Large-Scale and Small-Scale Turbulence Structures in the Intermediate Wake of a Two-Dimensional Normal Plate *

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Turbulence structure in the intermediate wake of a normal flat plate was studied experimentally by a conditional-sampling technique. The measurements were made at a section 8.0 heights downstream of the front face of the plate. Velocity fluctuations were decomposed into those produced by large-scale structures and those associated with small-scale structures: the former fluctuations were again decomposed into the phase-averaged fundamental component and the deviation from it (large-scale deviation). Velocity vector fields, contours of streamlines, vorticity of the fundamental component, together with contours of intensity and correlation of the large-scale deviation velocity, were presented and discussed. Contours of intensity, correlation and turbulence intermittency of the small-scale fluctuations were also presented in and around the large-scale structures. Large Reynolds shear stress associated with the large-scale deviation velocity was suggested to be the result of a stretching of vortex filaments in the saddle-point flow region.

Key Words : Turbulence, Wake, Coherent Structure, Conditional Sampling, Unsteady Flow, Saddle-Point Flow, Normal Plate

1. Introduction

In fluid engineering there are many cases where the wake of a body meets a downstream body; for example, cascade wings of multi-stage turbomachines affected by wakes of upstream cascades, pipe bundles in heat exchangers, complex tall chimneys, groups of buildings, etc. Small airplanes can be out of control or crash if they encounter the vortex wake of a huge airplane. In order to evaluate the fluctuating forces acting on downstream bodies, it is necessary to understand the structure of wakes of upstream bodies. Since in most applications the distance between upstream and downstream bodies is several to ten times the representative length of the bodies, the structure of wakes in this region (hereafter referred to as the intermediate wake) is practically important. The coherent structure in the intermediate wake of two-dimensional bodies has been studied by visualizations⁽¹⁾ and phase-averaging measurements⁽²⁾⁻⁽⁶⁾. In the latter the coherent structure is mostly described in terms of velocity-vector fields and contours of streamline, vorticity, turbulence intermittency, etc. Recently Cantwell & Coles⁽⁷⁾ obtained the coherent structure in the near wake of a circular cylinder and properties of random velocity fluctuations in and around the coherent structure by a flying hot-wire technique.

In a previous paper⁽⁸⁾ the authors divided the velocity fluctuation in the

intermediate wake of a circular cylinder into two components, i.e. one due to the coherent (large-scale) structure and one due to the small-scale structure to obtain the properties of these structures by a phase-averaging technique.

An instantaneous velocity Q can be written as

$$Q = \bar{Q} + \bar{q} + q'$$

where $\bar{Q}$ is the time-mean value, $\bar{q}$ is the velocity fluctuation due to the large-scale structure and q' denotes the velocity fluctuation due to the small-scale structure. If we denote phase-averaged quantities by $\langle \rangle$, the phase average of the large-scale structure $\langle \bar{q} \rangle$ is defined as

$$\langle \bar{q} \rangle = \langle Q \rangle - \bar{Q}$$

The difference $\bar{q} - \langle \bar{q} \rangle = \bar{q}'$ is non-zero because the circulation, the shape, the central position etc. differ from one realization to another of the large-scale structure; this feature is referred to as a jittering or dispersion. Cantwell & Coles⁽⁷⁾ defined $\bar{q}' + q' (= q^*)$, say as the random velocity and discussed its phase-averaged property. It should be noted, however, that the fluctuation q^* contains a component associated with the dispersion of the large-scale structure. Since, as will be shown later, $\bar{q}'$ is greater than q' in the intermediate wake at high Reynolds numbers, it is essential to sort out the component q' as correctly as possible in order to study the properties of the small-scale structure.

The purpose of this paper is to clarify the turbulence structure in the intermediate wake, in terms of the space-time distributions of the phase-averaged large-

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scale components $\langle \bar{q} \rangle$, the dispersions $\bar{q}'$ and the small-scale components q' . A normal plate was employed in this study as a wake-producing obstacle because its separation point is fixed at the edges, so that the coherent structure is expected to have a smaller dispersion than that in the wake of a round body such as a circular cylinder.

