A Study on Rolling Finish of Gear^{*}
(Theoretical Analysis of Number of Contact Points
and Plastic Deformations)

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To clarify the deformation mechanism in rolling finish of the gears, the number of the contact points and the tooth profile are theoretically calculated by taking the difference between the contact point at which the deflection of the tooth is considered and the geometrical contact point, and the calculated results are compared with the experimental results. In this calculation, the degree of the interference at the tip and root of the gears and the contact length in the direction along the tooth depth are considered.

This method of the analysis is applicable to estimate the condition of the tooth contact and the tooth profile after rolling finish.

Key Words: Gear, Forming, Rolling Finish, Theoretical Analysis, Plastic Deformation, End Effects

1. Introduction

It has been already pointed out by the previous studies⁽¹⁾⁽²⁾ that an accuracy of the tooth profile is unsatisfactory because a plastic indentation in the vicinity of the pitch point on the tooth flank occurs in rolling finish of the gears. It is necessary that the deformation mechanism be made clear, in order to improve the accuracy of the rolled gear and obtain a high accurate tooth profile. Therefore, the theoretical analyses have been carried out.⁽³⁾⁻⁽⁵⁾ Authors theoretically analyzed the deformation of the tooth flank with a tooth profile error in consideration of the elastic deflection of the tooth, the depths of the elastic and plastic indentations, and compared the theoretical results with the experimental results.⁽⁶⁾

However, in the previous analyses, the condition of the tooth contact during rolling finish has been simplified by using that of the tooth contact without load, and further the interference at tip and root of the gears has been neglected. An exact analysis for the actual tooth contact in rolling finish has not been made.

In this paper, a model of the tooth contact with a difference between the contact point at which the tooth deflection is considered and the geometrical point without load is proposed. The anal-

ysis of the tooth contact during rolling finish is carried out using the above-cited model and further introducing the contact conditions between the tooth flanks of the die and test gears. In this case an interference action occurs at the tip and root of the test gear. Particularly, the tooth profile provided with an undercutting zone to avoid the trochoidal interference at the tooth fillet is discussed. The effect of the contact length in the direction along the tooth depth is considered. The variations in the number of the contact points and in the tooth profile, which are considered important in clarifying the deformation of the tooth surface, are examined, and the calculated results are compared with the experimental results. Consequently, it is made clear that this method of the analysis is applicable to estimate the tooth contact during rolling finish and the tooth profile after rolling finish.

2. Analysis of Tooth Profile

2.1 Model of tooth contact

The tooth contact is geometrically performed on an identical line of action. However, the tooth is a cantilever and is assumed to be an elastic-plastic solid (the die gear is assumed to be an elastic solid). Therefore, in the actual tooth contact, the tooth is deflected, and the elastic and plastic indentations are produced at the contact point. Consequently, the test gear is slightly rotated from the geometrical position without load, and the contact points deviate from the geometrical line of action. The model of the tooth contact at an arbitrary contact point i is shown in Fig.1. In this figure, δ_{di} and δ_{ai} indicate the displacements at the contact point i of the teeth of the die and test gears (including the depth of the elastic and plastic indentations),

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respectively, and they are given by the displacement occurring tangentially (actual line of action) to the base circle through the contact point i and the displacement in the clockwise direction is assumed to be positive. E_{di} and E_{ai} indicate the distances between the meshing points (a and b) and the contact points on the base circles along the line of action, respectively, and r_{da} and r_{ga} indicate the radii of the base circle, respectively. Further, φ_{di} and φ_{ai} are defined by the angles shown in Fig.1, and hereafter may be termed "contact angle".

The forces acting on the contact point i are shown in Fig.2. The direction of action of the normal force W_i differs from the direction along the line of action without load. Moreover, the tangential force F_i is also acting. Therefore, r_{ga}^* and r_{da}^* indicate the distances between the centers of the gears and a point at which the common normal to both tooth flanks intersects a line perpendicular to the common normal through the centers of the gears, respectively. r_{ga}^* and r_{da}^* differ from r_{ga} and r_{da} , and they have different values at the simultaneous contact points. In Fig.2, E_{di}^* indicates the distance between the meshing point and a point at which the common normal to both tooth flanks is tangent to the circle with a radius of r_{ga}^* . ψ_{ai} is the angle shown in this figure. Moreover, $\Delta\theta$ indicates the difference between the meshing position without load and the position during rolling finish, and the reverse direction to rotation of the test gear is defined as the positive direction.

