A Study on NC Gear Cutting of Large-size Bevel Gears
3rd Report; A Method to Save Path Number of the Cutter
in Tooth Generation *

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The normal tooth profile of a bevel gear changes its form along the tooth trace. When this profile is generated by an end milling cutter of straight axial profile, the number of cutter paths to generate the tooth will surely become large. If the axial profile is suitably designed and the generating point is properly selected in accordance with the cutter profile relative to the tooth profile, the number of the paths will be reduced greatly. This report proposes a new cutting method based on this idea. It is a procedure to avoid the interference between the profiles and to select the generating points, giving also the unremoved volume on the generated tooth surface. The ratio of the unremoved volume in the case of three cutter paths by the curved profile cutter compared to that by the straight profile cutter is estimated to be about 1/20 for the pitch angle $\gamma = 15^\circ$, and about 1/7 for $\gamma = 75^\circ$.

Key words: Gear, Bevel Gear, NC Gear Cutting, Profile Curvature, Formed Cutter, Generating Method

1. Introduction

If we can get a large size bevel gear with a small cutter that is attached to a machine tool for general use, surely the cutting cost will be sured. In this case, the motions of the cutter are supposed to become complicated. But, this problem may be solved by the numerical control techniques. Thinking in this way, the authors have proposed a new cutting method^(1,2) in which the tooth surface of a bevel gear has a new tooth trace and is formed by the end milling cutter. It has been believed that the form-cutting method can't make the bevel gear tooth surface. Additionally, the authors have contrived a method⁽³⁾ to cut the tapered tooth of the bevel gear where the tooth depth is designed to agree with the tooth thickness under consideration of the tooth bending strength. Sometimes, the straight bevel gear is milled⁽⁴⁾ by the cutter the axial profile of which is formed after the inside tooth profile of bevel gear. The idea of this cutting method is also applicable for the spiral bevel gears. But in that case, the tooth profile will be overcut severely at both the tip and the root on the outside heel of the gear. The magnitude of the tooth gap is excessively large in comparison with that of the usual tooth modification.

The above mentioned methods require only one path of the cutter along the tooth trace to form the tooth surface. Thus, it requires only a short cutting time. But, these methods have many drawbacks such as the limitations on both the tooth traces

and the tooth profiles as reported previously. In contrast to these cutting methods, this paper contrives another method which is applicable to an arbitrary profile of the tooth surface with a little addition of the number of the cutting paths. There is another potential method, in which the end milling cutter has a straight edge of the axial profile and generates the tooth surface. In this method, unfortunately, the number of the cutter paths along the tooth trace becomes large to get an accurate profile of the tooth surface. By the way, if we give the cutter an axial profile that is almost the same as the tooth profile of the gear, we shall be able to reduce greatly the number of the cutting paths.

2. The Effect of the New Cutting Method

2.1 Interference between the cutter and the gear tooth

To simplify the relation between the cutter profile and the gear profile, they are assumed to be plane curves. Let the cutter profile contact with the tooth profile at a point O , and have the same magnitude of curvature as that of the tooth profile at the same point O . Then, the relative curvature of the profiles is zero. Now we take notice of the profile gap of the higher order. Then even if the relative curvature is not zero but has a small value, the two profile curves will generally intersect each other in the neighbourhood of the contact point O .

As an example of the problem mentioned above, let us give both the cutter and the tooth the involute curves respectively in a common plane. To distinguish these curves from each other, let us give the curve of

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$$(h_i/h_s)_m \approx 1/6(M=1), 1/11(M=2)$$

$$1/17(M=3), 1/22(M=4)$$

The effect of the curved profile cutter amounts to about 10 times. The height h of the unremoved blank is almost in proportion to the second power of the number N of the paths. Therefore, the effect on the number N of the generating paths caused by the difference in the profiles of cutters is about 3 times.

3. A Concrete Example

The previous discussions are based on the approximate relations. The magnitude of the tooth profile errors based on the approximations may be the same order of the unremoved volume. Therefore, by using a concrete example, the relations between the cutter and the tooth surface will be checked and the exponent constant M will be searched in this section.

