Study on Hydrodynamic Performance of Darrieus-type Cross-flow Water Turbine*

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For the purpose of utilization of extremely low-head water power, a small model of Darrieus-type cross-flow water turbine was produced and examined for the effects of channel height and number of blades on turbine performance. Then, time variation in generated torque in a revolution of runner was investigated experimentally and compared with the results of theoretical calculation. Performance tests showed that the best efficiency was obtained by double-blade runner under the narrowest channel height condition. Time variations in measured torque showed strong dissimilarity in their wave pattern and quantitative disagreement in their peak values to the theoretical results obtained under the assumption of a uniform steady relative flow. Theoretical analysis suggests that this difference arises from disregard for the effects of circular motion on a flow around the blade and the unsteady character of flow on a Darrieus blade in the above simple calculation.

Key Words : Hydraulic Machine, Water Turbine, Cross-flow, Darrieus Runner, High Specific Speed, Hydrodynamic Performance, Rotating Blade, Unsteady Flow

Introduction

The Darrieus rotor developed for a wind mill is a high-speed type which might be suitable to a water turbine for utilization of extremely low-head hydraulic power. The horizontal shaft tubular-type water turbine has been widely used for specific speeds lower than 900 [rpm, kW, m], although few special ones operated under considerably lower heads such as 1.28m⁽¹⁾ can be found. Therefore, development of a low-cost type turbine suitable for extremely low-head operation would be worth for utilization of extremely low-head natural water power or recovery of wasted low-head hydraulic power in industry, because the turbine efficiency lowers and the capital cost per unit generated power rises as the available head decreases.

The Darrieus-type cross-flow runner seems to be fit for this purpose because of its simple shape and structure. In the case of water turbine, the amount of extracted power can be increased by use of a runner casing which sustains a static pressure difference, in other words, a water level difference between up- and downstream sections of the runner, in contrast to the case of a wind mill where only a part of kinetic energy of the fluid

can be extracted. However, a study on the turbine performance of a Darrieus-type runner covered with a casing has never been tried.

In view of the circumstances above mentioned, model tests on the turbine performance of a Darrieus-type cross-flow runner placed in a straight water channel of rectangular cross-section was carried out for various values of channel height and blade number. Besides, the time variation in generated torque in a revolution of runner was investigated experimentally and compared with the results of theoretical calculation.

Experimental Apparatus

A horizontal-axis, two-dimensional Darrieus runner was adopted in order to facilitate precise finishing and setting of the blades and comparison of experimental results with theoretical values, whereas a ϕ -shape rotor is usually used for a wind mill for reasons of stress reduction. The runner is placed in a horizontal closed channel of rectangular cross-section as shown in Fig.1. The turbine ① is attached to the end of an open channel of 1.2m in depth and the flow-rate Q is measured by an orifice-meter fitted to the delivery pipe of the water supplying pump. The runner shaft is coupled with a flywheel ③ through a torque pick-up ② to drive a DC motor-generator ④ through a timing belt engaging with the flywheel. The amount of available head imposed on the runner is adjusted by manual control of a butterfly valve ⑤ in the draft tube, and evaluated by subtracting the head loss which arises between

* Received 30th March, 1983.

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retical values estimated by this method will be indicated by replacing the subscript th with i.

(2) The amount of circulation around the Darrieus blade always varies with time and its time variation corresponds to the sum of vortices shed at that moment from the blade as a vortex sheet. Induced velocities by the vortex sheet and the vortices bounded around the other blades should have influence on the flow at point A, in addition to the flow interference with the channel wall. Therefore, the relative velocity W and the angle of attack α at point A are corrected with consideration of their effects⁽²⁾, and then C_l and C_d are evaluated by the same method as the previous one. Values calculated by this method are indicated by replacing the subscript th with v.

Aerofoil characteristics for the profile NACA0030 were obtained by converting the wind tunnel data⁽³⁾ for blade aspect ratio of 6 into values for infinite blade length, referring to the data^{(4),(5)} on symmetric aerofoils thinner than NACA0030.