The analysis of the tooth contact during rolling finish is carried out using

Fig.1 Model of tooth contact

the model of the tooth contact mentioned above.

2.2 Conditions of contact between tooth flanks of die and test gears at contact point

The displacement in the direction along the line of action at the contact point except the depths of the elastic and plastic indentations from δ_{di} is given by $(\delta)_{di}$. The co-ordinates $(x)_{di}$ and $(y)_{di}$ at the point a whose position before deformation is varied by the displacement $(\delta)_{di}$ are

$$\begin{aligned} (x)_{di} &= \mp [r_{da} \sin \varphi_{di} - \{E_{di} \pm (\delta)_{di}\} \cos \varphi_{di}] \\ (y)_{di} &= r_{da} \cos \varphi_{di} + \{E_{di} \pm (\delta)_{di}\} \sin \varphi_{di} \end{aligned} \quad (1)$$

Therefore the gradient of the tangent $[d(y)_d/d(x)_d]_i$ to the tooth flank of the die gear is

$$[d(y)_d/d(x)_d]_i = \frac{\{E_{di} \pm (\delta)_{di}\} \cos \varphi_{di} \pm [d(\delta)_d/d\varphi_d]_i \sin \varphi_{di}}{\mp E_{di} - (\delta)_{di} \sin \varphi_{di} + [d(\delta)_d/d\varphi_d]_i \cos \varphi_{di}} \quad (2)$$

For the case of the test gear, the gradient of the tangent to the tooth flank at the point b whose position before deformation is varied by the displacement $(\delta)_{di}$ can be derived by a similar method. If the gradients at the meshing point of the die and test gears are assumed to be equal, we have

$$\begin{aligned} & \frac{\{E_{di} \pm (\delta)_{di}\} \cos \varphi_{di} \pm [d(\delta)_d/d\varphi_d]_i \sin \varphi_{di}}{\mp E_{di} - (\delta)_{di} \sin \varphi_{di} + [d(\delta)_d/d\varphi_d]_i \cos \varphi_{di}} \\ &= \frac{\{E_{ai} \pm (\delta)_{ai}\} \cos \varphi_{ai} \pm [d(\delta)_a/d\varphi_a]_i \sin \varphi_{ai}}{\mp E_{ai} - (\delta)_{ai} \sin \varphi_{ai} + [d(\delta)_a/d\varphi_a]_i \cos \varphi_{ai}} \end{aligned} \quad (3)$$

where the upper sign at the double sign corresponds to the case of the contact point where the tooth flank at the coasting side of the die gear mates with that at the followed side of the test gear. In contrast, the lower sign corresponds to the case of the contact point where the tooth flank at the driving side of the die gear mates with that at the driven side of the test gear. Hereafter, the meaning of the double sign is the same. As Eq. (3) holds at all of the simultaneous contact points, the number of the conditional equations of the tooth contact corresponding to the number of the contact points can be obtained.

2.3 Geometrical and dynamical conditions of tooth contact

Taking account of the tooth profile error e_{ai} of the test gear, the condition of the tooth contact in the x direction at the contact point i shown in Fig.1 is

$$x_{di} = \mp [r_{da} \sin \varphi_{di} - \{E_{di} \pm (\delta)_{di}\} \cos \varphi_{di}] - x_{ai} = \pm [r_{ga} \sin \varphi_{ai} - \{E_{ai} \pm \delta_{ai} - e_{ai}\} \cos \varphi_{ai}] \quad (4)$$

where e_{ai} is positive when the tooth thickness decreases. In the analysis, the tooth profile error of the die gear is considered to be zero. This equation, similar to Eq.(3), holds at all of the simultaneous contact points.

According to the condition of the tooth contact in the y direction, the center distance a_i at the contact point i is given by,

$$a_i = y_{di} + y_{ai} = [r_{ga} \cos \varphi_{di} + (E_{di} \pm \delta_{di}) \sin \varphi_{di}] + [r_{ga} \cos \varphi_{ai} + (E_{ai} \pm \delta_{ai} - e_{ai}) \sin \varphi_{ai}] \dots \dots \dots (5)$$

the center distance is equal at all of the simultaneous contact points,

$$a_i = \text{const.} \dots \dots \dots (6)$$

Equation (6) shows the relation between the two positions of the simultaneous contact points in the y direction. As for the conditions of the tooth contact in the y direction, the number of the equations which is one less than that of the contact points can be obtained. Equations (4) and (6) obtained above are derived from the geometrical conditions of the tooth contact. In the previous calculations, a part of these equations has been expediently used.

