Three-dimensional Jet Issuing from the Cruciform Nozzle*

(1st Report, Mean Flow Properties)

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Mean flow properties of air jet issuing from the cruciform nozzle with a sufficiently large aspect ratio into still air have been observed over a wide area of the flow field. From the experiment, it is clear that the streamwise variation of the both scales of velocity and length is slower than that of two-dimensional and axis-symmetric jets. The velocity contours show a symmetric, crisscross shape even at the most downstream section measured in this experiment, whilst in the upstream region the velocity profile exhibits a saddle-back shape for both symmetric slices of $y = 0$ and $z = 0$. In the region of $x/d \geq 75$ the velocity profiles can be expected to be geometrically similar in the entire flow field.

Key Words: Turbulence, Cruciform Nozzle, Jet, Mean Flow Properties, Characteristic Scale, Similarity

1. Introduction

A cruciform jet crossing the long axis of two plane jets with a sufficiently large aspect ratio has a three-dimensional structure due to the mutual interference between two intersecting plane jets. With this jet the velocity profiles in the flow field still maintain the crisscross shape well up to the downstream extent. Studies on the velocity profile in such flow field can be applied to some of engineering practices, such as the improvement of thrust at the combustion nozzle, the promotion of heat transfer due to the impinging jet and the air conditioning. The aim of the present investigation is to clarify the difference of mean flow properties between the cruciform jet and the two-dimensional or the axis-symmetric jet.

So far, many studies concerning the turbulent jet have been done with respect to the two-dimensional and axis-symmetric jets, and review papers by Abramovich⁽¹⁾ and Rajaratnam⁽²⁾ are presented. Recently Ishigaki⁽³⁾ reexamined a large volume of data based on the method of his theoretical consideration. Most jets used in general are three-dimensional ones originating from rectangular nozzles with various aspect ratios or different shaped nozzles. In previous studies concerning the three-dimensional jets, a square jet measured using

the conditional sampling method by Nakamura et al.⁽⁴⁾, jets issuing from rectangular nozzles with several aspect ratio by Sforza⁽⁵⁾ and Sfeir⁽⁶⁾, jets from various kinds of irregular shaped nozzles by Okitsu⁽⁷⁾ et al. and the jet from cruciform nozzles with three finite values of aspect ratio by Marsters⁽⁸⁾ are presented. In reference (2) a review about a three-dimensional jet is given and also a theoretical analysis of the three-dimensional jet using turbulent models is made by Rodi⁽⁹⁾.

Flow field chosen for the present study is a cruciform jet where a quasi two-dimensional jet remains up to the boundary of a sufficiently large control volume, and where three-dimensional regions appear due to the mutual interference between two intersecting plane jets and two-dimensional regions exist adjacent to each other. Therefore, the present cruciform jet is different from the three-dimensional jets which eventually become axis-symmetric jets, and it has not been investigated so far. In this paper the streamwise variation of various scales and the similarity of mean velocity profiles are discussed using the measured value of velocity profiles up to a distance 150 times the nozzle width in the downstream direction.

Nomenclature used in this study are listed below and shown in Fig.1.

- x, y, z : Cartesian coordinate system, x in the direction of free stream, y, z distances along two-nozzle axes from the origin
- y' : distance measured along $y = z$ slice from the x axis
- U_e : jet exit velocity
- U_{ox} : mean velocity on the jet centerline (x axis)
- U_o : maximum mean velocity on the jet centerline in the quasi two-dimensional region

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- U : x component of mean velocity
 b' : half width on $y = |z|$ slice
 b_m : distance from the x axis on $z = 0$ slice where $U = (U_{ox} - U_o)/2$
 b_o : half width for the quasi two-dimensional region
 l', l, l_o : Townsend's length scales on $y = |z|$ slice, $z = 0$ slice and for the quasi two-dimensional region, respectively
 η, ζ : nondimensional distance of y/b_o and z/b_o along both y and z axes, respectively
 P_{ox}, P : mean static pressures (gauge pressure) on the jet centerline and for the quasi two-dimensional region, respectively
 d : jet nozzle width
 Re : Reynolds number ($= U_e \cdot d / \nu$)
 ρ, ν : density and kinematic viscosity of air