Theoretical performances of a double-blade runner by the above methods are shown in Fig.4. It is seen that the vortex sheet shed from the blade decreases the power output, although this effect diminishes as the channel height is reduced. But, both the best efficiency and the value of $U/\bar{V}$ where the efficiency gives the maximum value are insensitive to the method employed. The maximum value of ef-

Fig.4 Theoretical blade efficiency and power coefficient

Fig.5 Theoretical performance of double-blade runner

iciency obtained is 0.646 for $U/\bar{V}=3.77$ by the method (1).

The overall performance of a double-blade runner by the method (1) is shown in a contour map, Fig.5 for comparison with experimental results which will be presented in the following. The theoretical head $\bar{H}_{th}$ is defined by the equation

$$\bar{H}_{th} = \bar{T}_{th} \omega / (\rho g B S \bar{V} \bar{n}_{th}) \quad (7)$$

where g is the gravitational acceleration and ω is the angular velocity of the runner.

Outlet Flow from Runner

In Fig.6, distributions of velocity components parallel to the channel axis at the runner outlet V_{a2} and drops in total head through the runner ΔH are shown for dimensionless channel height $S/D=1.33$. The figure shows that the water is pushed aside more and more by the rotating blade as the number of blades increases, and a considerable amount of water leaks through the clearance between the channel wall and the surface of revolution by the blades without contributing to the power generation by the runner, while a uniform distribution is observed at a state without runner excepting in the wake of the runner shaft. Theoretical distributions of V_{a2} calculated by the method (2) are shown by dotted lines and they agree well with the experimental results. Fig.7 shows the effect of channel height on the flow distribution in the downstream section. The leak water passing through the clearance without consumption of total head disappears in small clearance conditions.

Fig.6 Effects of blade number on outlet flow from runner

Fig.7 Effects of channel height on outlet flow from runner

Experimental Results on Turbine Performance

Fig.8 consisting of four figures shows a typical example of experimental results on runner efficiency, (a), (b) and (c) showing the effect of channel height and (c) and (d) the effect of blade number. The efficiency $\bar{\eta}$ shown here is defined by

$$\bar{\eta} = \bar{T}\omega / (\rho g Q H) \quad (8)$$

Fig.9 shows examples of available head and shaft torque characteristics for double- and quadruple-blade runners. The double-dot-dash lines in Fig.8 and Fig.9 represent the inception limit of cavitation obtained by visual observation. A sharp drop in efficiency takes place in the velocity range higher than this limit due to the growth of cavity.

Comparing these results with the theoretical performance shown in Fig.5, it is seen that they generally show a similar tendency except in the regions affected by cavitation, and iso-head and iso-torque lines have a fairly good similarity in their shapes. The best efficiency point obtained in the experiment is marked by the symbol * in Fig.8 and Fig.9. It is in the velocity range lower than the inception limit of cavitation so that it is guaranteed that data measured near the best efficiency point are free from cavitation. The maximum values of efficiency measured are 0.53, 0.67, 0.69 and 0.57 for each figure in Fig.8 in the order of (a) to (d), and the speed ratios U/V at these points are 3.87, 3.88, 3.86 and 3.00 respectively. In Fig.8, each iso-efficiency contour has two nearly straight parts in the upper and lower sides of the dot-dash line and their extension points at the origin. A ridge line formed between upper and lower parts agrees with a dot-dash line which connects the best efficiency point to the origin and, this line is very close to the theoretical best efficiency line in Fig.5 for the double-blade runner so that it may be concluded that the ex-

Fig.8 Efficiency characteristics of tested runners

Fig.9 Head and torque characteristics of tested runners

perimental results are in fairly good agreement with the theoretical results on their principal character.

From the measured maximum efficiency, it is clearly known that a reduction of clearance between the channel wall and the surface of revolution by the blades is effective to improve the turbine efficiency. But, it is still doubtful that the above conclusion is applicable up to the dimensionless channel height less than 0.95, because this tendency is not valid for a quadruple-blade runner with dimensionless channel height $S/D = 0.95$.

