the x-axis fixed on the upper body and the walking plane is expressed as follows:

$$\beta(\theta) = \varphi_H(\theta) - \varphi_K(\theta) + \varphi_A(\theta) + \varphi(\theta) \quad \dots\dots(1)$$

And, in the double-supporting phase, the next relation holds.

$$\beta(\theta) = \beta(\theta + \pi) \quad \dots\dots\dots(2)$$

Assuming that only both toes or heels do not contact with the walking plane in the double-supporting phase, the next relation holds because the vertical distances between the hip joint J_H and the two contact points are equal to each other.

$$\left. \begin{aligned} & [l_2 \sin \varphi(\theta) + l_3 \cos \varphi(\theta) + L_1 \sin \{\varphi_K(\theta) \\ & \quad - \varphi_A(\theta) - \varphi(\theta)\} + L_2 \sin \{\varphi_A(\theta) + \varphi(\theta)\}] \\ & - [-l_1 \sin \varphi(\theta + \pi) + l_3 \cos \varphi(\theta + \pi) \\ & \quad + L_1 \sin \{\varphi_K(\theta + \pi) - \varphi_A(\theta + \pi) - \varphi(\theta + \pi)\} \\ & \quad + L_2 \sin \{\varphi_A(\theta + \pi) + \varphi(\theta + \pi)\}] = 0 \end{aligned} \right\} \quad \dots\dots\dots(3)$$

Assuming that each foot does not slip against the walking plane in the double-supporting phase, the next relation holds.

$$\left. \begin{aligned} & [l_2 \cos \varphi(\theta) - l_3 \sin \varphi(\theta) - L_1 \cos \{\varphi_K(\theta) \\ & \quad - \varphi_A(\theta) - \varphi(\theta)\} + L_2 \cos \{\varphi_A(\theta) + \varphi(\theta)\}] \\ & - [-l_1 \cos \varphi(\theta + \pi) - l_3 \sin \varphi(\theta + \pi) \\ & \quad - L_1 \cos \{\varphi_K(\theta + \pi) - \varphi_A(\theta + \pi) - \varphi(\theta + \pi)\} \\ & \quad + L_2 \cos \{\varphi_A(\theta + \pi) + \varphi(\theta + \pi)\}] = l_1 + l_2 - S \end{aligned} \right\} \quad \dots\dots\dots(4)$$

In those equations $L_1 = \overline{J_H J_K}$, $L_2 = \overline{J_K J_A}$ and S is a stride.

Put the crank angles of the ankle-path-generator located forward at the beginning and end of the double-supporting phase as $\theta = 0$ and $\theta = \theta_1$, respectively. Because Eqs. (2)~(4) hold simultaneously in the phase $0 \leq \theta \leq \theta_1$, the foot dimension, stride, and the angles $\varphi(\theta)$, $\varphi(\theta + \pi)$, $\varphi_A(\theta)$ and $\varphi_A(\theta + \pi)$ are determined from those equations when $\beta(\theta)$ of Eq.(1) is given.

As for the single-supporting phase $\theta_1 < \theta < \pi$, $\varphi(\theta)$ is equal to zero within the phase $\theta_2 \leq \theta \leq \theta_3$, where both of the toe and heel are in contact with the ground. Though $\varphi(\theta)$ can be set arbitrarily in the non-contact phase $\theta_1 + \pi < \theta < 2\pi$, it is determined here so as to be a monotonously decreasing function of θ just before the foot lands as shown in Fig.3.

After $\varphi(\theta)$ is determined, an angular displacement of the foot about the ankle joint $\varphi_A(\theta)$ is determined from Eq.(1). In the case of the ankle-path-generator synthesized in the former paper, $\varphi_A(\theta)$ varies taking four extremal values as shown in Fig.4.

4. Synthesis of Foot-Driving Mechanism

4.1 Relation of angular displacements

In general, most part of walking may be made with a constant stride and a constant speed, and the stride and speed may be varied only for a short time when starting and finishing the walking or when avoiding obstacles. Therefore, even when the type (c) mechanism is adopted as a foot-driving mechanism and driven by servo-mechanisms,

Fig.3 Angle between foot and ground

Fig.4 Angular displacement curve of foot

it is desirable to determine the parameters of the mechanism such that the mechanisms can be driven synchronously with the ankle-path-generator at the same constant speed.