2. Experimental Apparatus and Techniques

The wind tunnel used in the present experiments is shown in Fig.1. The settling chamber had a square section whose length of diagonal line was 990 mm, four screens (open area ratio of $\beta \approx 0.62$), and honeycombs. The cruciform nozzle installed at the end of the tunnel was constructed of four acrylic resin plates which were machined with special caution, and the edges of crossed parts of nozzle were finished sharp. The nozzle had a rectangular shape of 3.0 mm width (with in accuracy of 0.05 mm) and 990 mm length, and the transverse section profile of the nozzle had a quarter circle of 10 mm radius as shown in Fig.1. The configuration of flow field, the coordinate system, and the nomenclature are also described in Fig.1.

Fig.1 Configuration of flow field and coordinate system

Mean velocity and static pressure were measured with a pitot-static pressure tube whose outer diameter was 1.5 mm (total pressure tube diameter 0.8 mm and static pressure tube diameter 0.3 mm). Special care was taken to suppress the vibration of support of the pitot-static tube in pursuing the experiments. The main measurement was made at 9 sections in the range of $x/d = 0$ to 150. For estimating the streamwise variation of physical quantities, the measurement was carried out at 19 stations in consideration of the accuracy.

The Reynolds number based on the nozzle width d and jet exit velocity ($U_e = 40$ m/s) was kept constant at 8×10^3 in the present experiment. The mean velocity at the nozzle exit shows a valley of profile (at most 2% of U_e) near the center of jet, while it shows the uniformity of profile for the other part. The magnitude of turbulent intensity is less than 1% of U_e at the nozzle exit. Now, we can select the point of $z/d = 0$, $y/d = 60$ as representative of quasi two-dimensional region. This region was determined from the contours of the mean velocity as mentioned later.

3. Experimental Results and Discussion

3.1 Configuration of overall flow field

To see the feature of the cruciform jet, mean velocity profiles on both the $z = 0$ slice (right hand side) and $y = z$ slice (left hand side) are shown in Fig.2. At first, the result for $z = 0$ slice will be discussed. At the section of $x/d = 10$ a decay of mean velocity on x axis is not found, and thus the existence of a potential core is indicated. This potential core region in the present jet is longer than that of a two-dimensional jet, which shows that the degree of mixing (entrainment) between the jet near the center region and the surrounding fluid is smaller compared with that of two-dimensional jet. In the extent of $x/d < 50$, the velocity profiles show a saddle-back shape (6~8) which has the minimum value in the region of $y/d = 7 \sim 15$. The existence of a saddle-back shape is the feature of a three-dimensional flow. In the far downstream extent $x/d \geq 75$, these profiles take a monotonous shape that shows a maximum value at x axis and gradually approaches a quasi two-dimensional value with an increasing y/d . On the other hand, considering

Fig.2 Mean velocity profiles for the $z=0$ slice and the $y=z$ slice

the mean velocity profiles on $y = z$ slices, they indicated a typical profile which takes a maximum value at jet centerline and decreases to zero with an increasing y'/d except for upstream extent. Therefore, the velocity profiles on both $y = 0$ slice and $y = z$ slice do not agree with each other even in the farthest downstream section measured in the present experiment and are different from the velocity profile of an axis-symmetric jet near the center region of jet. However, both profiles on $y = 0$ and $y = z$ slice seem to have a characteristic shape.