Next, the dynamical conditions may be obtained. One of the dynamical conditions is derived from the equilibrium condition of the forces acting in the direction of the rolling force, corresponding to the y direction. The relation between F_i and W_i is given by $F_i = \mu W_i$ (see Fig.2). In this analysis, the value of μ obtained with the rolling experiments of the rollers is used. Since μ differs by depending on the direction of sliding, $\mu = \mu_+ = 0.135$ when the specific sliding for the test rollers is positive. In contrast, $\mu = \mu_- = 0.167$ when the specific sliding is negative.

When the number of the simultaneous contact points is assumed to be k , the following equation is derived from the equilibrium of forces in the y direction.

$$P_i = \sum_{i=1}^k \{(\sin \psi_{ai} + \mu \cos \psi_{ai}) W_i\} \dots \dots \dots (7)$$

($\mu: \mp \mu_+$ or $\pm \mu_-$)

where P_i is the rolling force.

On the other hand, with respect to the other dynamical condition, the equilibrium for the torques can be considered.

Fig.2 Forces acting on contact point

If the torques acting on the test gear is defined by T_a , we have

$$T_a = \pm \sum_{i=1}^k \{(r_{ga}^* - \mu E_{ai}^*) W_i\} \dots \dots \dots (8)$$

where T_a is positive when the torques act in the reverse direction to rotation of the test gear. Particularly, T_a becomes zero when the test gear is free driven.

When the conditional equations of the tooth contact, Eqs. (4), (6) ~ (8), which are obtained from the geometrical and dynamical relations, and the conditional equation of the contact at the tooth flanks, Eq. (3), which is described in Section 2.2, are solved as the simultaneous equations, the tooth contact in rolling finish can be analyzed under more practical condition.

2.4 Deflection of tooth

The displacements δ_{ait} and δ_{aif} in the direction along the line of action at the contact point i can be obtained⁽⁶⁾ as the sum of the displacements due to the loads acting on the contact point i ,

$$\begin{aligned} \delta_{ait}, \delta_{aif} &= [\text{displacement due to elastic deformation}] + [\text{depth of elastic indentation}] + [\text{depth of plastic indentation}] \\ &= (\delta)_{ait} + [\delta_e]_{ait} + [\delta_p]_{ait}, \\ &(\delta)_{ait} + [\delta_e]_{ait} + [\delta_p]_{ait} \end{aligned}$$

and the displacement due to the loads acting on the point j at another flank of the same tooth,

$$\delta_{aij}, \delta_{aifj} = -(\delta)_{aij}, -(\delta)_{aifj}$$

where $[\delta_p]_{ait}$ is zero, because the tooth flank of the die gear is not plastically deformed. The displacement of the tooth due to the elastic deflection (δ) is calculated by Ishikawa's equation of the tooth deflection.⁽⁷⁾ In this calculation, the tooth deflection due to the tangential load is also considered.

The depth of the elastic indentation $[\delta_e]$ is calculated by Videky's equation which was used in the previous report.⁽⁸⁾

$$\begin{aligned} [\delta_e] &= 4(1-\nu^2)(W_i/b)/(\pi E) \\ &= 5.626 \times 10^{-3} (W_i/b) \quad \text{mm} \end{aligned}$$

where E is Young's modulus GPa, ν is Poisson's ratio, W_i is normal tooth load kN and b is face width mm.

