Analysis of Stability of Externally Pressurized, Gas Lubricated Floating Bush Journal Bearings*

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The threshold speed of whirl instability induced in an externally pressurized, gas lubricated floating bush journal bearing is analyzed theoretically with regard to the effect of rotation of the bush. The authors' previous reports presented at the 4th and 5th Gas Bearing Symposium, the Univ. of Southampton, U.K., are revised and supplemented in relation to that effect. The following facts are found anew; (1) Rotation of the bush in the same direction as the journal produces a reduction in the whirl threshold speed. (2) Rotation of the bush at a low speed in the reverse direction produces an increase in the threshold speed, but its rotation at an excessive speed in this direction produces a drastic reduction in that because of the occurrence of a backward precession.

Key Words: Bearing, Gas Bearing, Externally Pressurized Gas Bearing, Journal Bearing, Floating Bush, Whirl, Stability.

1. Introduction

Elastic supporting of the bearing bush is quite efficacious to stabilize gas lubricated journal bearings. At the 4th and 5th Gas Bearing Symposium, the University of Southampton, U.K., the authors proposed two types of externally pressurized, gas lubricated journal bearings in which the bearing bushes were floated on externally pressurized gas films[1],[2]. To use the gas films in place of elastic members realizes operations at extremely high or low temperature. Their construction is shown in Fig. 1. The upper figure shows the first type in which the pressurized gas to the inside film is fed through a flexible feeder apart from the feed to the outside one. In this type, the outside-to-inside films' stiffness ratio can be controlled by changing each supply pressure. The lower figure shows the second type in which the gas to the inside film is directly fed through the circumferential groove on the outside bearing surface. This type has an advantage in its simple construction, but it is almost impossible to vary the films' stiffness ratio after machining. Besides such features, there is one more difference between them. In the second type, the bearing bush is rotatable, and, if any stoppers are not provided, it rotates in the same direction as the journal rotation with a viscous traction of the inside film. Moreover, it can be driven in the reverse direction by means of some kinds of gas turbines or electric motors. The analysis in the previous reports had already taken up the effect of

bush rotation, but in a wrong way. The authors had realized their mistake, and have been looking for a timely chance for publication of its correction. Fortunately, in Japan, experimental works on the effect of bush rotation in such a bearing have recently been reported by S. Takagi, et al.[3] and A. Sasaki, et al.[4] in series. Taking this opportunity, here, the mistake in the previous reports is corrected, and the effect of bush rotation is theoretically examined anew.

2. Analysis of stability

2.1 Film forces

It is of primary importance to know the characteristics of the reaction forces generated in the inside and outside films.

Fig. 1 Schematics of floating bush bearings

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Fig. 2 Coordinates system

In the first place, the inside film shall be taken up. Coordinates are shown in Fig. 2. The axial coordinate is represented by z .

In a fixed bush bearing for high speed rotation, the clearance is set somewhat large, and the supply pressure is set as high as possible. Under such a condition, inertia forces of the supplied gas play an important role in determining the stability [5], [6]. However, in a floating bush bearing, high speed rotation can be achieved even for a small clearance and a low supply pressure. Under this condition, the film pressure can be governed by the conventional Reynolds equation:

$$\frac{\partial}{\partial \zeta} (H^3 P \frac{\partial P}{\partial \zeta}) + \frac{\partial}{\partial \theta} (H^3 P \frac{\partial P}{\partial \theta}) = \lambda (\omega_1 + \omega_2 - 2 \frac{\partial \phi}{\partial t}) \frac{\partial}{\partial \theta} (PH) + 2 \lambda \frac{\partial}{\partial t} (PH) \quad (1)$$

$$H = h/C_r, P = p/p_a, \zeta = z/R, \lambda = \frac{6\eta}{p_a} (\frac{R}{C_r})^2$$

where R is the bearing radius, C_r the radial clearance, p_a the ambient pressure and η the viscosity. ω_1 and ω_2 are angular speeds of rotation of the journal and the bush. $\partial \phi / \partial t$ represents the angular speed of whirl. Their directions are shown in Fig. 2.

For simplification of the problem, it is assumed that both centers of the journal and bush are concentric to the center of the outside bearing surface under stable operations, because the radial load can be supposed to be small enough in a high speed machine. It can also be supposed that, at the marginal stability, both centers of the journal and bush whirl steadily with small radii. Under these suppositions, the second term of the right-hand side of Eq. (1) can be ignored, and $\partial \phi / \partial t$ can be replaced by a constant speed, Ω . The right-hand side can then be simplified as;

$$\lambda (\omega_1 + \omega_2 - 2\Omega) \frac{\partial}{\partial \theta} (PH) \quad (2)$$

The film pressure which is hydrodynamically generated by the journal displacement is characterized by this term. Referring to Fig. 2, the film thickness is given by $H = 1 + (r_1/C_r) \cos \theta$, so that;

$$\frac{\partial H}{\partial \theta} = - (r_1/C_r) \sin \theta \quad (3)$$

This informs us that the hydrodynamical film force on the journal has a component perpendicular to the displacement, and that its magnitude is proportional to $\lambda (\omega_1 + \omega_2 - 2\Omega) r_1$.