Specific speeds at the best efficiency point of the double-blade runner are 1317, 1071 and 1017 [rpm, kw, m] for channel heights $S/D = 1.33$, 1.05 and 0.95 respectively.

With regard to the effects of blade number, the data show that the quadruple-blade runner is inferior to the double-blade runner in efficiency. This is the same result as that of the theoretical calculation by the method (2) and it means that the Darrieus runner is a high-speed type runner. Comparison of both figures in Fig.9 shows that the dot-dash line drawn by connecting the points where the maximum torque is obtained on each iso-head line shifts to the low speed-ratio side as the blade number is increased. This may be due to the following circumstances. As the blade number is increased, the distortion of flow distribution along the channel height is magnified, and the actual oncoming flow velocity concerned in torque generation lowers and, in consequence, the apparent flow velocity $\bar{V}$ for the maximum-torque point increases.

Fig.10 illustrates the variation in power coefficient and efficiency with speed ratio for the best efficiency flow-rate of double-blade runner, comparing with the theoretical results shown in Fig. 4. Herein, $\bar{C}_p$ is the averaged value of measured power coefficient in the same definition as C_{pth} .

Now, if the runner efficiency is cal-

Fig.10 Comparison of experimental results with calculated performances

culated taking the decrement of hydraulic power flux between the up- and down-stream sections of the runner, each of which is obtained by integrating the flow distribution measured by yawmeter traversing, as the input power supplied to the runner instead of $\rho g Q H$, maximum values of efficiency higher by 30, 8 and 4% than those given by the performance test are obtained for double-blade runners with $S/D = 1.33$, 1.05 and 0.95 respectively though certain problems in accuracy still remain about the flow value measured by a yawmeter because it is an indefinite time average of a periodic flow. Subtracting the efficiency drop corresponding to the loss of kinetic energy necessary for equalizing the distorted velocity distribution at the runner outlet from the above efficiency gap, remainders the balances are still 22, 5 and 2% in a decreasing order of S/D . These values amount to 20~30% of the increment of velocity head near the channel wall by flow distortion and, therefore, it is likely that this gap is caused by an increase in power loss due to the wall friction. Anyhow, these results reveal that the distinct drop in efficiency under the channel height $S/D = 1.33$ is not attributed to the drop in blade efficiency but to the increase in energy consumption in the downstream region of the runner due to a magnified flow distortion.

The measured power coefficient has a value larger by 20% than that calculated by the method (2) at the best efficiency speed ratio for both of $S/D = 1.33$ and 1.05.

Measurement of Time Variation in Torque

The flow passing through the Darrieus blade is time-dependent so that the shaft torque varies with time with the same period as the rotation of runner. Therefore, investigations on the time variation in torque are necessary for further improvement of turbine performance and a reliable strength design of the blade. The instantaneous torque generated by the blades was obtained by adding the mechanical resistance torque exerted on the runner shaft to the reading of the torque pickup. Since the shaft, runner, torque pickup, couplings and flywheel compose a vibration system, a torsional vibration may be induced by resonance with the periodic variation of the driving torque and lead to a noticeable error in the measured torque. The natural frequency of the system was only 40Hz, though the side disks of the runner were made of aluminium, and the measured torque was influenced by resonance. Therefore, the measured value was corrected by expanding the torque curve in a Fourier series and dropping the harmonics above 30Hz, after confirming the fact by spectrum analysis that the torque curve almost consists of the fundamental component and its harmonics.