The depth of the plastic indentation is calculated by using the experimental equation obtained from the rolling experiments of the rollers.⁽⁹⁾ For the region in which the end effect of the tooth on the depth of the indentation is considered to be significant, that is the tip and root of the test gear, the test results obtained from the compression tests⁽⁹⁾ of

the die roller and rectangular bars are used. The depth of the plastic indentation $[\delta_p]$ mm including the end effect is expressed as follows:

$$[\delta_p]/[\delta_{p0}] = 10^{\alpha} + 10^{\beta\eta}$$

where

$$\begin{aligned}\alpha &= 0.85\{(W_i/b)/W_0\} + 0.38\{(b/b_0) - 1.58\}^2 \\ &\quad + 0.24\{(H_v/H_{v0}) - 5.26\}^2 - 5.38 \\ \beta &= 0.17\{(W_i/b)/W_0\} - 0.079 \\ \eta &= 0.15\{(W_i/b)/W_0\} - 0.044\end{aligned}$$

In this equation, H_v is a Vickers hardness of the test gear. This equation is expressed in terms of the nondimensional equation using the standard values; $W_0 = 0.980$ kN/mm, $b_0 = 10$ mm, $H_{v0} = 200$ and $[\delta_{p0}] = 7.6 \times 10^{-3}$ mm (except the depth of the plastic indentation due to the end effect), which is calculated by using the values of W_0 , b_0 and H_{v0} . The term of $10^{\beta\eta}$ in this equation expresses the end effect and it is replaced with zero when $(W_i/b) < 0.288$ kN/mm.

2.5 Calculation of conditional equations of tooth contact

The difference between the contact angle φ_{di} or φ_{ai} at the contact point i of the die or test gear and the operating pressure angle α_{00} at the pitch point without load is given by $\Delta\varphi_{di}$ ($=\alpha_{00}-\varphi_{di}$) or $\Delta\varphi_{ai}$ ($=\alpha_{00}-\varphi_{ai}$), respectively. When the angle $\Delta\theta$ corresponding to the difference between the tooth positions without load and that during rolling finish is expressed in terms of the length of the line of action, $\Delta E' = r_{ga}\Delta\theta$. E_{di} and E_{ai} can be expressed by the relations of $E_{di} = E'_{di} - r_{ga}\Delta\varphi_{di}$ and $E_{ai} = E'_{ai} \mp \Delta E' - r_{ga}\Delta\varphi_{ai}$, respectively. Therefore, substituting these equations into Eqs. (3), (4) and (6) and neglecting the small terms of higher than the second order, the following equations can be obtained. Condition of contact between the tooth flanks;

$$\begin{aligned}&(C_{d5}C_{a1} - C_{d6}C_{a1})\Delta\varphi_{di} + (C_{d1}C_{a6} - C_{d4}C_{a5}) \\ &\quad \times \Delta\varphi_{ai} + (C_{d1}C_{a2} - C_{d4}C_{a3})\Delta E' \\ &= C_{d4}C_{a1} - C_{d1}C_{a4} + C_{d7}C_{a1} - C_{d1}C_{a7} \\ &\quad \pm C_{d4}C_{a8} \mp C_{d6}C_{a4} \dots \dots \dots (9)\end{aligned}$$

where

$$\begin{aligned}C_{m1} &= r_{gm}(E'_{mi}/r_{gm})\{1 + (E'_{mi}/r_{gm})^2\} \\ &\quad \times \cos \alpha_{00} \\ C_{m2} &= \{1 + 3(E'_{mi}/r_{gm})^2\} \sin \alpha_{00} \\ C_{m3} &= \mp \{1 + 3(E'_{mi}/r_{gm})^2\} \cos \alpha_{00} \\ C_{m4} &= \mp C_{m1} \tan \alpha_{00} \\ C_{m5} &= \mp C_{m4} \pm r_{gm}C_{m3} \\ C_{m6} &= \pm (C_{m1} + r_{gm}C_{m2}) \\ C_{m7} &= [d(\delta)_m/d\alpha_m] \cos \alpha_{00} - (\delta)'_{mi} \\ &\quad \times \sin \alpha_{00} \{1 + (E'_{mi}/r_{gm})^2\} \\ C_{m8} &= [d(\delta)_m/d\alpha_m] \sin \alpha_{00} + (\delta)'_{mi} \\ &\quad \times \cos \alpha_{00} \{1 + (E'_{mi}/r_{gm})^2\}, \quad (m = d, a)\end{aligned}$$

Condition of tooth contact in the r direction;

$$\begin{aligned}&(E'_{di}\Delta\varphi_{di} + E_{ai}\Delta\varphi_{ai}) \tan \alpha_{00} \mp \Delta E' \\ &= \mp \delta'_{di} \mp \delta'_{ai} + e_{ai} \dots \dots \dots (10)\end{aligned}$$

Condition of tooth contact in the y direction;

$$E'_{di}\Delta\varphi_{di} + E_{ai}\Delta\varphi_{ai} = \text{const.} \dots \dots \dots (11)$$

where α_d and α_a are pressure angles.