Thus, we try to drive the cranks of two foot-driving mechanisms with a phase angle of π at a constant speed. Because the angular displacement $\varphi_A(\theta)$ varies taking four extremal values as shown in Fig.4, a mechanism shown in Fig.5 is adopted instead of the mechanism of Fig.2(c). Denote by θ_0 the angle of the link $J_H J_1$ to the x -axis when the crank angle of the ankle-path-generator θ is zero, and denote moreover by Θ and Φ respectively the angles of the links $J_H J_1$ and $J_K J_6$ to the link $J_H J_K$. Then, the crank angle of the foot-driving mechanism Θ becomes as follows:

$$\Theta = \theta - \varphi_H(\theta) + \pi - \theta_0 \quad (5)$$

And denoting the angle between the link $J_K J_6$ and the extension of the link $J_A J_K$ by ψ , the angle Φ is expressed as follows:

$$\Phi = \psi - \varphi_K(\theta) - \gamma_K + \pi \quad (6)$$

In which $\gamma_K = \angle J_5 J_K J_6$.

4.2 Synthesis

Given the attitude angle of the upper body $\beta(\theta)$ based on an ideal gait, the foot dimension, stride and the angular displacement of the foot about the ankle joint may be determined from Eqs.(1)~(4). However, mechanisms with a single degree of freedom such as Fig.5 cannot generate precisely the required angle $\varphi_A(\theta)$. Thus, the parameters of the mechanism shall be determined here by specifying the attitude angle of the upper body and the angle between the foot and the walking plane at important instants in a period of gait, for example at instants when the toe and heel land and take off.

First, assuming that the four-bar loop $J_K J_6 J_7 J_A$ in the mechanism of Fig.5 is a parallel-crank mechanism, that is $J_K J_6 = J_A J_7$, and $J_K J_A = J_6 J_7$, the angle Φ is expressed as follows from Eq.(6) because $\psi = \varphi_A$.

$$\Phi = \varphi_A(\theta) - \varphi_K(\theta) - \gamma_K + \pi \equiv \Phi_P \quad (7)$$

Substituting $\varphi_H(\theta)$ and $\varphi_K(\theta)$ determined by the ankle-path-generator and $\varphi_A(\theta)$ shown in Fig.4 into Eqs.(5) and (7), and specifying γ_K , the angular displacement $\Phi_P = \Phi_P(\theta + \theta_0)$ is obtained, which varies taking four extremal values as shown by the broken line

in Fig.6. $\Phi_P = \Phi_P(\theta + \theta_0)$ should be generated by the six-bar mechanism $J_H J_1 J_2 J_3 J_5 J_K$. In the case where the output angular displacement of the mechanism takes four extremal values, their two minimum values or two maximum values are equal to each other⁽²⁾. However, $\Phi_P(\theta + \theta_0)$ does not satisfy in general this condition as shown in Fig.6.

Thus, we modify the link lengths of the loop $J_K J_6 J_7 J_A$ so that the minimum values or maximum values of Φ may become equal to each other as shown by the full line in Fig.6.

Because the angular relationship of links at the instants of landing and taking-off of the foot and the transmissibility of motion are essentially important, the six-bar loop $J_H J_1 J_2 J_3 J_5 J_K$ is synthesized by the following method instead of a method of specifying the two equal extremals which has been reported by K. Ogawa⁽³⁾.