The variation in measured torque in a revolution of single-blade runner is presented in Fig.11 for different channel height conditions and in Fig.12 for differ-

ent speed ratios, in comparison with the theoretical results calculated by the methods described before. The following two facts are obvious, comparing the measured data with the theoretical results: (1) The measured torque has a much larger value than the theoretical values in the upstream half of the circular motion of the blade. (2) The torque measured in the downstream half of the blade motion has a much less value than that obtained in the upstream course. In addition to the effects of the wake behind the shaft, the flow distortion due to the blade rotation mentioned before may be one of the reasons for the decrease in torque generation in the downstream course. This inference is supported by the fact that the torque generated in the downstream course increases as the channel height is reduced as shown in Fig.11. This increase in generated torque is cancelled considerably by the increase in resistant torque due to the effects of channel wall. Fig.12 shows that, as the speed ratio is increased, the difference in generated torque between the up- and downstream courses of the blade motion increases and the angular position where the blade gives the maximum torque shifts in the direction of increasing θ .

Consideration on Fluid Force Acting on Darrieus Blade

The following inference was deduced from the fact that a considerable difference was observed between the measured and the calculated torque in their magnitude and character of time variation as shown in Figs.11 and 12. It might be inap-

Fig.11 Torque variation of single-blade runner at different channel heights

Fig.12 Torque variation of single-blade runner at different speed ratios

propriate to assume that the magnitude and the direction of relative velocity were approximately identical on a blade section and equal to the values of W and α at point A, and the fact that the magnitude and the direction of peripheral velocity are different at each point on a blade section rotating along a circular path, if the blade dimension is finite, should be considered. Then, a flow analysis was carried out with a potential flow around a rotating blade with boundary conditions that local relative velocities are tangent to the blade surface at every reference point, together with the same assumption as used in the previous calculation that the flow is steady at every instant; and the results were compared with the results obtained by the previous method.

In the numerical procedure of flow analysis, vortices given by the modified Glauert series are distributed on a line drawn in equi-distance from the blade surface and the camber line, and the relative velocity on the blade surface is calculated as a resultant vector of the velocity induced by these vortices, the uniform absolute velocity $\bar{V}$ and the local peripheral velocity of the rotating motion, and then the pressure distribution along the blade surface is computed. Thereafter the blade torque is calculated by integrating the circumferential component of a pressure force acting on the blade surface. In this calculation, the effects of free-vortices shed from the blade are not taken into consideration.

Time variations in torque coefficient in a revolution of a blade obtained by this calculation are presented in Fig.13. Herein, the value in parenthesis () is a value in which the effects of lack of uniformity of relative velocity on the blade section are taken into account. C_{t10} shown by a dotted line is the theoretical torque coefficient calculated by the method (1) with the lift coefficient for a blade in a uniform inviscid flow. The peak values of C_{t10} are equal to each other in the up- and downstream course of the blade similarly to the case of C_{t1} , while (C_{t1}) has a value higher than C_{t10} in the upstream course and a value lower than C_{t10} in the downstream course regardless of the speed ratio and the channel height, similarly to the experimental re-

Fig.13 Torque variation of Darrieus blade by potential-flow analysis

sults.

From the above results, we may conclude that the disagreement between the measured and the calculated torque is caused by the difference between the flow field around a blade in a rectilinear motion and the flow field around a blade in a circular motion. However, as aerofoil data by wind tunnel tests are all for a blade in a rectilinear motion with constant speed, it is necessary to introduce a method which relates the wind tunnel data to the aerofoil characteristics of a blade in a circular motion in order to predict the torque performance of a Darrieus rotor.

The difference between both flows may be decomposed into the following three effects; the shear flow effect arising from the fact that the relative flow on a blade in a circular motion results in a shear flow in the direction of blade thickness; the camber effect owing to the circumstances that the camber line of the blade is out of the pitch circle; and the incidence effect which takes place when the camber line touches the pitch circle at a point other than the mid-chord. Neglecting the shear flow effect because the value of $2l/D$ is small enough, the remaining two effects are evaluated quantitatively in the form of the increment of circulation around the blade.