With respect to the dynamical conditions, as the amount of the deviation of the direction of action of the load W_i from the direction along the line of action without load is negligible, it can be considered that there is little effect of the deviation on the condition of the tooth contact. Therefore, the amount of the deviation is neglected. Putting $\varphi_{ai} \approx \alpha_{00}$, Eqs. (7) and (8) are rewritten as follows:

$$P_i = \sum_{i=1}^n \{(\sin \alpha_{00} + \mu \cos \alpha_{00}) W_i\} \dots \dots \dots (12)$$

$$T_a = \pm \sum_{i=1}^n \{(r_{ga} - \mu E'_{ai}) W_i\} \dots \dots \dots (13)$$

where the symbol $\cdot$ indicates the values at the contact point without load. When the conditional equations of the tooth contact Eqs. (9)~(13) are solved as the simultaneous equations, the condition of the tooth contact during rolling finish can be approximately estimated.

2.6 Determination of initial point of interference at tip and root of test gear

As the degree of the interference at the tip and root has a significant effect on the normal tooth loads acting on the simultaneous contact points, the position of the initial point of the interference has to be exact. Therefore, in the case of the calculation of the position of the initial point of the interference, the conditional equation of the tooth contact is not transformed into an approximate equation as described in the conditional equation of the tooth contact. Figure 3 shows a diagram for the interference at the tip and root. The degree of the interference of the teeth can be estimated by evaluating the position of the r co-ordinate x_c at the tip c_r and root c_R and that of the r co-ordinate x_d of the point at which the tooth flank of the die gear intersects the line perpendicular to the y

(a) Interference at root (b) Interference at tip

Fig.3 Diagram for interference at tip and root

axis through the points C_r and C_R . When an interference of the tooth occurs, the relations of $|x_c| \geq |x_d|$ at the tip and $|x_c| \leq |x_d|$ at the root are given. If the contact angles at the points c and d are indicated by φ_{ac} and φ_{ad} , respectively and the positions at these points on the line of action are indicated by E_{ac} and E_{dd} ($=E_{dd}-r_{gd}(\alpha_{b0}-\varphi_{dd})$), respectively, x_c and x_d are expressed as follows.

$$x_c = \mp \{r_{ga} \sin \varphi_{ac} - (E_{ac} \pm \delta_{ac} - e_{ac}) \cos \varphi_{ac}\}$$

$$x_d = \mp \{r_{gd} \sin \varphi_{ad} - (E_{dd} \pm \delta_{dd}) \cos \varphi_{ad}\}$$

With respect to the condition of the interference, the following equation is given.

$$\begin{aligned} & \{r_{gd} \sin \varphi_{ad} - (E_{dd} \pm \delta_{dd}) \cos \varphi_{ad}\} \\ & + \{r_{ga} \sin \varphi_{ac} - (E_{ac} \pm \delta_{ac} - e_{ac}) \cos \varphi_{ac}\} \leq 0 \\ & \dots\dots\dots (14) \end{aligned}$$

The relation between the center distance a and the values of the y co-ordinates y_c and y_d at the points c and d is given by $a = y_c + y_d$. Therefore,

$$\begin{aligned} a = & \{r_{ga} \cos \varphi_{ad} + (E_{dd} \pm \delta_{dd}) \sin \varphi_{ad}\} \\ & + \{r_{ga} \cos \varphi_{ac} + (E_{ac} \pm \delta_{ac} - e_{ac}) \sin \varphi_{ac}\} \end{aligned}$$

By substituting the value of φ_{ad} calculated from the above equation into Eq.(14), it can be estimated whether an interference of the tooth is produced at the tip and root of the test gear or not. In this calculation, $\delta_{dd}=0$, when the interference of the tooth occurs at the tip, and $\delta_{ac}=0$, when it occurs at the root.