As for the four-bar loop $J_H J_1 J_2 J_3$, the next relation holds from Fig.7.

$$(a_1 + a_2 \cos \Theta - a_4 \cos \Psi)^2 + (a_2 \sin \Theta + a_4 \sin \Psi)^2 = a_3^2 \quad (8)$$

Specifying the crank angles $\Theta = \Theta_1$ and the transmission angle $\mu = \mu_1$ when Ψ takes its minimum value Ψ_1 , the next equations hold.

$$\left. \begin{aligned} \Psi_1 &= \pi - \Theta_1 - \mu_1 \\ (a_3 - a_2) \sin \Theta_1 - a_4 \sin \Psi_1 &= 0 \\ (a_3 - a_2) \cos \Theta_1 + a_4 \cos \Psi_1 &= a_1 \end{aligned} \right\} \quad (9)$$

Moreover specifying another arbitrary angular relation (Θ_2, Ψ_2) , the link lengths $a_2 \sim a_4$ are determined from Eqs.(8) and (9), where a_1 is specified arbitrarily as a unit of

Fig.6 Φ - Θ curve

Fig.5 Foot-driving mechanism

Fig.7 Four-bar loop $J_H J_1 J_2 J_3$

length.

After the link lengths of the four-bar loop are determined, the angular displacement ψ of the link J_3J_2 can be calculated as a function of θ from Eq.(8). This means that the relation of the angular displacement between the links J_3J_2 and J_4J_3 of the 2nd four-bar loop $J_3J_2J_4J_3$ in Fig.5 is determined. Therefore the link lengths of the loop can be determined by adopting any method of synthesis of a four-bar function generator, for example the precision-point method⁽⁴⁾.

If the generated function $\phi(\theta+\theta_0)$ has a great error, the above synthesis is repeated by changing the value of ψ_2 .

5. Example of Synthesis and Discussions

5.1 Example of synthesis

The type F six-bar mechanism shown in Fig.8 is adopted as an ankle-path-generator. The curve in the figure is the trajectory of the ankle joint J_A . The hip-joint angle $\varphi_H(\theta)$ and the knee-joint angle $\varphi_K(\theta)$ vary as shown in Fig.9.

Assuming that the toe and heel contact simultaneously with the walking plane at the end of double-supporting phase, set $\theta_1 = \theta_2 = \pi/6$ and specify the following values:

$$\varphi(\theta_1) = 0, \varphi(\pi) = -\pi/18, \varphi(\pi + \theta_1) = -\pi/6$$

$$\beta(0) = \beta(\theta_1) = 0$$

Moreover specifying the stride as $S=280$ mm, the foot dimension and the angle between the foot and the walking plane when the foot lands are determined as follows from Eqs.(1)~(4):

$$l_1 = 107.07, l_2 = 55.40, l_3 = 50.66(\text{mm})$$

$$\varphi(0) = 0.0778\pi$$

Then, the angular displacement $\varphi(\theta)$ becomes the full line shown in Fig.10, where the heel is assumed to leave the ground at $\theta_3 = 7\pi/9$ and the curve except in the phase $\theta_1 \leq \theta \leq \theta_3$ is determined by connecting smoothly the points marked by o.

Next, assuming that the upper body never oscillates, that is $\beta(\theta) = 0$, the angular displacement $\varphi_A(\theta)$ is determined from Eq.(1) as the chain line shown in Fig.10. And assuming that the four-bar loop

Fig.8 Ankle-path-generator

$J_KJ_3J_4J_A$ in Fig.5 is a parallel-crank mechanism, and setting $\gamma_K = 0$, the angular displacement $\phi_P(\theta)$ is determined from Eq.(7) as the broken line shown in Fig.10, which becomes also the broken line shown in Fig.11 as a function of θ . Because the minimum values or maximum values of $\phi_P(\theta)$ are not equal to each other, the link length $\overline{J_KJ_4}$ is changed so that the minimum values may become equal to each other. Concretely, setting $\overline{J_KJ_4} = \overline{J_4J_7} = 180$ mm and $\overline{J_4J_7} = 40$ mm, the link length is obtained as $\overline{J_KJ_6} = 37.70$ mm, and $\phi(\theta)$ becomes the full line shown in Fig.11.

Now, the six-bar loop $J_HJ_1J_2J_3J_4J_K$ shown in Fig.5 is synthesized by the method described in the section 4.2.