The co-ordinate system and the definition of main symbols are shown in Fig. 1. The x-axis of the co-ordinate is taken in the longitudinal direction, the y-axis parallel to the front face of the plate with a height h and the z-axis normal to the x- and y-axes so as to form a right-handed system; the origin is set at the forward stagnation point at the mid-span. The velocity components in the x- and y-directions are denoted by U, V , the corresponding time-mean components by $\bar{U}, \bar{V}$, and the fluctuating components by u, v . The velocity of the main flow is written as U_0 .

Fig. 1 Definition of the co-ordinate system and main symbols.

2. Experimental apparatus and method

2.1. Experimental apparatus. The experiments were conducted in an open-return wind-tunnel with a 30 cm high, 30 cm wide and 101 cm long test section. The free-stream turbulence intensity $(\bar{u}^2)^{1/2}/U_0$ was 0.2% or less at $U_0=16.4$ m/sec. The test plate (manufactured of brass) had a height $h=(20.0 \pm 0.1)$ mm and a thickness 4.3 mm, the edges being bevelled to 30° (see Fig. 3). The plate spanned the whole width of the test section, so that the aspect ratio was 15.0 and the tunnel-wall blockage was 6.7%. No end plates were used. Reynolds number based on the free-stream velocity U_0 and the height of the plate h was 2.3×10^4 . The vortex-shedding frequency f_k was $0.146 U_0/h$ at this Reynolds number.

The velocities were measured by constant-temperature hot-wire anemometers with X-probes at many positions y along a vertical line ($x/h=8.0, z/h=0.0$), which will hereafter be referred to as the line of measurement. Reference or conditioning signals for phase-averaging were velocity waveforms detected at a fixed position ($x/h=8.0, y/h=0.0, z/h=0.25$). The hot-wires were tungsten wires of $5 \mu\text{m}$ diameter with a working length of about 1 mm, the distance between two wires in the X-array being about 1 mm. Outputs of the anemometers were linearized and recorded on four channels of an analogue tape recorder with a DC-10 kHz frequency response. The time span of signals employed in phase-averaging was 10 s; this was long enough to yield steady

averaged values. The signals were digitized (12 bit) with a sampling frequency 20 kHz and later analyzed by a large computer (HITAC M-280H).

The turbulence intensities $(\bar{u}^2)^{1/2}/U_0, (\bar{v}^2)^{1/2}/U_0$ at the centre of the wake were nearly constant within a spanwise distance $|z| \leq 5h$, the deviation being 3.5% or less. Accordingly the flow in the wake was fairly two-dimensional.

2.2. Large-scale and small-scale structures. As mentioned in Chapter 1, the instantaneous velocities are decomposed in the following forms

$$U = \bar{U} + u, V = \bar{V} + v$$

$$u = \langle \bar{u} \rangle + \bar{u}' + u', v = \langle \bar{v} \rangle + \bar{v}' + v'$$

In this paper the components $\langle \bar{u} \rangle, \langle \bar{v} \rangle$ are called the fundamental velocity fluctuation and the corresponding structure of flows the fundamental structure; the components $\bar{u}', \bar{v}'$ the large-scale velocity deviation and the corresponding structure of flow the large-scale deviation structure; the components u', v' the small-scale velocity fluctuation and the corresponding structure of flow the small-scale structure (see Fig. 2).