Fig.3 shows the streamwise variation of mean static pressures on both the x axis and the quasi two-dimensional region. In the neighborhood of nozzle exit, the static pressure on the x axis is positive, while on the quasi two-dimensional region it is nearly equal to zero. The streamwise variation of static pressure on the x axis is smaller than that on the quasi two-dimensional region, and its value does not still recover to the atmospheric pressure in the farthest section measured in the present experiment. Here, let's compare the present results with those about the axis-symmetric jet by Okamoto⁽¹⁰⁾ and the two-dimensional jet by Hussain et al.⁽¹¹⁾ The static pressure profile of the present jet on the x axis has a different tendency from that of an axis-symmetric jet, and the negative value of the present jet is smaller than that of the axis-symmetric jet. However, this negative value remains far downstream. The static pressure profile of the present jet for the quasi two-dimensional region has a little smaller absolute value than that of the two-dimensional jet, but the tendency of static pressure profile is similar. It is a distinctive feature of present flow field that the asymptotic process of the static pressure on the x axis to the atmospheric pressure is extremely calm.

Fig.3 Streamwise variation of mean static pressure profiles

Fig.4 Velocity scales

3.2 Scales of flow field

Fig.4 shows the streamwise variation of the velocity scales of the maximum velocity on the x axis and on the quasi two-dimensional region. The maximum velocity U_{0x}/U_e on the x axis is almost equal to unity in the extent of $x/d \leq 10$. Therefore, existence of the potential core region is confirmed. U_{0x}/U_e decays as $(x/d)^{-0.46}$ in the extent of $x/d \geq 75$. The decay rate is evidently smaller than that of the axis-symmetric jet as shown in Fig.4. Meanwhile, the maximum velocity U_{0z}/U_e for the quasi two-dimensional region is equal to unity in the extent of $x/d \leq 4$. In the wide extent of $x/d \geq 8$, U_{0z}/U_e decays as $(x/d)^{-0.50}$, and the decay rate agrees with that of a two-dimensional jet by Miller et al.⁽¹²⁾ As mentioned above, the decay rate of the velocity scale on the x axis is smaller than that of the quasi two-dimensional region. This tendency is also recognized in the wake of a cruciform circular cylinder.⁽¹³⁾ Meanwhile, the maximum velocity of a jet issuing from the cruciform nozzle with aspect ratio of 12 by Marsters⁽⁸⁾ decays as $(x/d)^{-1.01}$ in the extent of $x/d \geq 75$. Because this flow field eventually becomes an axis-symmetric jet, the nature of jet is essentially different from the present jet.

Next, Fig.5 shows the streamwise variation of both the half width and Townsend's length scale⁽¹⁴⁾ as the representative length scale. As we can not define the half width on the $y = 0$ slice, the half width both on the $y = z$ slice and the quasi two-dimensional region only are described in the figure. The half width b' on the $y = z$ slice increases as $(x/d)^{0.91}$ in the extent of $x/d \geq 30$, and is slightly smaller than that of the axis-symmetric jet⁽¹⁰⁾. The half width b_0 for the quasi two-dimensional region increases as $(x/d)^{1.0}$ in the extent of $x/d \geq 10$, and agrees quite well with that of the two-dimensional jet by Miller et al.⁽¹²⁾ in the same manner as the tendency of the velocity scale. On the other hand, Townsend's length scales both on $z = 0$ slice and $y = z$ slice are defined as

$$l^2 = \frac{\int (\partial U / \partial y) y^2 dy}{\int (\partial U / \partial y) dy} \quad (z=0 \text{ 面})$$

$$l'^2 = \frac{\int (\partial U / \partial y') y'^2 dy'}{\int (\partial U / \partial y') dy'} \quad (y=z \text{ 面})$$

..... (1)