Let θ_1 and θ_2 correspond to $\theta = \theta_1 = \pi/6$ and $\theta = \pi$ respectively, and put $\theta_1 = \pi/18$, $\mu_1 = 5\pi/6$ and $\psi_2 = 4\pi/9$, then the next link lengths of the loop $J_HJ_1J_2J_3$ are obtained from Eqs.(8) and (9).

$$a_1 = 100.00, a_2 = 24.41, a_3 = 92.82$$

$$a_4 = 34.73 \text{ (mm)}$$

The initial angle of the crank J_HJ_1 is obtained as follows from Eq.(5):

$$\begin{aligned} \theta_0 &= \theta_1 - \varphi_H(\theta_1) + \pi - \theta_1 \\ &= 0.7445\pi \end{aligned}$$

Fig.9 Hip-joint and knee-joint angles

Fig.10 $\varphi-\theta$, $\varphi_A-\theta$ and $\varphi_P-\theta$ curves

Next, specifying the angular displacements (ψ, ϕ) at $\theta=0, \theta=\theta_1=\pi/6; \theta=\pi, \theta=\theta_1+\pi=7\pi/6$, the parameters of the four-bar loop $J_3J_4J_5J_6$ are obtained as follows by the four-point synthesis⁽⁴⁾:

$$a_5=100.00, a_6=38.94, a_7=66.36$$

$$a_8=30.15 \text{ (mm)}, \gamma_3=0.573 \pi$$

5.2 Discussions

By connecting the foot-driving mechanism obtained just above with the ankle-path-generator shown in Fig.8, the leg-mechanism shown in Fig.12 is obtained. In the figure, the full line, the chain line and the broken line are respectively the trajectories of the ankle J_A , toe P and heel Q relative to the upper body.

If both of the toe and heel contact with the walking plane in the single-supporting phase $\theta_1 < \theta < \pi$, the attitude angle of the upper body β varies as shown by the full line and broken line in Fig.13. Actually however, β may vary in the phase $\theta_1 < \theta < \pi$ as shown by the chain line in the figure because the walking machine is rotated about the toe by a certain moment so that $\phi(\pi) = -\pi/18$. Then, the positions of the legs and upper body relative to the walking plane at every interval of the crank angle of $\pi/6$ in a half period of gait become as shown in Fig.14.

Fig.11 ϕ - θ curves

Fig.12 Synthesized leg-mechanism

In the double-supporting phase, the distance between the heel of the front leg and the toe of the rear leg varies as shown in Fig.15. The distance means a slip between the foot and the walking plane. Though it may be possible to decrease the slip by making the double-supporting phase short, the foot-driving mechanism has to be driven non-uniformly to remove the slip perfectly.

A walking machine has been manufactured by combining the synthesized leg-mechanisms with a mechanism for compensation of moments due to the gravitational and inertial forces, and an experiment on the machine has been carried out. Rollers have been attached at the heels and the foot-driving mechanism and the ankle-path-generator have been driven at the same constant speed, though the slip of about 4mm may be negligibly small in comparison with the stride $S=280$ mm. An experimental gait at 92 steps/min is shown in Fig.16, which is confirmed to be almost the same with the stick diagram shown in Fig.14.

Fig.13 Attitude angle of upper body

Fig.14 Stick diagram

Fig.15 Slip between foot and ground

Fig.16 Gait of experimental machine (Walking speed: 92 steps/min,
Photo. speed: 5 frames/sec)

6. Conclusions

For the development of biped walking machines whose leg-mechanisms have only revolute pairs and few degrees of freedom, a dimensional synthesis of the foot-driving mechanism which generates the required angular motion of the foot about the ankle joint has been carried out, yielding the following results:

(1) The dimension of the foot which is attached to the ankle-path-generator obtained in the former paper can be determined by specifying the stride and the angles of the foot and the upper body to the walking plane.

(2) Also the required angular displacement of the foot relative to the shank in a period of gait can be determined by specifying the phase in which both of the toe and heel are in contact with the walking plane.

(3) In order to generate the required angular displacement of the foot, a foot

-driving mechanism which is composed of a Watt-type six-bar mechanism fixed on the thigh and a four-bar mechanism fixed on the shank has been proposed and synthesized so that its crank may be driven at constant speeds.

(4) A biped walking machine has been manufactured by combining the synthesized ankle-path-generator and foot-driving mechanism with a moment compensator. The machine has been confirmed to walk straight on flat grounds satisfying the given conditions.

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