On the Static Running Accuracy of Ball Bearings*

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The quasi-static vibratory analysis of a rotor supported by ball bearings is presented. The system is an ideal ball bearing in which the inner ring moves in the radial plane with two degrees of freedom under a constant radial load. The motion of the inner ring centre due to ball revolution, which has been known as the Perret-Meldau problem since the 1950s, is analyzed in detail. The results show that the inner ring motion has complicated features and changes drastically with the design and operating conditions. All computed results are arranged in a numerical table and in charts. They represent a complete package for the estimation of the magnitudes and characteristics of the inner ring motion, and may be used in the design process to examine the rigidity, critical speed and vibratory nature of a rotor.

Key Words: Mechanism, Dynamics of Machinery, Machine Element, Vibration, Vibration of Rotating Body, Bearing, Ball Bearing, Running Accuracy.

1. Introduction

In a ball bearing in which the balls are arranged discretely, excitation is generated by ball passage and a rotor supported by ball bearings makes a radial vibration. This is common knowledge⁽¹⁾ nowadays, but if we desire more details about this problem, we can mention only some numerical examples, namely, the papers written by Perret or Meldau and other authors. It is difficult to understand the relation between the predominant factors and the magnitudes of vibrations. The analysis of these items is not yet adequate and the procedure to obtain a solution to this problem is not complete.^{(1),(2)}

In 1950 Perret^{(3),(4)} analyzed a model which was composed of an ideal ball bearing of which the outer ring was stationary, the inner ring was imposed with a constant radial load and the balls rotated slowly. Perret calculated the inner ring motion in the radial plane. The vibration in this model is assumed to be due to ball passage with the inertia and frictional forces neglected.⁽²⁾ Therefore, we refer to this problem as a static running accuracy problem.

Meldau⁽⁵⁾⁻⁽⁸⁾ also analyzed this problem in detail and published many papers. The results are interesting and we refer^{(1),(2)} frequently to his results.⁽⁶⁾ Other authors, e.g. Tamura⁽⁹⁾⁻⁽¹¹⁾ et al., have written papers on this problem. The problem is completely fit for analysis using a powerful weapon, the electronic computer. This is essentially a complicated

nonlinear problem and the method of solution must be a trial method. It is not sufficient to have the results of some numerical examples,^{(6),(10)} but we wish to work out a method to obtain promptly a solution for arbitrary combinations of the conditions. Examples represented by Meldau, Tamura and others may not either sufficiently cover most of the cases which occur.

The results which express concisely and precisely the relation between the predominant factors and the resulting phenomena, are interesting and important. It is necessary to present these relations for the discussion of the vibration problem of a rotor supported by ball bearings.

This paper has examined and discussed the problem carefully and completely, and established a procedure to calculate a solution to the problem. If we have a pocket calculator, by using the charts and numerical tables contained in this paper it will be easy to get an approximate solution to the problem.

2. Fundamental Equations^{(9),(10)}

Nomenclature (see Figs. 1,2)

C_s : elasticity factor of the ball bearing⁽¹⁾

N : total number of balls

W : constant radial load applied to the inner ring

X, Y : $-x, -y$ components of the spring forces acting on the inner ring (restoring forces)

x, y : co-ordinates in the radial plane, also the components of the inner ring motion

Γ : nondimensional characteristic parameter
 $= \gamma_0 (NC_s/W)^{2/3}$

Γ_c : critical condition of two balls loaded (when $\Gamma_c < \Gamma$, only two balls loaded)
 $= N^{2/3} \{ \cot^2(\pi/N) - 1 \} / 2$

$2\gamma_0$: mean radial clearance

θ_j : the revolution angle of the ball "j"
 $= \theta_1 + (2\pi/N)(j-1) \quad (j=1, 2, \dots, N)$

$[]_+ = \max([], 0)$

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C_s is a constant which can be calculated using the geometrical dimensions and the material of the ball bearing components (in the papers^{(9),(10)}, $\bar{P}_n$ and $\bar{d}_n$ are used and the relation between them and C_s is, $C_s = \bar{P}_n / \bar{d}_n^{1.5}$), and the procedure to get C_s is shown in Appendices 1 and 4.