The components $\bar{u}, \bar{v}$ were at frequencies of the order of the vortex-shedding frequency f_k whereas the components u', v' were at frequencies higher than f_k . Accordingly $\bar{u}, \bar{v}$ were obtained by passing the signals u, v through a digital low-pass filter (with a cut-off frequency $4f_k$) and u', v' by passing u, v through a digital high-pass filter (with a cut-off frequency $4f_k$). The choice of this cut-off frequency is by the author's intuition. It should be noted, however, that this frequency is twice the frequency of the longitudinal velocity fluctuation at the wake axis. This implies that vortices of length scale less than roughly $1/4$ of the shed vortices are classified as small-scale structures. The small-scale structure thus means a structure of flow contributing to fluctuations of length scales approximately by a quarter less than the scale of the periodically shed vortices.

Fig. 2 Decomposition of velocity fluctuations into three components.

2.3. Reference signal and definition of phase. The phase was determined from signals (u_r, v_r , say) which were obtained by respectively passing the velocity fluctuations u, v at the reference point through a narrow band-pass digital filter with a central frequency $2f_k$ and f_k . Figure 3 shows an example of simultaneous waveforms of u_r, v_r, u, v and u at a height $y/h=3.0$ on the line of measurement,

together with the phase angle φ . The phase φ is defined by

$$\varphi = \pi \left(\frac{t - t_{1,i}}{t_{2,i} - t_{1,i}} \right) \quad (t_{1,i} \leq t \leq t_{2,i})$$

$$\varphi = \pi \left(\frac{t - t_{2,i}}{t_{1,i+1} - t_{2,i}} \right) + \pi \quad (t_{2,i} \leq t \leq t_{1,i+1})$$

where $t_{1,i}$ is the i -th time (measured from an arbitrary origin $t=0$) when $v_f=0$ and $dv_f/dt > 0$, $t_{2,i}$ the i -th time when $v_f=0$ and $dv_f/dt < 0$.

Fig. 3 Example of waveforms of velocity fluctuations u , u_r , v_r and reference signals u_f , v_f and definition of phase angle φ .

As mentioned previously, the large-scale structures passing the line of measurement have various length scales, circulations and vertical positions of the centre. The fluctuations in the length scale are supposed to have a strong influence on the period T_b of v_f . On the other hand, if the centre of the large-scale structures suffers no change in the vertical y -position, the period T_u of u_f is exactly a half the mean vortex-shedding period $T_k (=1/f_k)$. As seen in Fig. 3 (note the inserted arrows), the waveform of u shows high-frequency random fluctuations at a time when the signal u_f becomes irregular. This is probably caused by an outward shift of a large-scale structure.

In order to remove the large-scale structures with extremely large dispersion (i.e. extremely large changes in the length scales, circulations, vertical positions of the centre) from the phase-averaging, the signals were checked during a half period $T_u/2$, $T_b/2$ of u_f , v_f and those satisfying the following conditions were sampled:

$$0.85 T_k/2 \leq T_u/2 \leq 1.15 T_k/2$$

$$0.3 T_k/4 \leq T_b/2 \leq 1.7 T_k/4$$

The waveforms satisfying the conditions of T_b and T_u were respectively 92% and 85% of all the waveforms; the waveforms satis-

fying simultaneously the two conditions were 75%, the actual number of the waves being 880. The phase-averaging was made at equally-spaced 120 phases of one period for two consecutive periods.

3. Results and discussion

3.1. Intensity of fluctuating velocities. An example of the fluctuating velocity waveforms is shown in Fig. 4; distributions of various intensities and correlations of the velocity fluctuations are given in Fig. 5. The energy of the large-scale deviation $\langle \bar{u}^2 \rangle$ occupies about 80% of the overall energy of the longitudinal-velocity fluctuation u^2 . $\langle \bar{u}^2 \rangle$ of the fundamental velocity fluctuation is much smaller than its vertical component $\langle \bar{v}^2 \rangle$. On the other hand, the energy of the small scale velocity fluctuation is distributed equally in longitudinal and vertical directions. The correlation $\bar{u}'\bar{v}'$ of the large-scale deviation is nearly equal to the global correlation uv . It is clear from the above results that the large-scale deviation is the major component of velocity fluctuations in the intermediate wake.