Fig.5 Length scales

respectively. Thus, l on $z = 0$ slice increases as $(x/d)^{0.91}$ beyond $x/d = 20$, while l' on $y = z$ slice increases as $(x/d)^{0.99}$ beyond $x/d = 30$. Therefore, the rate of spread is slightly larger than that of b' . l_0 determined from equation (1) for the quasi two-dimensional region increases as $(x/d)^{0.98}$ in the extent of $x/d \geq 10$, and this rate of spread is equal to that of b_0 . So far, a reversal of the length scale values with respect to the typical two axes of a rectangular jet is observed in the cases of the three-dimensional jet such as the rectangular jets by Sforza et al.⁽⁵⁾ and Sfeir⁽⁶⁾ and Tsuchiya et al.⁽¹⁵⁾ However, in the present experiment the reversal of that for both $z = 0$ slice and $y = z$ slice is not observed. Thus l' on $y = z$ slice is always larger than l on $z = 0$ slice. In the wake of a cruciform circular cylinder by Osaka et al.⁽¹³⁾ a reversal of the length scales of l and l' occurs.

3.3 Isotach plot

To show the distribution of the mean velocities in the flow field, the isotach plot of that is described based on the measurements at 1029~1244 points in the cross section. Figures 6, 7 and 8 show the contours of the mean velocity at the section of the $x/d = 20, 75$ and 150 respectively. At $x/d = 20$, the whole aspect of this isotach plot indicates a symmetric crisscross shape. Considering the streamwise variation of the velocity and the length scales discussed earlier, it can be said that the result in the larger region of $|y/d|$ and $|z/d|$ becomes a quasi two-dimensional jet. The present jet shows a velocity defect region, namely a saddle-back shape mentioned above, in the vicinity of $y/d = z/d = \pm 8$. This characteristic shape can be observed in the upstream extent in the wake of a cruciform circular cylinder,⁽¹³⁾ and its origin is explained by the existence of a secondary flow. However, in the present jet the distortion of isotach plot is not clearly recognized even in the corner of the crisscross shape of mean velocity. At $x/d = 75$ the flow field spreads further, but the tendency of the isotach plot is almost equal to that of upstream section. However, the velocity defect region which was observed both on the y and the z axes at $x/d = 20$ can not be found. At the farthest down-

stream section of $x/d = 150$ measured in the present experiment, the spread of the flow field becomes larger, though the isotach plot still takes a crisscross shape. From this fact, we can expect the similarity of the mean velocity profile in the entire

Fig.6 Isotach plot of mean velocity ($x/d = 20$)

Fig.7 Isotach plot of mean velocity ($x/d = 75$)

Fig.8 Isotach plot of mean velocity ($x/d = 150$)

Fig.9 Similarity of mean velocity for the $z = 0$ slice and the $y = z$ slice

region of the cross section. The existence of the velocity defect region (so-called saddle-back shape) is also observed in other three-dimensional flows, such as the rectangular jet by Sforza et al.⁽⁵⁾ and Sfeir⁽⁶⁾ (at the distance of about 30 times the short axis) and in the wake of a cruciform circular cylinder by Osaka et al.⁽¹³⁾ (about 150 times the diameter).

3.4 Similarity

So far, it is reported that in the two-dimensional or axi-symmetric jets the self-preservation was exactly valid. Realization of the similarity of three-dimensional jet, however, is not distinct, and it is very difficult to derive a self-preserving solution even in the representative slice. Here, the similarity of mean velocity profile is discussed both on the representative slices ($z = 0$ slice and $y = z$ slice) and on the whole cross section, based on the experimental result. The similarity of the mean velocity profiles normalized using U_{ox} and l, l' on both the $z = 0$ slice and $y = z$ slice is shown in Fig. 9. The velocity profile on the $z = 0$ slice shows the similarity in the region beyond $x/d = 75$, while the profile on the $y = z$ slice becomes similar in the region beyond $x/d = 50$. Comparing the similarity profiles for the two-dimensional jet⁽¹⁶⁾ and for the axi-symmetric jet,⁽¹⁷⁾ the present U/U_{ox} takes a lower value in the region of $y'/l' < 0.75$, and a higher value in the region of $y'/l' > 0.75$. This fact means that the present flow field can not be expressed simply as the sum of two intersecting plane jets and that to understand the present jet we have to consider the mean flow kinetic energy flux in the cross section of $x/d = \text{const.}$. From the results mentioned above, it may be said that the similarity of the mean velocity profiles on both representative slices is valid in the region beyond $x/d = 75$ at least. From the survey with respect to the effect of Reynolds number made in the preliminary experiments, it can be said that the variation of Reynolds number does not affect the similarity of mean velocity.