When the inner ring motions x and y , and the revolution angle θ_i are known, the restoring forces X and Y are given by

$$\begin{bmatrix} X \\ Y \end{bmatrix} = C_s \sum_{j=1}^N (x \cos \theta_j + y \sin \theta_j - \gamma_0)^{1.5} \begin{bmatrix} \cos \theta_j \\ \sin \theta_j \end{bmatrix} \quad (1)$$

and the incremental stiffnesses are

$$\begin{bmatrix} \partial X / \partial x \\ \partial Y / \partial y \\ \partial X / \partial y \end{bmatrix} = \frac{3}{2} C_s \sum_{j=1}^N (x \cos \theta_j + y \sin \theta_j - \gamma_0)^{0.5} \begin{bmatrix} \cos^2 \theta_j \\ \sin^2 \theta_j \\ \cos \theta_j \sin \theta_j \end{bmatrix}, \quad \text{where } \frac{\partial Y}{\partial x} = \frac{\partial X}{\partial y} \quad (2)$$

The static runing accuracy problem is to solve the equation (1) under the conditions that $X = W (> 0, \text{ constant})$, $Y = 0$ to get $x, y = f(\theta_i; W, \gamma_0, N, C_s)$.

3. A Case of Two Balls Loaded, Analytical Exact Solutions, $\Gamma_c < \Gamma$

When two balls are loaded, the solutions are expressed as follows:^{(10),(11)}

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{W}{NC_s} \right)^{2/3} \left[\frac{\Gamma}{\sin(2\pi/N)} \begin{bmatrix} \sin \bar{\theta}_2 + \sin \theta_1 \\ \cos \bar{\theta}_2 - \cos \theta_1 \end{bmatrix} + \frac{N^{2/3}}{\sin^{5/3}(2\pi/N)} \begin{bmatrix} \sin \bar{\theta}_2 & \sin \theta_1 \\ \cos \bar{\theta}_2 & -\cos \theta_1 \end{bmatrix} \begin{bmatrix} \sin^{2/3} \bar{\theta}_2 \\ \sin^{2/3} \theta_1 \end{bmatrix} \right] \quad (3)$$

$$\begin{bmatrix} \partial X / \partial x \\ \partial Y / \partial y \\ \partial X / \partial y \end{bmatrix} = \frac{\{W(NC_s)^2\}^{1/3}}{N^{2/3} \sin^{1/3}(2\pi/N)} \frac{3}{2} \begin{bmatrix} \cos^2 \theta_1 \cos^2 \bar{\theta}_2 (\sin^{1/3} \theta_1 / \cos^2 \theta_1 + \sin^{1/3} \bar{\theta}_2 / \cos^2 \bar{\theta}_2) \\ \sin^{1/3} \theta_1 \sin^{1/3} \bar{\theta}_2 (\sin^{5/3} \theta_1 + \sin^{5/3} \bar{\theta}_2) \\ \sin^{1/3} \theta_1 \sin^{1/3} \bar{\theta}_2 (\cos \theta_1 \sin^{2/3} \bar{\theta}_2 - \cos \bar{\theta}_2 \sin^{2/3} \theta_1) \end{bmatrix} \quad (4)$$

$$\text{where } \theta_1 = 0 \sim 2\pi/N \text{ (see Fig. 2), } \bar{\theta}_2 = 2\pi/N - \theta_1 = 2\pi/N - \theta_1 \quad (5)$$

x , $\partial X / \partial x$, $\partial Y / \partial y$ are even functions with respect to the independent variable $\theta_1 - \pi/N$, and y , $\partial X / \partial y$ are odd functions. $\partial X / \partial x$, $\partial Y / \partial y$ are maximum when $\theta_1 = \bar{\theta}_2 = \pi/N$ and minimum when $\theta_1 = 0$ or $\bar{\theta}_2 = 0$. The minimum value of $\partial Y / \partial y$ is zero. The values of $\partial X / \partial x$, $\partial Y / \partial y$ and Γ_c , for the cases of $N = 6$ to 14 are given in Table 1.