It is reasonable to relate the aerofoil characteristics of a blade in a circular motion to those in a rectilinear motion with W and α at the point of quarter chord of the blade in a circular motion, because the center of pressure is near the quarter chord in most cases. Circulations around a blade in contact with the pitch circle at quarter chord shown by solid line in Fig.14(a) are represented by Γ and Γ_0 for the blades in a circular motion and a rectilinear motion respectively. Also the circulation around a blade in contact with the pitch circle at mid-chord and in circular motion is denoted by Γ' as shown by a broken line. Since the deviation of camber line from the pitch circle is symmetrical chordwise, in the last condition, we assume that the camber effect only exists in this case and the increment of circulation due to the camber effect is defined by $\Delta\Gamma_c = \Gamma' - \Gamma_0$. $\Delta\Gamma_\alpha = \Gamma - \Gamma'$, that is the increment of circulation by shifting the contact point from mid-chord to quarter chord, is defined as the incidence

Fig.14 Illustration of camber effect and incidence effect

effect. The reason for this name may be understood by recalling that the peripheral velocity at the mid-chord is in the same direction as the chord line for the blade illustrated by a solid line, while it makes a certain angle to the chord line for the blade illustrated by a broken line in Fig.14(a). Then, the circulation around a blade in circular motion Γ is decomposed into three parts as $\Gamma = \Gamma_0 + \Delta\Gamma_c + \Delta\Gamma_\alpha$. The results of potential flow analysis for channel height $S/D = \infty$ indicate that $\Delta\Gamma_c$ and $\Delta\Gamma_\alpha$ are almost constant independently of angular position θ and blade setting angle δ as shown in Fig.14(b), where δ denotes the angle between the chord line and the tangent of the pitch circle drawn at the contact point. The increments of lift coefficient and angle of attack, ΔC_l and $\Delta\alpha$, corresponding to the increments of circulation due to the camber effect and the incidence effect $\Delta\Gamma_c$ and $\Delta\Gamma_\alpha$ can be expressed as follows

$$\Delta C_l = 2\Delta\Gamma_c / lW \quad (9)$$

$$\Delta\alpha = 2\Delta\Gamma_\alpha / \{lW(dC_l/d\alpha)\} \quad (10)$$

where $(dC_l/d\alpha)$ is the slope of the lift coefficient curve calculated for a blade in a rectilinear motion.

Fig.15 is a comparison of theoretical distributions of pressure coefficients $k = (p - p_\infty) / (\rho W^2 / 2)$ on the blade surface at angular position $\theta = \pi/2$ by various methods of calculation for an inviscid flow of channel height $S/D = \infty$. k^* is a value derived straightforward by taking the effects of circular motion into account, or a value derived by the calculation shown by a solid line in Fig.13, which should be regarded as the basis for the comparison of each method. k_0 is a value for a blade immersed in a uniform flow of relative velocity W and angle of attack α , or a value derived by the calculation shown by a dotted line in Fig.13. k_0^* is a value of k_0 for a blade with correction ΔC_l in camber and correction $\Delta\alpha$ in angle of attack which is obtained by the method proposed above. k_{0c} is a value of k_0 for a blade with a camber given by the assumption that the effects of circular

Fig.15 Comparison of pressure distributions on Darrieus blade by various methods of calculation

motion can wholly be represented by a correction in camber. The definition of $k_{0\alpha}$ is the same as that for k_{0c} if the word "camber" is replaced by "angle of attack". A close agreement between k^* and k_0^* proves the validity of the proposed method for evaluation of a force acting on a blade in circular motion.

Subsequently, this method is applied to the calculation of aerofoil characteristics of a real blade. The following relation is used for evaluation of the torque coefficient generated by the real blade.

$$(C_{tv})_s = (\ell/2D)(W/V)^2 [C_{\ell}(\alpha+\Delta\alpha) + m\Delta C_{\ell}] \times \sin\alpha - C_d(\alpha+\Delta\alpha) \cos\alpha \quad (11)$$

where m denotes the ratio of $(dC_{\ell}/d\alpha)$ in unstalled region obtained by wind tunnel tests to that obtained by potential flow analysis, and the value in parenthesis $()_s$ is a value calculated by the above method. The time variation in the torque coefficient for the single-blade runner by the above method is shown by a dot-dash line in Fig.16. The difference between the experimental and calculated results is still large, although the value of $(C_{tv})_s$ is higher in the upstream course than in the downstream course.