3.2. Fundamental structure and turbulence. The velocity-vector field of the fundamental structure ($\langle \bar{u} \rangle + \bar{U} - U_c, \langle \bar{v} \rangle$) is shown in Fig. 6. Here U_c is the convection velocity of the large-scale structure (i.e. the celerity of streamwise vortex

Fig. 4 Example of waveforms of three components of velocity fluctuations.

Fig. 5 Energy and correlation of velocity fluctuations divided by U_c^2 . The abscissa is multiplied by 100.

centre) Figure 6 is the vector field seen by an observer moving downstream with the vortex.

The convection velocity U_c was evaluated from $U_c = 2\pi\Delta x f_K / \theta$ where $\Delta x (=0 \sim 0.9h)$ is the streamwise distance between two hot-wire probes, θ is the phase difference of the measured crossflow-velocity components at the mean vortex-shedding frequency f_K . The convection velocity is generally a function of the vertical y position. This functional relation was measured previously and used to obtain the location of the centre of the large-scale structure and its convection velocity. Assuming a particular y position as a tentative centre, we can construct a velocity-vector field such as Fig. 6. If the centre of the vector field coincided with the assumed y position, this was taken as the true centre. If the two positions were not the same, iteration was rehearsed until a coincidence was attained. The centre of the large-scale structure was found to be at $y/h=0.50$ and the convection velocity at the centre was $U_c/U_0=0.815$. The phase angle ϕ can be transferred to the x -coordinate by the relation $x=U_c\phi/2\pi f_K$.

Figure 7 shows the streamline pattern seen by an observer travelling with the large-scale structure. Here the stream function $\langle\bar{\psi}\rangle$ was calculated from

$$\langle\bar{\psi}\rangle = \langle\bar{\psi}_0\rangle + \int \{(\langle U\rangle - U_c)dy - \langle V\rangle dx\}$$

where $\langle\bar{\psi}_0\rangle$ is an arbitrary constant.

Figure 8 presents contours of the

Fig. 6 Vector field $\langle(U) - U_c, \langle V\rangle$ of fundamental structure.

Fig. 7 Stream lines of fundamental structure. $\bullet$: vortex centre, $\times$: saddlepoint, ---: turbulent boundary.

phase-averaged turbulence intermittency function I . The function I was defined as unity (or zero) when a short-time average of $(du'/dt)^2$ is greater (or less) than a threshold e_{th} . The averaging time was one-tenth of the vortex-shedding period and the threshold level was 0.2% of $(du'/dt)^2$. When v' was used instead of u' to define I , the result was practically the same as Fig. 8.

Figure 9 shows contours of frequency at which the intermittency function I drops from 1 to 0, the direction of flow being from left to right. Accordingly, the probability of the flow being turbulent is high on the right-hand side of the ridges of the contour lines whereas it is low on the left-hand side of the ridges. The ridges will hereafter be called the turbulent boundaries.

Fig. 8 Phase-averaged turbulence intermittency function (contour interval=0.1).

Fig. 9 Contours of frequency at which the turbulence intermittency function drops from 1 to 0. $\times$: ridge of contourlines.

Fig. 10 Vorticity of fundamental structure (contour interval=0.2)

(a) Overall fluctuation; $\langle u'^2 \rangle / U_0^2 \times 10^3$
(contour interval=8)

(b) Fundamental structure; $\langle \tilde{u}^2 \rangle / U_0^2 \times 10^3$
(contour interval=2)

(c) Large-scale deviation; $\langle \tilde{u}^2 \rangle / U_0^2 \times 10^3$
(contour interval=5)

(d) Small-scale structure; $\langle u'^2 \rangle / U_0^2 \times 10^3$
(contour interval=1)

Fig. 11 Energies of longitudinal velocity fluctuations.