4. Mean Spring Properties

The solutions in a general form to equation (1) and the corresponding equation (2) are as follows:⁽⁹⁾ [this is a general form of equations (3),(4)]

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{W}{NC_s} \right)^{2/3} f(\theta_i; \Gamma, N) \quad (6)$$

$$\begin{bmatrix} \partial X / \partial x \\ \partial Y / \partial y \\ \partial X / \partial y \end{bmatrix} = \{W(NC_s)^2\}^{1/3} f(\theta_i; \Gamma, N) \quad (7)$$

$f(\theta_i; \Gamma, N)$ is a nondimensional function and when more than two balls are loaded, we have to use a numerical method to get the solution. There are many algorithms to get the numerical solution, especially the approximation using the Newton-Raphson method; it can be noticed that the coefficients of the simultaneous linear equations to be solved coincide directly with the incremental stiffnesses. The general trial method is not so good in convergence for the case of two balls loaded and we have to use the expression shown in Chapter 3, but this is applicable only to the case of ideal ball bearings.

Generally speaking, the fluctuating parts of x and y are small compared with the mean value of x , while the mean value of y is zero. The fluctuating parts of $\partial X / \partial x$ and $\partial Y / \partial y$ are smaller than their mean values, and the mean value of $\partial X / \partial y$ is zero. The mean values of x , $\partial X / \partial x$ and $\partial Y / \partial y$ are neces-

Table 1. Some numerical values concerning the state of only two balls loaded.

N	6	7	8	9	10	11	12	13	14
Γ_c	3.30	6.06	9.66	14.17	19.66	26.21	33.88	42.74	52.85
$\partial X / \partial x$ min	0.454	0.410	0.375	0.347	0.323	0.303	0.286	0.271	0.258
$\partial Y / \partial y$ max	0.567	0.547	0.522	0.496	0.472	0.449	0.429	0.410	0.393
$\partial X / \partial y$	0.536	0.510	0.484	0.458	0.435	0.413	0.394	0.376	0.360
$\partial X / \partial x$ max	0.189	0.127	0.0895	0.0657	0.0498	0.0387	0.0308	0.0249	0.0205
$1/\Phi_1(\Gamma_c)$	0.153	0.105	0.0744	0.0551	0.0417	0.0324	0.0258	0.0209	0.0172

sary for the estimation of the mean spring constants and natural frequencies in the x and y directions.

These approximate values are given as follows:⁽⁹⁾

$$x = x_0 = (W / NC_s)^{2/3} \Phi_1(\Gamma) \quad (8)$$

$$\begin{bmatrix} \partial X / \partial x \\ \partial Y / \partial y \end{bmatrix}_{x=x_0} = \{W(NC_s)^2\}^{1/3} \begin{bmatrix} \Phi_2(\bar{\Gamma}) \\ 1/\Phi_1(\bar{\Gamma}) \end{bmatrix} \quad (9)$$

$$\text{where } \bar{\Gamma} = \min(\Gamma, \Gamma_c)$$

Here $\Phi_1(\Gamma)$ is a solution to the equation

$$\frac{1}{2\pi} \int_0^{2\pi} (\Phi_1 \cos \theta - \Gamma)^{1.5} \cos \theta d\theta = 1 \quad (10)$$

and $\Phi_2(\Gamma)$ can be obtained, by using the above $\Phi_1(\Gamma)$, from the equation

Fig. 1. Geometrical dimensions of the ball bearing.

Fig. 2. A system of co-ordinates x, y, θ_i .

Fig. 3. Inner ring motion in the radial plane, vertical direction $\xi(\theta_1)$, horizontal direction $\eta(\theta_1)$, ($N=6$).

Fig. 5. The relation between Γ and the Fourier coefficients ξ_0 , ξ_x , ξ_y , p-p values η_{pp} , η_{xx} , η_{yy} , and distortion factors ϵ_x , ϵ_y , of the inner ring motion, ($N=8$).