Performance Predicted with Consideration on Dynamic Stall

Effects of an unsteady flow on the aerodynamic characteristics of a blade have not been taken into account thus far. But, it seems to be possible that a fairly large difference between the experimental and calculated results shown above is caused by this effect. Accordingly, the effect of an unsteady flow on the blade force of a Darrieus runner is investigated by applying the semi-empirical theory proposed by Kimura^{(6),(7)}. In its application, the equivalent angle of attack α_e only is taken into consideration, and the values of α_e and $\Delta\theta$, the delay in angular position where the dynamic stall arises, are calculated by the following equations

$$\alpha_e = (\alpha+\Delta\alpha) + (\omega\ell/4W)\{d(\alpha+\Delta\alpha)/d\theta\} \quad (12)$$

$$\Delta\theta = 1.6 \sqrt{\omega\ell/2W} \quad (13)$$

It is also assumed that, after the dynamic stall has arisen, the drag coefficient $C_{de}(\theta)$ approaches $C_{dd}(\theta)$ which is given by

the equation

$$C_{dd}(\theta) = C_d[\alpha_e(\theta-\Delta\theta)] \quad (14)$$

exponentially conforming to the relation

$$dC_{de}/d\theta = (C_{dd} - C_{de})/\Delta\theta. \quad (15)$$

Values obtained by this method are marked by $()_d$.

According to the result of consideration on dynamic stall, the drag coefficient describes a large hysteresis curve in a revolution of the blade as shown in Fig. 17. The turbine performance obtained by this calculation is illustrated in Fig.18 comparing the experimental result with the results of calculation by the previous methods. Fig.16 and Fig.18 show that the consideration on dynamic stall yields the values of torque coefficient, power coefficient and efficiency passably close to the experimental results. But, the agreement is still insufficient and further investigations on an unsteady flow around the blade of Darrieus runner will be necessary to establish a reliable method for prediction of turbine performances and blade stresses.

Fig.17 Periodic variation in lift and drag coefficients of Darrieus blade

Fig.16 Comparison of time variations in torque by various methods of calculation

Fig.18 Comparison of turbine performance calculated with consideration of dynamic stall with experimental results

Conclusions

For the purpose of utilization of extremely low-head hydropower, a small Darrieus-type cross-flow water turbine was produced and examined; besides a theoretical method for performance prediction was investigated. The results are summarized as follows :

(1) Since the Darrieus-type runner is essentially a high speed type runner, a higher blade efficiency is obtained in lower head conditions. Therefore, it is recommendable to place the runner in an open channel or a loosely closed channel when the blade efficiency is a matter of concern. However, the best turbine efficiency was obtained by placing the runner in a tightly closed channel, because the additional losses due to the occurrence of flow distortion induced by the runner decreased with the reduction of clearance between the channel wall and the surface of revolution by the blades.

(2) The common features of a high speed runner were recognized in the experimental results, namely that the double-blade runner provides a higher efficiency than the quadruple-blade runner in every examined channel height.

(3) The time variation in blade torque in a revolution of the runner, calculated under the conventional assumption that a uniform relative flow is incoming to the blade at every instant, did not provide a reasonable prediction of measured torque. It was found that this disagreement across from disregarding of the difference between flow fields around a blade in a rectilinear motion and one in a circular motion, and then a method to correct these effects by modification of camber and angle of attack was proposed.

(4) Considerations on dynamic stall yielded calculated results passably close to the experimental results. But, the agreement is still not satisfactory and further investigations on an unsteady flow around the blade of Darrieus runner are needed for reliable performance- or strength-design.

This research was supported financially by the Grant-in-Aid for Energy Research from the Ministry of Education, Science and Culture, Japan. The authors are indebted to Mr. N. Sakai and Mr. A. Sato for collaboration in fabricating the experimental apparatus and for assistance in experimental works.

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