Using a complex plane, the vectorial expression of this hydrodynamical film force can be as follows;

$$c_1 (1 - \frac{\omega_1 + \omega_2}{2\Omega}) i\Omega \mathbf{r}_1 \quad (4)$$

$$\mathbf{r}_1 = r_1 e^{i\Omega t} \quad (5)$$

where c_1 corresponds to the damping coefficient which comes from so-called squeeze action. The hydrostatical variation in the film pressure caused by the journal displacement generates a centripetal force proportional to the displacement. This is given by $k_1 \mathbf{r}_1$ where k_1 represents so-called hydrostatical film stiffness. The hydrodynamical component in this direction is small enough compared with this, so that the resultant film force is given as;

$$\mathbf{F}_1 = c_1 (1 - \frac{\omega_1 + \omega_2}{2\Omega}) i\Omega \mathbf{r}_1 + k_1 \mathbf{r}_1 \quad (6)$$

$i\Omega \mathbf{r}_1$ signifies the rotational speed of the vector $\mathbf{r}_1$. Since, at the marginal stability, it is equivalent to $\dot{\mathbf{r}}_1$, the inside film's force can finally be given by

$$\mathbf{F}_1 = c_1 (1 - \frac{\omega_1 + \omega_2}{2\Omega}) \dot{\mathbf{r}}_1 + k_1 \mathbf{r}_1 \quad (7)$$

where $\dot{}$ represents differentiation with respect to time. In the previous papers, $(\omega_1 + \omega_2)$ was replaced by $(\omega_1 - \omega_2)$. This is wrong, and should be corrected as above.

In the same manner as mentioned above, the outside film's force caused by the bush displacement is given as follows:

$$\mathbf{F}_2 = c_2 (1 - \frac{\omega_2}{2\Omega}) \dot{\mathbf{r}}_2 + k_2 \mathbf{r}_2 \quad (8)$$

where c_2 and k_2 are the damping coefficient and stiffness of the outside film.

2.2 Characteristic equation

Using the film forces given by Eqs. (7) and (8), equations of motion of the journal and bush can be obtained as follows;

$$m_1 (\ddot{\mathbf{r}}_1 + \ddot{\mathbf{r}}_2) + c_1 (1 - \frac{\omega_1 + \omega_2}{2\Omega}) \dot{\mathbf{r}}_1 + k_1 \mathbf{r}_1 = 0 \quad (9)$$

$$m_2 \ddot{\mathbf{r}}_2 - c_1 (1 - \frac{\omega_1 + \omega_2}{2\Omega}) \dot{\mathbf{r}}_1 + c_2 (1 - \frac{\omega_2}{2\Omega}) \dot{\mathbf{r}}_2 - k_1 \mathbf{r}_1 + k_2 \mathbf{r}_2 = 0 \quad (10)$$

where m_1 is the mass of the journal reduced to the bearing section, and m_2 is the mass of the bush. Substituting $\mathbf{r}_1 = r_1 e^{st}$ and $\mathbf{r}_2 = r_2 e^{st}$ into Eqs. (9) and (10), the characteristic equation of the system can be obtained as follows:

$$s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0 \quad (11)$$

$$a_1 = \frac{c_1}{m_1} \{ \frac{1+m^*}{m^*} [1 - R(1+\alpha)] + \frac{c^*}{m^*} (1-R\alpha) \}$$

$$a_2 = \frac{k_1}{m_1} \{ \frac{1+m^*+k^*}{m^*} + \frac{c^* \bar{c}^2}{m^*} (1-R\alpha) [1 - R(1+\alpha)] \}$$

$$a_3 = \frac{c_1}{m_1} \frac{k_1}{m_1} \{ \frac{k^*}{m^*} [1 - R(1+\alpha)] + \frac{c^*}{m^*} (1-R\alpha) \}$$

$$a_4 = \frac{k^*}{m^*} \frac{k_1^2}{m_1^2}$$

where $m^* = m_2/m_1$, $k^* = k_2/k_1$, $c^* = c_2/c_1$,
 $\bar{c} = c_1/\sqrt{m_1 k_1}$, $\alpha = \omega_2/\omega_1$, $R = \omega_1/2\Omega$.

2.3 Threshold of whirl instability

Applying the Routh criterion to Eq. (11), we know that the following conditions should be satisfied for stable operations;

$$a_1, a_2, a_3, a_4 > 0 \quad (12)$$

$$a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 > 0 \quad (13)$$

In these coefficients, the parameters m^* , k^* , c^* and $\bar{c}$ are fixed at the stage of design. Therefore, if the value of α is supposed beforehand, only R is the unknown variable related to the rotational speed of the journal. The value of R at threshold of instability is given by some real root of $a_1, a_2, a_3, a_4 = 0$ and $a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 = 0$. The journal is supported stably under non-rotating condition. It is accelerated continuously from zero to the threshold speed, so that the absolute value of R grows continuously from zero to the threshold one. Taking account of such facts, the value of R at the threshold must be a real root whose absolute value is the smallest. This shall be denoted by R_{on} . Then, the rotational speed of the journal at the threshold of whirl, ω_{on} , is obtained from the relation of $\omega_{on} = 2R_{on}\Omega$. The value of Ω , the angular speed of whirl at marginal stability, can be obtained by substituting $s = i\Omega$ into Eq. (11), that is;