(a) Overall fluctuation; $\langle v'^2 \rangle / U_0^2 \times 10^3$
(contour interval=8)

(b) Fundamental structure; $\langle \tilde{v}^2 \rangle / U_0^2 \times 10^3$
(contour interval=8)

(c) Large-scale deviation; $\langle \tilde{v}^2 \rangle / U_0^2 \times 10^3$
(contour interval=3)

(d) Small-scale structure; $\langle v'^2 \rangle / U_0^2 \times 10^3$
(contour interval=1)

Fig. 12 Energies of vertical velocity fluctuations.

Fig. 13 Velocity correlation.

Figure 10 shows contours of the vorticity $\langle \tilde{\omega} \rangle = \partial \langle \tilde{v} \rangle / \partial x - \partial \langle \tilde{u} \rangle / \partial y$. The vorticity attains a maximum at the centre of the large-scale structure, the contours being approximately concentric circles. The circulation of the vortex Γ evaluated by the surface integral of the vorticity is $\Gamma / U_0 h = 2.1$. The circulation shed from the separation point during one period of the vortex shedding, Γ_0 , is $(1/2)(1 - C_{pb})U_0^2 / f_K$ (C_{pb} : the base-pressure coefficient), the ratio Γ / Γ_0 is equal to 0.3 if C_{pb} is assumed to be $-1.08^{(9)}$. This value of Γ / Γ_0 is close to 0.26 for a D-shaped cylinder (Davies⁽²⁾) and 0.35 for a circular cylinder at 8 diameters downstream (Cantwell & Coles⁽⁷⁾).

3.3. Phase-averaged energy of fluctuating velocities. Contours of energy of fluctuating velocities in the longitudinal and vertical directions are shown in Figs. 11 and 12. The phase-averaged energy of the global longitudinal-velocity fluctuation $\langle u^2 \rangle$ attains a maximum near the centre of the vortex; their contours seem to generally extend towards a nearby saddle point. On the other hand, it is worth noting that the energy of the global vertical-velocity fluctuation attains a maximum at different positions, i.e. nearly at the middle of two consecutive vortex centres.

The energies of the fundamental structures $\langle \tilde{u}^2 \rangle$, $\langle \tilde{v}^2 \rangle$ (see figures labelled (b)) take a primary maximum at a position fairly far from the vortex centre. The location of maxima of the vertical component $\langle \tilde{v}^2 \rangle$ almost coincides with that of $\langle v^2 \rangle$; this is probably because $\langle \tilde{v}^2 \rangle$ occupies the largest part of the time mean overall energy v^2 (see Fig. 5).

The energies of the large-scale deviations $\langle \tilde{u}''^2 \rangle$, $\langle \tilde{v}''^2 \rangle$ are shown in figures labelled (c). The longitudinal component $\langle \tilde{u}''^2 \rangle$ attains a maximum at the vortex centre whereas in the vertical component $\langle \tilde{v}''^2 \rangle$ a maximum appears at a position slightly off the vortex centre. The contour lines of $\langle \tilde{u}''^2 \rangle$ and $\langle \tilde{v}''^2 \rangle$ generally seem to extend towards a saddle point in the vertical and longitudinal directions, respectively.

The energies of the small-scale structures $\langle u'^2 \rangle$, $\langle v'^2 \rangle$ (figures labelled (d)) show very similar contour lines for the two components whose energies are also of the same magnitude. This suggests that the energy associated with the small-scale structure is equally distributed in the two components. Moreover, the contour lines are also similar to those given by Cantwell & Coles⁽⁷⁾.

3.4. Correlation of fluctuating velocities. Figures 13(a)–(d) show correlation of fluctuating velocities. Contours of correlation of the global velocity fluctuations $\langle uv \rangle$ (Fig. 13(a)) show maxima very close to the saddle points. The correlation of the large-scale deviation $\langle \tilde{u}'' \tilde{v}'' \rangle$ also attains maxima at the saddle points (see Fig. 13(c)). This is reasonable because, as seen in Fig. 5, the large-scale deviation occupies most of the overall correlation uv . The above feature suggests that the large-scale deviation

structure is stretched in the direction of out-going separatrices of the saddle points. Such a stretching can yield a significant correlation $\langle \tilde{u}'\tilde{v}' \rangle$ near the saddle points. In fact Cantwell & Coles⁽⁷⁾ suggest that the stretching is the most important and even universal mechanism of generating turbulent Reynolds stresses in free shear flows.