Fig. 4. The relation between Γ and the Fourier coefficients ξ_0 , ξ_x , ξ_y , p-p values η_{pp} , η_{xx} , η_{yy} , and distortion factors ϵ_x , ϵ_y , of the inner ring motion, ($N=6$, * corresponding to Fig. 3).

$$\Phi_2 = \frac{3}{4\pi} \int_0^{2\pi} (\Phi_1 \cos \theta - \Gamma)^{2.5} \cos^2 \theta d\theta \quad \dots (11)$$

The values of $\Phi_1(\Gamma)$ and $\Phi_2(\Gamma)$ are given in Table 3 in Appendix 5, and the values of $1/\Phi_1(\Gamma_c)$ and $\Phi_2(\Gamma_c)$ which are functions of N , are given in Table 1.

5. Characteristic Values of the Running Accuracy

x and y in equation (6) can be expressed

as follows:

$$\begin{cases} \begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{W}{NC_s} \right)^{2/3} \begin{bmatrix} \xi \\ \eta \end{bmatrix}, \text{ and,} \\ \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \xi_0 \\ 0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} \xi_k \cos kN\theta_1 \\ \eta_k \sin kN\theta_1 \end{bmatrix} \end{cases} \quad \dots (12)$$

where ξ_0 , ξ_k , $\eta_k = f(\Gamma, N)$

ξ and η are nondimensional functions. The computed values of ξ and η as functions of θ_1 are shown in Fig. 3 for the case of $N=6$. In the figure, the magnification factors

Fig. 7. The relation between Γ and the Fourier coefficients ξ_0 , ξ_k , η_k , p-p values ξ_{pp} , η_{pp} , and distortion factors ϵ_x , ϵ_y , of the inner ring motion, ($N=10$, *corresponding to Meldau's results).

of ξ and η coincide, but when Γ is different, the magnification factor is also different. Generally, the greater value of the p-p of ξ or η for each Γ is given inside a frame. The curve without the frame is a Lissajous figure of ξ and η , where the p-p spans of ξ and η figures are equal. The effect of Γ is very complicated and also the effect of N is important.

The characteristic p-p amplitudes ξ_{pp} , η_{pp} and the distortion factors ϵ_x , ϵ_y are defined as follows:

$$\left[\begin{matrix} \xi_{pp} \\ \eta_{pp} \end{matrix} \right] = \max_{\theta_1} \left[\begin{matrix} \xi \\ \eta \end{matrix} \right] - \min_{\theta_1} \left[\begin{matrix} \xi \\ \eta \end{matrix} \right] \quad \dots \dots \dots (13)$$

$$\left[\begin{matrix} \epsilon_x \\ \epsilon_y \end{matrix} \right] = \left[\frac{\sqrt{1 - \xi^2 / \sum_k \xi_k^2}}{\sqrt{1 - \eta^2 / \sum_k \eta_k^2}} \right] \times 100\% \quad \dots \dots \dots (14)$$

The Fourier coefficients ξ_0 , ξ_k and η_k , the p-p values ξ_{pp} and η_{pp} , and the distortion factors ϵ_x and ϵ_y as functions of Γ are shown in Fig. 4, where the asterisk * corresponds to the results in Fig. 3. We can get an approximate pattern of $x(\theta_1)$ - and $y(\theta_1)$,

and estimate the p-p amplitudes easily by using these figures and equation (12). The cases of number of balls $N=8$ to 10 are shown in Figs. 5 to 7, and the asterisk * in Fig. 7 indicates the conditions for Meldau's examples.

By discussing and examining these figures, we can find the global characteristics and nature of the static running accuracy problem as follows:

(1) The p-p curves have maximum and minimum values and the number of the maximum (or minimum) parts is equal to $(N-3)$. The values of Γ which correspond to the maximum and the minimum points in both ξ_{pp} and η_{pp} curves are different (the maxima and/or the minima of ξ_{pp} and η_{pp} with respect to Γ do not always coincide with those of p-p values of $x(\theta_i)$ and $y(\theta_i)$ with respect to W).