3.1 Crown gear tooth surface

Let the tooth surface of the crown gear be the ruled surface. The tooth trace of the crown gear is a logarithmic spiral, and the mother line of the surface is perpendicular to the tooth trace and inclines at the angle α to the crown axis. Now, let us introduce the coordinate system $O_c X_c Y_c Z_c$, where the origin O_c is set at the intersecting point of crown gear axis with the pitch plane, the X_c axis is laid on the base line fixed on the pitch plane and the Z_c axis coincides with the axis of crown gear. And, let i_c, j_c, k_c be the unit vectors corresponding to the axes X_c, Y_c, Z_c respectively. Let l_i be the cone distance of the intersecting point I of the tooth trace with the X_c axis and l be that of the point B on the tooth trace. The angle Φ between the base line and the radial line passing through the point B can be defined in the clockwise direction by Eq.(1),

$$\Phi = \tan \Psi \ln(l/l_i) \quad (1)$$

where Ψ is the helix angle which is positive in the case of right hand helix and negative in the case of left hand helix.

The following are defined

b : $\vec{O_c B}$,

t : unit vector tangent to the tooth trace,

a : unit vector parallel to the mother line of the ruled surface of the crown gear tooth,

m : distance of the point S_c on the tooth surface from the tooth trace along the mother line,

r_c : $\vec{O_c S_c}$.

These vectors can be expressed by the following Eqs.

$$b = l \cos \Phi i_c - l \sin \Phi j_c \quad (2)$$

$$t = \cos(\Psi + \Phi) i_c - \sin(\Psi + \Phi) j_c \quad (3)$$

$$a = \cos \alpha k_c + \sin \alpha t \times k_c \quad (4)$$

$$r_c = \{l \cos \Phi - m \sin \alpha \sin(\Psi + \Phi)\} i_c - \{l \sin \Phi + m \sin \alpha \cos(\Psi + \Phi)\} j_c + m \cos \alpha k_c \quad (5)$$

3.2 Tooth surface of the bevel gear

Figure 3 shows that the crown gear tooth surface contacts with the bevel gear tooth surface at the point $S_c(S_g)$ at the angular position φ_c of the crown gear which is the angle of the base line to the axis of the instantaneous motion of the gears.

Let us introduce another Cartesian coordinate $OXYZ$ where the origin O is set at the point O_c , the axis Z coincides with the axis Z_c and the axis X is laid on the instantaneous axis, and let i, j, k be the unit vectors corresponding to the axes X, Y, Z respectively. Additionally, define as follows

γ : pitch angle of the bevel gear,

ζ : unit vector parallel to the angular velocity.

The surface normal n of the crown gear tooth at the point S_c intersects with the pitch plane at the point P . When the crown gear rotates by an angle φ_c , the point P comes on the instantaneous axis. The position of the point P is given by the following expression.

$$\vec{OP} = b + t \times k_c m / \sin \alpha$$

Using Eqs.(2) and (3), the angle φ_c can be expressed by Eq.(6).

$$\tan \varphi_c = \frac{\{l \sin \Phi \sin \alpha + m \cos(\Psi + \Phi)\}}{\{l \cos \Phi \sin \alpha - m \sin(\Psi + \Phi)\}} \quad (6)$$

And, the angle φ_g of the bevel gear is given by

$$\varphi_g = \varphi_c / \sin \gamma \quad (7)$$

The tooth surface ($\vec{OS_c} \equiv r$) of crown gear and its tooth normal n are expressed by Eqs.(8) and (9).

$$r = \{l \cos(\Phi - \varphi_c) - m \sin \alpha \sin(\Psi + \Phi - \varphi_c)\} i - \{l \sin(\Phi - \varphi_c) + m \sin \alpha \cos(\Psi + \Phi - \varphi_c)\} j + m \cos \alpha k \quad (8)$$

$$n = \cos \alpha \sin(\Psi + \Phi - \varphi_c) i + \cos \alpha \cos(\Psi + \Phi - \varphi_c) j + \sin \alpha k \quad (9)$$