Next, the similarity of the mean velocity profile in the whole cross section will be discussed. In the cross section normal to the jet centerline (x axis), we see the similarity of the profile in the whole cross section in that the mean velocity profile

Fig. 10 Length ratio among three representative scales

normalized using the characteristic velocity scale and length scale becomes similar in the whole flow field as discussed in the three-dimensional wall jet by Koso et al.⁽¹⁸⁾ Then, if the similarity is kept, it can be expected that the ratio of length scales on both the representative slices will be constant, being independent of x/d . Figure 10 shows the streamwise variation of the ratio of length scales for both the $z = 0$ slice and $y = z$ slice and both the quasi two-dimensional region and $y = z$ slice. With the ratio of the length scale for both $z = 0$ slice and $y = z$ slice, the ratio of half width b_m/b' has a constant value in the region beyond $x/d = 50$, while the ratio of Townsend's length scale l/l' is also constant in the region beyond $x/d = 100$. Meanwhile, the ratio of the length scale for the quasi two-dimensional region and the $y = z$ slice (half width b_0/b' and Townsend's length scale l_0/l'), has an approximately constant value in the region beyond $x/d = 75$. From the result described above, it can be suggested that the similarity of mean velocity in the whole cross section is approximately valid in the region beyond $x/d = 75$.

Now, let's consider the empirical formula of mean velocity. Deriving the self-preserving solution from the equation of motion was difficult, so the mean velocity formula utilized the experimental result. In deriving the experimental formula, the following conditions were assumed.

(1) In the quasi two-dimensional region, the velocity profile agrees with that of a two-dimensional jet.

(2) The half width on both the $y = 0$ and the $z = 0$ slices varies along an exponential curve of $[\exp(-0.693\eta^q)]$ from b_0 to b_m as z is increased in the region from the fringe of jet flow to the center of the jet.

(3) The value calculated with the empirical formula becomes equal to the measured value of $U/U_{ox} = 0.5$ on the $y = z$ slice.

Considering the well known solution for the two-dimensional jet⁽¹⁹⁾ and the empirical conditions described above, the following equation is obtained,

$$\frac{U}{U_{ox}} = \frac{1}{2} \exp[-0.693\{(1 + A \exp(-0.693\eta^q))^2 \zeta^2\}] + \frac{1}{2} \exp[-0.693\{(1 + A \exp(-0.693\zeta^q))^2 \eta^2\}] \quad \dots\dots\dots (2)$$

where the value of index q and empirical constant A in formula (2) can be determined from the condition (3). Figures 11(a), (b) show comparison of the empirical formula (2)

Fig. 11 Comparison of empirical equation (2) with experimental results

with the isotach plot obtained by the experiments for the first quadrant of two cross sections of $x/d = 75$ and 150 . Taking $q = 4.0$ and $A = 0.35$ in the formula, the calculated results described by the solid lines in the figure agree well with the experiments in the whole cross section at $x/d = 75$. Furthermore, at $x/d = 150$ the agreement of the two mentioned above is fairly good in the whole cross section. Next, to examine the validity of the empirical formula (2) in detail, comparison between the similar profile and formula (2) was made for both the $z = 0$ slice and the $y = z$ slice. The curve of the formula (2) with the values of q and A at $x/d = 150$ is shown with solid lines in Fig.9. As can be seen from Fig.9, the curve of formula (2) gives an overestimate compared with the measured value for the $z = 0$ slice except near the jet centerline region and the quasi two-dimensional region, while for the $y = z$ slice it is valid.