(2) When $\Gamma \rightarrow \infty$, which implies a case of two balls loaded, the pattern of $\xi(\theta_i)$ and $\eta(\theta_i)$ converges to a constant pattern i.e. parallel lines of ξ_k and η_k and constant ε_x and ε_y . The Fourier coefficients have negative sign and $|\xi_k|$ is almost proportional to k^{-2} , which implies a pattern with corners, and also $|\eta_k|$ is almost proportional to k^{-1} , which implies a discontinuous pattern, both with respect to θ_i . These typical patterns $f(\theta_i)$ are represented by the case of $\Gamma=50$ in Fig. 3.

(3) When Γ decreases from a large value, the sign of the Fourier coefficients changes periodically. The speed of changing the sign is almost proportional to k , and the value of the minimum part of the p-p almost corresponds to the zero value of the fundamentals.

(4) For $\Gamma < \Gamma_0$, all ξ_0 , η_{pp} and $|\eta_k|$ increase monotonically with Γ ; ξ_{pp} has only one small value and all $|\xi_k|$ have zero value once and then increase monotonically with Γ .

(5) For $\Gamma < -1.245$, all ξ_{pp} and η_{pp} decrease rapidly and monotonically with a decrease of Γ , and $\xi(\theta_i)$ and $\eta(\theta_i)$ asymptotically approach simple harmonic waves because the distortion factor is rapidly decreased. The critical value of $\Gamma = -1.245$ implies that all the balls, even when $W \neq 0$, are loaded.

(6) The line levels corresponding to k , which connect the peak values of $|\xi_k|$ and $|\eta_k|$ respectively, decrease monotonically with an increase of the order number k .

(7) The patterns of $\xi(\theta_i)$, $\eta(\theta_i)$ [accordingly $x(\theta_i)$, $y(\theta_i)$] are determined only by N and Γ . The magnitudes of x and y are proportional to $(W/NC_r)^{2/3}$. Similarly for the incremental stiffness, the pattern with respect to θ_i is decided by N and Γ , and the magnitude is proportional to the factor $\{W/(NC_r)^2\}^{1/3}$.

6. Conclusion

We have derived a general expression for a solution to the so-called static running accuracy problem of the ball bearing generated by ball passage. An analysis is made of the relation between the predominant factors and the resulting characteristics, namely, Fourier coefficients, p-p amplitudes and distortion factors. The computed results are arranged in charts and tables which can be made generally use of. It is very easy to obtain a solution of the inner ring motion for a given condition by using these results. The procedure is appropriately arranged and a detailed discussion is made.

Appendix

1. Calculation of C_r

Nomenclature (see Fig. 1)

- D_i : the diameter to locus of inner raceway groove curvature centres
- D_o : the diameter to locus of outer raceway groove curvature centres
- d : ball diameter
- E : modulus of elasticity
- r_i : inner raceway groove curvature radius
- r_o : outer raceway groove curvature radius
- ν : Poisson's ratio

The suffixes:

- i : inner ring (together $\cdot = i, o$)
- o : outer ring

$$\left. \begin{aligned} \lambda_i &= 2r_i/d-1 \\ \rho_i &= d/D_i \end{aligned} \right\} \dots\dots\dots (15)$$

$$\phi_i = 1 \mp (1 + \lambda_i) \rho_i \quad \text{where } \left(\begin{array}{l} - : \cdot = i \\ + : \cdot = o \end{array} \right) \dots\dots\dots (16)$$

$$\varphi_i = \left\{ \frac{1 + 2\lambda_i \phi_i}{(1 + \lambda_i) \phi_i} \right\}^{1/3} \delta^* \left(\frac{1}{1 + 2\lambda_i \phi_i} \right) \dots\dots\dots (17)$$

$$C_s = \frac{2E\sqrt{d}}{3(1-\nu^2)} / (\varphi_i + \varphi_o)^{1.5} \dots\dots\dots (18)$$

for the case of steel bearing,
($E=207\,760$ MPa, $\nu=0.3$)

$$C_s = \frac{152\,205\sqrt{d}}{(\varphi_i + \varphi_o)^{1.5}} \left[\frac{N}{\text{mm}^{1.5}} \right] \dots\dots\dots (19)$$

where x , y , r_0 , d are in mm.