By the way, when we turn the vector A about the axis C by an angle φ we have a new vector B given as follows,

$$B = C(A \cdot C) + \cos \varphi (A - C(A \cdot C))$$

$$+ \sin \varphi C \times A \quad (10)$$

where $C^2 = 1$. Let us express this relation in a short form as

$$B = R(C, \varphi) A$$

Fig.3 Contact point between the crown gear tooth surface and the bevel gear tooth surface

Using this expression, the point $\mathbf{I}_g$ of the bevel gear tooth surface and its normal $\mathbf{n}_g$ of the angular position can be expressed by Eqs.(11) and (12).

$$\mathbf{r}_g = R(\xi, \varphi_g - \varphi_a) \mathbf{r} \dots\dots\dots (11)$$

$$\mathbf{n}_g = R(\xi, \varphi_g - \varphi_a) \mathbf{n} \dots\dots\dots (12)$$

For convenience, let us call the axial profile of the end milling cutter that contacts with the bevel gear the formed cutter profile. We probably are able to analyze the correct curvature of the formed cutter profile. But, we can freely select the generating point on the formed cutter profile. So, we shall approximate the curvature of the formed cutter profile with that of the tooth profile of the equivalent spur gear. The pitch radius R_{el} of the equivalent spur gear at the point B of the cone distance l is expressed by Eq.(13). Using Eq.(13), the curvature ρ_l of the tooth profile at the point S_g is derived as Eq.(14).

$$R_{el} = l \tan \gamma / \cos^2 \Psi \dots\dots\dots (13)$$

$$\rho_l = R_{el} \sin \alpha + m \cot \alpha \dots\dots\dots (14)$$

3.3 Axial profile of the end milling cutter

Let us define the axial profile of the cutter such that the cutter contacts along a line with the outside tooth surface of bevel gear. And, let us express the cutter profile by modifying the involute profile of the equivalent spur gear.

Let us consider the equivalent spur gear at the pitch point P_e of the cone distance l_e . And, let O_e be the axis center of the equivalent spur gear. Let us set the axis of the cutter parallel to the line $\overline{O_e P_e}$ and distant x_0 from this line and set it in a plane that is perpendicular to the tooth trace. Let the point O_E be a projection of the point O_e to the axis of the cutter. Let us introduce the coordinate system $O_e \cdot X_e Y_e Z_e$ where the axis Z_e is parallel to the line $\overline{O_e P_e}$ and the axis X_e to the line $\overline{O_e O_e}$. And, let the unit vectors $\mathbf{i}_e, \mathbf{k}_e$ be parallel to the axes X_e, Z_e respectively.

When the involute profile of the equivalent spur gear passes through the point P_e , the point T_E laid on the profile is given with a parameter θ by the following equations.

$$R_e = l_e \tan \gamma / \cos^2 \Psi \dots\dots\dots (15)$$

$$R_0 = R_e \cos \alpha \dots\dots\dots (16)$$

$$\begin{aligned} \overline{O_E T_E} &\equiv \mathbf{r}_e \\ &= [R_0 \{ \sin(\alpha - \theta) + (\theta - \tan \alpha) \cos(\alpha - \theta) \} + x_0] \mathbf{i}_e \\ &\quad + [R_0 \{ \cos(\alpha - \theta) - (\theta - \tan \alpha) \sin(\alpha - \theta) \}] \mathbf{k}_e \end{aligned} \dots\dots\dots (17)$$

And, the point O_E fixed on the cutter axis is given by

$$\overline{O O_E} = (l_e + x_0 \sin \Psi) \mathbf{i} + x_0 \cos \Psi \mathbf{j} - R_e \mathbf{k} \dots\dots\dots (18)$$

We shall define the profile of the cutter in accordance with its contact line that passes through the point P_e .