$$\Omega = \pm \sqrt{a_3(R=R_{on})/a_1(R=R_{on})} \quad (14)$$

Non-dimensional threshold speed normalized by the resonant speed of the journal-bush system, $\sqrt{k_1/m_1}$, can thus be obtained as follows:

$$\begin{aligned} \bar{\omega}_{on} &= \frac{\omega_{on}}{\sqrt{k_1/m_1}} \\ &= \pm 2R_{on} \sqrt{\frac{\frac{k^*}{m^*}[1-R_{on}(1+\alpha)] + \frac{c^*}{m^*}(1-R_{on}\alpha)}{1 + \frac{m^*}{m^*}[1-R_{on}(1+\alpha)] + \frac{c^*}{m^*}(1-R_{on}\alpha)}} \end{aligned} \quad (15)$$

If R_{on} is solved as a negative value, backward precessional instability can be observed, and then (-) sign should be used in the above equations.

3. Numerical results and discussions

As found above, the normalized threshold speed, $\bar{\omega}_{on}$, can easily be calculated when the bush-journal mass ratio, m^* , the outside-inside films' stiffness ratio, k^* , the outside-inside films' damping coefficient ratio, c^* , the damping ratio of the bush-journal system, $\bar{c}$, and the bush-journal rotational speed ratio, α , are set at given values. In the following examples, the values of m^* and $\bar{c}$ are set at 0.5 and 0.5.

Firstly, results for $\alpha = 0$, without the bush rotation, are considered briefly. Fig. 3 shows variation in the normalized threshold speed with c^* . The normalized threshold speed for any value of k^* is at its maximum when $c^* \approx 2$. This optimal value of c^* tends to be larger for a larger value of m^* and a smaller value of $\bar{c}$. Fig. 4 shows variation

in the normalized threshold speed with k^* . There also exists the optimal value of k^* . However, in contrast with the variation with c^* , the normalized threshold speed drastically reduces if k^* becomes larger than the optimal value. This critical value is given by $1 + m^*$. Such a drastic reduction is attributable to the fact that the equation $a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 = 0$ has one real root when $k^* < 1 + m^*$, two equal real roots when $k^* = 1 + m^*$, and two different real roots when $k^* > 1 + m^*$.

Next, the effect of the bush rotation on the threshold speed shall be examined. The calculated results are shown in Fig. 5. As considered above, the normalized threshold speed for a non-rotating bush becomes maximal around $c^* = 2$. This value is realizable. The value of k^* must not be larger than $1 + m^*$. This is also realizable without any difficulties. Taking account of these facts, c^* is set at 1, 2 and 3, and k^* at 0.1, 0.5 and 1.

Fig. 3 Normalized threshold speed (variation with c^*)

Fig. 4 Normalized threshold speed (variation with k^*)

Fig. 5 Effect of bush rotation on the threshold speed

Fig. 5(a) shows the results for $c^* = 1$, which is a little below the optimal condition for a non-rotating bush. Under this condition, in the range of bush rotation in the same direction as the journal, $\alpha > 0$, the normalized threshold speed monotonically reduces with the rotational speed of the bush. Natural rotation caused by the tractional force of the inside film is in this direction. In the range of bush rotation in the reverse direction, $\alpha < 0$, the threshold speed increases with the rotational speed of the bush along the same curve. However, it reduces drastically when the rotational speed of the bush becomes higher than a certain value. This is shown by the curve going down steeply leftward. Such a drastic reduction is caused by the occurrence of a backward precession, in which R_{0n} is calculated as a negative value.

Fig. 5(b) is for $c^* = 2$, which is near the optimal condition for a non-rotating bush. Variation in the threshold speed with α for $k^* = 1$ is not monotonic in the range of forward precession. The normalized threshold speed reaches its maximum around $\alpha = -0.1$.

Fig. 5(c) is for $c^* = 3$. Under this condition, the variation with α in the range of forward precession becomes more complicated not only for $k^* = 1$ but also $k^* = 0.5$. The normalized threshold speed reaches its maximum around $\alpha = 0.1$, and its minimum around $\alpha = -0.2$ before its mode changes to backward precession. The effect of bush rotation around $\alpha = 0$ differs from that for $c^* = 1$ or 2.

4. Concluding remarks

The afore-mentioned results qualitatively coincide with the experimental results by A. Sasaki, et al. [4]. Quantitative discussions cannot be performed, because details of the parameters are not described clearly in their report. However, the feature of their experimental curves of the threshold speed versus α is identical with the curves for $k^* = 0.1$ in Figs. 5(a) and 5-(b). This fact indicates the reasonability of the analysis presented above.

Finally, the authors should apologize for having left their mistake uncorrected for a long time.

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