$\delta^*(F)$, the Hertz coefficient is generally taken from the numerical table^{(12),(13)}. It is sufficient to use the approximate expressions shown in Appendix 2 or 3.

2. Approximate formula for $\delta^*(F)$

The relative error is 0.01%.

(a) $F=0 \sim 0.5$:

$$\delta^*(F) = (1 - 1.04F^2)^{0.214} \dots\dots\dots (20)$$

(b) $F=0.5 \sim 0.999\,999\,999$:

$$\left. \begin{aligned} G &= 1/(1-F) \\ X_1 &= \ln(16G) - 2 \\ X_2 &= 1/(X_1 + \ln X_1) \end{aligned} \right\} \dots\dots\dots (21)$$

$$X = G / (X_2 X_1^{X_1})^{1/(1-X_2)} \dots\dots\dots (22)$$

$$\delta^*(F) = \frac{0.370\,07 \ln(16X)}{(X - 2.8/X^{3.12} + 0.234)^{1/3}} \dots\dots\dots (23)$$

3. The numerical table of $\delta^*(F)$ (Table 2)

The relative error of the linear interpolation is less than 0.1%, especially the cases for $F \rightarrow 1$ are given in detail.

4. The approximate formula for C_r

The next values are defined according to equation (15)

$$\left. \begin{aligned} \lambda &= (\lambda_i + \lambda_o)/2, \quad (0.002 < \lambda < 0.2) \\ \rho &= ((\rho_i + \rho_o)/2)^{1.04}, \quad (0 < \rho < 0.3) \\ \mu &= \log_{10}(\lambda_o/\lambda_i), \quad (|\mu| < 1) \end{aligned} \right\} \dots\dots\dots (24)$$

C_r N/mm^{1.5}, the approximate formula is just like equation (25), where d is in mm.

Table 2. The values of δ^*

F	$\delta^*(F)$	F	$\delta^*(F)$
0.0	1.00000	0.9990	0.20590
0.1	0.99777	0.9991	0.19996
0.2	0.99096	0.9992	0.19350
0.3	0.97921	0.9993	0.18642
0.4	0.96182	0.9994	0.17855
0.45	0.95066	0.9995	0.16965
0.50	0.93759	0.99955	0.16469
0.55	0.92233	0.99960	0.15932
0.60	0.90447	0.99965	0.15342
0.65	0.88350	0.99970	0.14688
0.70	0.85916	0.99974	0.14104
0.74	0.83516	0.99978	0.13450
0.78	0.80757	0.99980	0.13090
0.80	0.79181	0.99982	0.12702
0.82	0.77444	0.99984	0.12282
0.84	0.75512	0.99986	0.11822
0.86	0.73341	0.99988	0.11312
0.88	0.70866	0.99990	0.10735
0.90	0.67992	0.99991	0.10414
0.91	0.66361	0.99992	0.10067
0.92	0.64565	0.99993	0.096869
0.93	0.62566	0.99994	0.092650
0.94	0.60310	0.99995	0.087889
0.95	0.57715	0.999955	0.085246
0.955	0.56253	0.999960	0.082382
0.960	0.54651	0.999965	0.079247
0.965	0.52877	0.999970	0.075772
0.970	0.50885	0.999974	0.072677
0.974	0.49089	0.999978	0.069219
0.978	0.47057	0.999980	0.067318
0.980	0.45930	0.999982	0.065276
0.982	0.44709	0.999984	0.063063
0.984	0.43377	0.999986	0.060643
0.986	0.41907	0.999988	0.057961
0.988	0.40263	0.999990	0.054937
0.990	0.38390	0.999991	0.053260
0.991	0.37343	0.999992	0.051444
0.992	0.36202	0.999993	0.049459
0.993	0.34946	0.999994	0.047259
0.994	0.33543	0.999995	0.044780
0.995	0.31949	0.9999955	0.043405
0.9955	0.31060	0.9999960	0.041918
0.9960	0.30091	0.9999965	0.040291
0.9965	0.29027	0.9999970	0.038490
0.9970	0.27840	0.9999974	0.036888
0.9974	0.26778	0.9999978	0.035101
0.9978	0.25586	0.9999980	0.034119
0.9980	0.24927	0.9999982	0.033065
0.9982	0.24218	0.9999984	0.031925
0.9984	0.23446	0.9999986	0.030678
0.9986	0.22600	0.9999988	0.029298
0.9988	0.21657	0.9999990	0.027744