2.7 Consideration of contact length in the direction along tooth depth

In Sections 2.1~2.6, the analysis is carried out under the assumption that the deformation of the tooth occurs at the contact point on the tooth flank and the deformation occurring at the contact point does not affect that at the point neighboring on the contact point. However, since the contact extends in the direction along the tooth depth for the case of the actual tooth contact, it is considered that the contact length affects the tooth profile after rolling finish. Especially, it is undesirable that the tooth profile becomes steplike when the load shared at each tooth significantly decreases from a high load to a low load during rolling finish. Therefore, the tooth profile of each tooth flank at the driven and followed sides of test gear, in which the number of contact points ranges from one and two (from three to four and from two to three on both tooth flanks), is modified along the profile of the tooth flank of the die gear. In the calculation of the tooth profile, it is assumed that the tooth flank within the contact zone of the test gear is deformed along the profile of the tooth flank of the die gear.

2.8 Method of calculation

Considering the effects of the interference at the tip and root and the contact length and using the method described

in Section 2.5, the tooth profile after rolling finish is calculated. The calculation is carried out from the initial point of the interference between the tip on an arbitrary tooth flank at the driven side of the test gear and the tooth flank at the driving side of the mating tooth of the die gear to that between the tip on the next tooth flank at the driven side of the test gear and the next tooth flank at the driving side of the die gear. In this calculation the distance of division on the tooth flank is 0.25 mm in length of the line of action. As the conditional equations of the tooth contact are non-linear, the depth of the plastic indentation at the meshing point is estimated as follows; the depth of the plastic indentation and the tooth profile error are primarily calculated by using the tooth profile error before rolling finish, and then similarly calculated by using the tooth profile error obtained by the first calculation. Further, the calculation is carried out using the preceding results. This work is repeated until the depth of the plastic indentation after calculation agrees with that used as the initial condition. The tooth profile obtained by neglecting the small protuberances on the measured tooth profile curve is used as the tooth profile before rolling finish.

3. Comparison of Experimental Results with Calculated Results

The calculated results obtained under the conditions of the free driven test gear and the constant rolling force using the method described in Chap. 2 are compared with the experimental results. In this calculation, the pitch error is assumed to be zero.

3.1 Experimental method and conditions

The tests are carried out using the gear rolling machine used in the previous rolling experiments⁽⁶⁾ and the machine is improved to reduce the frictional resistance between the guides and the beds. The test gear is free driven. The details of the die and test gears are given in Table 1. The test gears are hobbled by a protu-

Table 1 Gear design specifications

	Die gear	Test gear	
Material	SKH 9	S45C	
Module	4		
Pressure angle deg.	20		
Number of teeth	36	28	
Pitch diameter mm	144	112	
Addendum modification coefficient	+0.2	+0.488	
Face width mm	14	5	10
Outside diameter mm	154.2	124.0	124.2
Hardness	Hv 741	Hv177 ± 3	Hv195 ± 5
Method of finish	Heat treatment after hobbled and then Maag ground	Reishauer ground after hobbled	Hobbled

berance hob to avoid the trochoidal interference. The rotational frequency of the test gear is 0.1 rpm. The machine oil (ISO VG 46) is smeared on the tooth flanks of the die and test gears. The rolling forces P of 0.392(40), 0.588(60) and 0.784(80), [kN/mm(kgf/mm)] are applied and the number of rolling passes is one. The measurement of the variation in the number of contact points is carried out using the strain gages stuck on the root at the side of the measured tooth of the test gear and on the roots at the sides of the four teeth of the die gear set at both sides of the measured tooth.

3.2 Experimental results and considerations

The variation in the number of contact points of the ground gears with the rotation of the die gear is shown in Fig.4. θ_d is the angle of rotation of the die gear. When the tip on the tooth flank at the driven side in the measured tooth of the test gear mates with the tooth flank at the driving side of the die gear, $\theta_d = 0$. The figure shows the variation in the number between $\theta_d = 0$ and the position ($\theta_d = 0.174$ rad) where the tip on the next tooth flank at the driven side of the test gear mates with the next tooth flank at the driving side of the die gear. In this figure, a dotted line indicates the experimental results. The number of contact points varies in order of $4 \rightarrow 3 \rightarrow 4 \rightarrow 3$ with the rotation of the die gear in the case of the face widths of 5 and 10 mm ($4 \rightarrow 3 \rightarrow 2 \rightarrow 3$ in the case of the face width of 5 mm and $P = 0.392$ kN/mm), and the length of the zone with four contact points increases and the length of the zone with three contact points decreases with an

Fig.4 Variation in number of contact points(Reishauer ground gears)

increase of the rolling force. This variation in the length of each contact zone due to the rolling force is attributed to the variation in the center distance due to the rolling force. For example, in the case of a large rolling force, the center distance is reduced by a large deflection of the tooth, and the interference at the tip and root can be easily produced.