The correlation of the small-scale structure $\langle u'v' \rangle$ is so small (less than the other correlations by an order of magnitude) that the contours show few definite features. It appears, however, that the contours near the saddle points are generally stretched in the direction of the out-going separatrix. This might imply that the small-scale structure also suffers a stretching at saddles.

The correlations attain a large value when the fluctuating velocity vectors are inclined by 45° with respect to the longitudinal direction; they are nearly zero if the velocity vectors are parallel or normal to the longitudinal direction. Accordingly, in order to better understand the nature of the velocity fluctuations, it is useful to know space-time distributions of the average directions of the velocity fluctuations. For this purpose, we defined a parameter R as follows. Let a set of fluctuating velocity vectors be denoted by $(v_i, i=1, N)$. As shown in Fig. 14, the components (u, v) of v_i in the co-ordinate (x, y) are transformed into (u_0, v_0) if the axis of co-ordinate is rotated by an angle θ to have a new co-ordinate (x_0, y_0) . The phase-averaging of u_0^2 and v_0^2 ($=u_0^2 + v_0^2$) was obtained to calculate the ratio of $\langle u_0^2 \rangle^{1/2}$ ($=E_0$) to $\langle u^2 + v^2 \rangle^{1/2}$ ($=E_c$), E_0/E_c is denoted by r_0 . The parameter R is defined as

$$R = 1 - r_{\min} / r_{\max}$$

where $\theta_{\max}$ and $\theta_{\min}$ are respectively the angles θ at which r_0 attains a maximum and minimum. It easily follows that $r_{\max} + r_{\min} = 1$ and $|\theta_{\max} - \theta_{\min}| = \pi/2$. R is unity where the velocity v_i is always in one direction and zero when the direction of the velocity changes randomly from 0 to 2π . Figure 15 shows the space-time distributions of vectors whose modulus and argument are respectively R and $\theta_{\max}$. The modulus R for the large-scale deviation (Fig. 15(a)) is largest near the saddles where the argument is almost $\pi/4$. This

Fig. 14 Definition of rotation of co-ordinate system.

feature clearly suggests that the velocity fluctuations associated with the large-scale deviation are the result of a stretching of vortex filaments. It is interesting to note that the turbulence boundaries are fairly normal to the vector $(R, \theta_{\max})$. On the other hand, the vector field for the small-scale structure (Fig. 15(b)) shows few significant features.

4. Conclusions

In this paper, the properties of the velocity fluctuations in the wake of a normal flat plate were studied at 8 heights downstream of it by decomposing the fluctuations into three components; the fundamental fluctuation, the large-scale deviation, and the small-scale fluctuation. Reynolds number based on the free-stream velocity and the plate height was 2.3×10^4 . Main results of the present study can be summarized as follows:

- (1) The energy of the large-scale deviation is much greater than that of the small-scale fluctuation.
- (2) The energy of the large-scale deviation attains maxima at the centre of the fundamental structure (i.e. the coherent vortex). The Reynolds shear stress associated with this deviation also attains maxima near the saddle points; this demonstrates that the large-scale deviation is the major component of the velocity fluctuations in the intermediate wake. More-

(a) Large-scale structure deviation

(b) Small-scale structure

Fig. 15 Vector fields showing directivity of fluctuating velocity vector.

over, the maximum of the shear stress at the saddles is probably the result of a stretching of vortex filaments there.

(3) The energy of the small-scale structure is the smallest among the three components of the velocity fluctuations. Its maximum appears at a position off the vortex centre. Contour lines of this energy are approximately concentric.

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