3.5 Momentum conservation for the representative slice

It is difficult to estimate the momentum flux over a sufficiently large control volume, because the present jet is a three-dimensional flow. So only the representative slice ($y = z$ slice) will be discussed.

Considering the equation of motion for the coordinate system of x, y', z' and applying it to $z' = 0$ ($y = z$) slice, the following equation is obtained using the boundary layer approximation with the pressure term neglected,

$$U \frac{\partial U}{\partial x} + V' \frac{\partial U}{\partial y'} = - \frac{\partial \overline{uw'}}{\partial y'} - \frac{\partial \overline{uw'}}{\partial z'} \quad \dots\dots(3)$$

where z' is a coordinate normal to the $x-y'$ plane, V' the mean velocity in the y' direction, and u, v' and w' are the components of turbulent intensity in the x, y' and z' directions respectively. Integrating the equation (3) along y' gives

$$\begin{aligned} \frac{d}{dx} \int_0^\infty U^2 dy' &= - \int_0^\infty U \frac{\partial W'}{\partial z'} dy' \\ &+ \frac{d}{dz'} \int_0^\infty (-\overline{uw'}) dy' \quad \dots\dots\dots(4) \end{aligned}$$

where W' is the mean velocity in the z' direction and is determined by the continuity equation. As the right hand side of equation (4) does not appear for the two-dimensional and the axi-symmetric jets, it is not necessary to consider them in the far downstream region, because the three-dimen-

sional jet eventually settles to an axi-symmetric jet. However, in the present flow field the effect of the right hand side of equation (4) still appears even in the far downstream region, thus it is necessary to consider them. Because of the lack of the experimental data, we can not discuss them in the present paper. Therefore, the validity of equation (5) was surveyed, for convenience, in which the right hand side of equation (4) is neglected

$$\int_0^\infty U^2 dy' = \text{const.} \quad \dots\dots\dots(5)$$

Fig.12 shows the streamwise variation of local momentum flux M normalized with the momentum flux Me at $x/d = 5$. The result gives a constant value in the region beyond $x/d = 75$. Hence in the far downstream region where the similarity of the profile in the whole flow field can be expected to hold, it is likely that equation (5) can make an approximate prediction. The fact that the magnitude of M/Me is small in the upstream region comes from neglect of the streamwise pressure gradient and the right hand side of equation (4).

4. Conclusions

The mean flow properties were measured for a three-dimensional jet issuing from a cruciform nozzle with a sufficiently large value of aspect ratio. From the results the following are revealed:

(1) The mean velocity profile shows the existence of a velocity defect region, namely a saddle-back shape, on both the $y = 0$ slice and the $z = 0$ slice in the upstream region. It is confirmed that the entire mean velocity profile still maintains a crisscross shape up to the downstream cross section of $x/d = 150$.

(2) The velocity scale U_{0x} decays as $(x/d)^{-0.46}$ in the region beyond $x/d = 75$. The rate of the decay is smaller than those of the two-dimensional and the axi-symmetric jets. For the length scales, b' increases as $(x/d)^{0.91}$ and l' as $(x/d)^{0.99}$ on the $y = z$ slice, while l increases as $(x/d)^{0.91}$ on the $z = 0$ slice. Therefore, the development of the length scales is slower than those of the two-dimensional and the axi-symmetric jets.

(3) The similarity of mean velocity profile on the representative slices is valid on the $y = z$ slice in the region beyond $x/d = 50$, and on the $z = 0$ slice in the region beyond $x/d = 75$. From the fact that the ratio of length scale for the representative slices is constant in the region beyond $x/d = 75$, it can be expected that the similarity of the flow field will be valid in the whole cross section. The empirical formula (2) is obtained with the typical profile of mean velocity in this similarity region.

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Fig.12 Momentum budget for the $y=z$ slice

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