A solid line indicates the calculated results. From this figure it can be found that the calculated results for the case of the face widths of 5 and 10 mm well agree with the experimental results. Therefore, using this method of the analysis, the condition of the tooth contact between the die and test gears during rolling finish can be estimated with a high accuracy.

The variation in the number of the contact points for the case of the hobbed gears with the tooth profile error is shown in Fig.5. The general tendency with the hobbed gears is almost similar to that with the ground gears and the calculated results well agree with the experimental results.

Next, in order to clarify the degree of the interference at the tip and root on the tooth flank, the calculated results about the number of contact points at each position E_a on the line of action are shown in Fig.6. The given conditions for the calculation in Fig.6 are similar to those in Fig.4(b). In this figure, D_r and F_r indicate the zone in which the interference at the tips on the tooth flanks at the driven and followed sides occurs, respectively. In contrast, D_r and F_r indicate the zone in which the interference occurs at the roots. Since the lengths of both the interferential zones at the tip and root increase with an increase of the rolling force, the number of contact points varies. Particularly, in the vicinity of $E_a = 25$ mm, it can be clearly observed that the length of the zone with two contact points significantly decreases with an increase of the length of the zone with three contact points and a zone with four contact points occurs. Thus the interference at the root has a comparatively large influence on the variation in the number of contact points. Moreover, the interferential zone considerably extends on the tooth flank with an

Fig.5 Variation in number of contact points(Hobbed gears)

increase of the rolling force. From these results it is clear that the interference at the tip and root is an important factor with a large influence on the tooth profile after rolling finish.

The variation in the tooth profile of the ground gear after rolling finish under $P=0.784 \text{ kN/mm}$ is shown in Fig.7. Figures 7(a), (b) and (c) show the tooth profiles before rolling finish, after rolling finish and those by the calculation, respectively. The indentation is observed on both tooth profiles of the driven and followed sides after rolling finish, and the shape of the indentation is different between the driven and followed sides. This fact is attributed to the variations in the load shared at each side and in the direction of the plastic flow due to the difference in the direction of the frictional force between both sides. The calculated variation in the tooth profile at

both sides well agrees with the experimental one. Therefore, this method of the analysis of the tooth profile is applicable to estimate the tooth profile after rolling finish with a sufficient accuracy. In Fig. 7(c), a dotted line indicates the tooth profile neglecting the contact length in the direction along the tooth depth. The shape of the tooth profile considering the contact length in the direction along the tooth depth is more similar to that of the tooth profile with the experiments. From this figure it can be seen that the contact length is a factor affecting the tooth profile.

The tooth profile of the hobbed gear under $P=0.784 \text{ kN/mm}$ is shown in Fig.8. The shape of the tooth profile in the

(a) Driven side

(b) Followed side

Fig.6 Variation in number of contact points on tooth flank (Reishauer ground gears, $b=10 \text{ mm}$)

(i) $b=5 \text{ mm}$

(ii) $b=10 \text{ mm}$

Fig.7 Tooth profile curve [Reishauer ground gears, $P=0.784 \text{ kN/mm}$ (80 kgf/mm)]

Fig.8 Tooth profile curve [hobbed gears, $P=0.784 \text{ kN/mm}$ (80 kgf/mm)]

experiments differs from that of the tooth profile in the calculation. This difference is attributed to the influence of a plastic flow. Therefore, if the effect of the plastic flow on the tooth profile is considered, the tooth profile more similar to that obtained in the experiments will be able to be evaluated. Consideration of this matter will be made in the future.

4. Conclusions

The condition of the tooth contact in rolling finish of the gears is theoretically analyzed, and the analytical results for the variation in the number of the contact points and the tooth profile are compared with the experimental results. The results may be summarized as follows:

- (1) The calculated results well agree with the experimental results. Therefore, this method of the analysis is applicable to estimate the condition of the tooth contact during rolling finish and the tooth profile after rolling finish.
- (2) The interference at the tip and root has an influence on the length of each contact zone, and the length of the zone of interference increases with an increase of rolling force.

- (3) In the analysis of the tooth profile, it is necessary that the effect of the contact length in the direction along the tooth depth be considered.

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