Let angle φ_g be the angular position of the bevel gear where the tooth profile includes the point P_e . The angle φ_g is given by putting the conditions $m = \theta$ and

$l = l_e$ for Eq.(6) and using Eq.(7). The vector $\mathbf{I}_g$ defined by Eq.(11) shows the gear tooth surface with the parameters (l, m) and Eq.(12) containing the angle φ_g shows the tooth normal $\mathbf{n}_g$.

Let us define the vector $\mathbf{I}_f$ that is pointing from O_E to $\mathbf{I}_g$ on the tooth surface.

$$\mathbf{r}_f = \mathbf{r}_g - \overline{O O_E} \dots\dots\dots (19)$$

At the point of the contact line between the surface produced by the edges of the end milling cutter and the tooth surface of the gear, the three vectors, i.e., vector $\mathbf{I}_f$, surface normal $\mathbf{n}_g$ and cutter axis $\mathbf{k}_c$, must be laid in a same plane. Then we get,

$$(\mathbf{n}_g \cdot \mathbf{r}_f \times \mathbf{k}_c) = 0 \dots\dots\dots (20)$$

The point $\mathbf{I}_f$ defined by Eq.(20) gives the axial profile L_c of the cutter. Let us express the point $\mathbf{T}_E$ on the profile L_c in the following form.

$$\begin{aligned} \mathbf{r}_E &\equiv \mathbf{r}_e + R_0 \{ \cos(\alpha - \theta) \mathbf{i}_e \\ &\quad - \sin(\alpha - \theta) \mathbf{k}_e \} E(\theta) \end{aligned} \dots\dots\dots (21)$$

where $E(\theta)$ is a factor corresponding to the profile modification, and is given by the following equations.

$$\mathbf{r}_K = (\mathbf{r}_f \cdot \mathbf{k}_e) \dots\dots\dots (22)$$

$$r_n = |\mathbf{r}_f - \mathbf{k}_e (\mathbf{r}_f \cdot \mathbf{k}_e)| \dots\dots\dots (23)$$

$$\theta_a = \tan^{-1} \{ (r_n - x_0) / r_K \} \dots\dots\dots (24)$$

$$\theta_0 = \tan^{-1} \{ \sqrt{(r_n - x_0)^2 + r_K^2} / R_0 \} \dots\dots\dots (25)$$

$$\theta = \alpha - \theta_a - \theta_0 \dots\dots\dots (26)$$

$$E(\theta) = \tan \alpha - \theta - \tan \theta_0 \dots\dots\dots (27)$$

In the numerical calculation, $E(\theta)$ is given as discrete values. For convenience of differentiation, let us approximate the value $E(\theta)$ by the polynomial expression.

$$E(\theta) \approx c_0 + c_1 \theta + \dots + c_n \theta^n \dots\dots\dots (28)$$

Let t_1 be the $\mathbf{i}_e$ component of the tangent vector $d\mathbf{T}_E/d\theta$ of the axial profile L_c of the cutter, and t_2 be the $\mathbf{k}_e$ component.

$$d\mathbf{r}_E/d\theta \equiv t_1 \mathbf{i}_e + t_2 \mathbf{k}_e \dots\dots\dots (29)$$

Then, the profile normal $\mathbf{n}_E$ directed to the cutter axis is expressed by Eq.(30).

$$\mathbf{n}_E = t_2 / \sqrt{t_1^2 + t_2^2} \mathbf{i}_e - t_1 / \sqrt{t_1^2 + t_2^2} \mathbf{k}_e \dots\dots\dots (30)$$

In estimating the curvature ρ_0 of the cutter profile L_c , let us set the order of the accuracy of the estimation as high as that for the tooth profile curvature ρ_l . Using the profile curvature of the equivalent spur gear, let us approximate the value ρ_0 by the following equation.

$$\rho_0 \approx R_e (\sin \alpha - \theta \cos \alpha) \dots\dots\dots (31)$$

3.4 The position of the cutter axis

The radius ρ_l of the curvature at the point S_g on the bevel gear tooth surface is expressed by Eq.(14) and the radius ρ_0 of the contact point S_g of the cutter is given by Eq.(32).

$$\rho_0 = \rho_l (l_0 / l)^{1/3} \dots\dots\dots (32)$$

Substituting the value ρ_0 into Eq.(31), we can get the parameter θ and fix the contact point.