$$C_s = \frac{34.594(1+0.17|\mu|^{1.81})\sqrt{d}/\lambda^{0.355}}{\left[1.44\lambda^{0.0523} - 0.63\lambda^{0.3702} + 0.08\lambda^{0.14}\rho(\rho - 0.77 \operatorname{sgn}(\mu)|\mu|^{0.93})\right]^{1.5}}$$

(accuracy 0.3%) (25)

When $1 \leq \lambda_0/\lambda_i \leq 7$ and $\rho=0$ (approximation), the accuracy is 1% and if all the denominators are replaced by unity, the accuracy is 5%.

5. The numerical table of Φ_1 , Φ_2 (Table 3)

$\Phi_1(\Gamma)$, $\Phi_2(\Gamma)$ which are necessary for equations (8) and (9), are given in Table 3. The error of the linear interpolation is 0.1%. For the values of Γ outside the tables, the next approximate formulae are effective with an accuracy of 0.01%.

(a) $\Gamma \leq -50$:

$$\left. \begin{aligned} \Phi_1 &= 4/3\sqrt{-\Gamma} \\ \Phi_2 &= 1/\Phi_1 \end{aligned} \right\} \dots\dots\dots (26)$$

(b) $\Gamma \geq 8000$:

$$\left. \begin{aligned} \Phi_1 &= \Gamma + 1.945\Gamma^{1/4} \\ \Phi_2 &= 1.030/(\Gamma^{1/4} + 1.23/\sqrt{\Gamma}) \end{aligned} \right\} \dots\dots\dots (27)$$

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Table 3. The values of Φ_1 , Φ_2 .

Γ	$\Phi_1(\Gamma)$	$\Phi_2(\Gamma)$	Γ	$\Phi_1(\Gamma)$	$\Phi_2(\Gamma)$
-50.0	0.18856	5.3033	0.0	2.6730	0.56117
-48.0	0.19245	5.1961	0.500	3.2760	0.56012
-46.0	0.19659	5.0867	1.000	3.8692	0.55721
-44.0	0.20101	4.9749	2.000	5.0311	0.54851
-42.0	0.20574	4.8605	3.000	6.1685	0.53860
-40.0	0.21082	4.7434	4.000	7.2886	0.52874
-38.0	0.21630	4.6233	6.000	9.4925	0.51069
-36.0	0.22222	4.5000	8.000	11.663	0.49501
-34.0	0.22867	4.3732	10.000	13.812	0.48141
-32.0	0.23570	4.2426	12.000	15.943	0.46950
-30.0	0.24343	4.1079	14.000	18.062	0.45895
-28.0	0.25198	3.9686	16.000	20.171	0.44953
-26.0	0.26149	3.8242	18.000	22.271	0.44102
-24.0	0.27217	3.6742	20.000	24.364	0.43330
-22.0	0.28427	3.5178	22.000	26.452	0.42622
-20.0	0.29814	3.3540	24.000	28.534	0.41971
-19.0	0.30589	3.2691	26.000	30.612	0.41369
-18.0	0.31427	3.1819	28.000	32.686	0.40809
-17.0	0.32338	3.0922	30.000	34.757	0.40286
-16.0	0.33334	2.9999	35.000	39.920	0.39115
-15.0	0.34427	2.9046	40.000	45.068	0.38102
-14.0	0.35636	2.8061	45.000	50.203	0.37211
-13.0	0.36981	2.7040	50.000	55.329	0.36418
-12.0	0.38491	2.5978	55.000	60.446	0.35705
-11.0	0.40203	2.4872	60.000	65.553	0.35059
-10.0	0.42166	2.3713	65.000	70.659	0.34468
-9.500	0.43262	2.3112	70.000	75.757	0.33925
-9.000	0.44448	2.2495	75.000	80.850	0.33424
-8.500	0.45737	2.1860	80.000	85.939	0.32958
-8.000	0.47146	2.1206	85.000	91.023	0.32523
-7.500	0.48693	2.0531	90.000	96.105	0.32117
-7.000	0.50403	1.9833	95.000	101.18	0.31735
-6.500	0.52308	1.9110	100.000	106.26	0.31375
-6.000	0.54447	1.8357	105.000	111.33	0.31035
-5.500	0.56873	1.7571	110.000	116.40	0.30713
-5.000	0.59655	1.6748	115.000	121.47	0.30407
-4.800	0.60889	1.6407	120.000	126.53	0.30117
-4.600	0.62203	1.6058	125.000	131.60	0.29840
-4.400	0.63606	1.5701	130.000	136.66	0.29575
-4.200	0.65109	1.5336	135.000	141.72	0.29322
-4.000	0.66725	1.4961	140.000	146.78	0.29079
-3.800	0.68448	1.4575	145.000	151.84	0.28840
-3.600	0.70357	1.4179	150.000	156.89	0.28607
-3.400	0.72414	1.3770	155.000	161.94	0.28378
-3.200	0.74644	1.3347	160.000	166.99	0.28148
-3.000	0.77141	1.2908	165.000	172.04	0.27917
-2.800	0.79888	1.2452	170.000	177.09	0.27684
-2.600	0.82958	1.1975	175.000	182.14	0.27448
-2.400	0.86424	1.1473	180.000	187.19	0.27209
-2.200	0.90383	1.0940	185.000	192.24	0.26967
-2.000	0.94976	1.0368	190.000	197.29	0.26722
-1.900	0.97571	1.0062	195.000	202.34	0.26475
-1.800	1.0041	0.97408	200.000	207.39	0.26226
-1.700	1.0354	0.93987	205.000	212.44	0.25975
-1.600	1.0703	0.90294	210.000	217.49	0.25722
-1.500	1.1097	0.86218	215.000	222.54	0.25467
-1.450	1.1314	0.83974	220.000	227.59	0.25210
-1.400	1.1549	0.81531	225.000	232.64	0.24951
-1.350	1.1805	0.78801	230.000	237.69	0.24689
-1.300	1.2087	0.75591	235.000	242.74	0.24425
-1.280	1.2210	0.74067	240.000	247.79	0.24159
-1.260	1.2339	0.72266	245.000	252.84	0.23891
-1.250	1.2407	0.71144	250.000	257.89	0.23621
-1.240	1.2478	0.69588	255.000	262.94	0.23349
-1.230	1.2553	0.68451	260.000	267.99	0.23075
-1.220	1.2630	0.67519	265.000	273.04	0.22799
-1.200	1.2792	0.65976	270.000	278.09	0.22521
-1.180	1.2961	0.64709	275.000	283.14	0.22241
-1.160	1.3138	0.63634	280.000	288.19	0.21959
-1.140	1.3321	0.62708	285.000	293.24	0.21675
-1.120	1.3510	0.61903	290.000	298.29	0.21389
-1.100	1.3704	0.61199	295.000	303.34	0.21099
-1.050	1.4209	0.59792	300.000	308.39	0.20805
-1.000	1.4738	0.58764	305.000	313.44	0.20508
-0.950	1.5286	0.58010	310.000	318.49	0.20208
-0.900	1.5850	0.57454	315.000	323.54	0.19905
-0.850	1.6425	0.57051	320.000	328.59	0.19599
-0.800	1.7010	0.56755	325.000	333.64	0.19289
-0.700	1.8199	0.56388	330.000	338.69	0.18975
-0.500	2.0624	0.56127	335.000	343.74	0.18657
0.0	2.6730	0.56117	340.000	348.79	0.18335

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