Let us set the cutter axis parallel to the crown axis, and let D_2 be the $\mathbf{k}_e$ component of the surface normal $\mathbf{n}_E$ of the cutter.

$$D_2 = (n_e \cdot k_e) \dots \dots \dots (33)$$

At the contact point between the gear and the cutter, the surface normal of the gear agrees with that of the cutter. To satisfy this condition, the bevel gear should rotate by the angle λ from the base position where the point S_c of the crown gear surface contacts with the bevel gear. At the base position, using Eq.(9) that expresses the normal n of the point S_g of the gear tooth surface, the values defined by Eqs.(34)-(37) respectively are derived. Then, according to the condition that the k component of the surface normal of the cutter is the same in magnitude as that of the gear, the angle λ is given by Eq.(38).

$$D_1 = (k \cdot \xi)(n \cdot \xi) \dots \dots \dots (34)$$

$$D = D_1 - D_2 \dots \dots \dots (35)$$

$$A = (K \cdot \xi \times n) \dots \dots \dots (36)$$

$$B = (k \cdot n) - (k \cdot \xi)(n \cdot \xi) \dots \dots \dots (37)$$

$$\tan \frac{\lambda}{2} = \frac{-A \pm \sqrt{A^2 + B^2 - D^2}}{(D - B)} \dots \dots \dots (38)$$

At the angular position λ of the bevel gear, the position r_λ of the point S_g of the gear tooth surface and the surface normal n_λ of the point are given by Eqs.(39) and (40) respectively.

$$r_\lambda = R(\xi, \lambda)r \dots \dots \dots (39)$$

$$n_\lambda = R(\xi, \lambda)n \dots \dots \dots (40)$$

Table 1 Dimensions of gears for numerical analyses

Pitch Angle	γ	15°, 45°, 75°
Outer Cone Distance	l_o	150 mm
Inner Cone Distance	l_i	100 mm
Spiral Angle	ψ	30°
Pressure Angle	α	20°
Addendum (Out-side)		6 mm
Dedendum (Out-side)		6 mm
Cutter Profile Offset	x_c	6 mm

$\gamma=45^\circ, l_o=150 \text{ mm}, l_i=150 \text{ mm},$
 $M=2, N=3$

Fig.4 Tooth profile interfering with the cutter profiles

$\gamma=45^\circ, l_o=153 \text{ mm}, l_i=150 \text{ mm},$
 $M=2, N=3$

Fig.5 Generated tooth profile without interference

The cutter axis is included in a plane that passes through the point S_g and is parallel to both the vectors n_λ and k . The vector r_e starting from the point O_E to the point of the parameter θ of the cutter profile can be derived by combining Eqs.(41) and (42) with Eq.(21).

$$i_e = k \times (k \times n_\lambda) / |k \times (k \times n_\lambda)| \dots \dots \dots (41)$$

$$k_e = k \dots \dots \dots (42)$$

Using these relations, the point O_E fixed on the cutter axis is expressed by Eq.(43).

$$\overrightarrow{OO_E} = r_\lambda - r_e \dots \dots \dots (43)$$

3.5 The unremoved volume

The previous sections describe the procedures of deriving the axial profile L_c of the cutter that forms the outside profile of the gear. Using the imaginary profile L_{ci} of the cutter that forms the inner side profile of the gear, we shall discuss the unremoved volume.

The start position of the rotation of the gears is given by the condition that the base line fixed on the pitch plane of the crown gear lies on the instantaneous axis. When the bevel gear rotates by an angle $\varphi_{g\lambda}$,

$$\varphi_{g\lambda} = \varphi_g + \lambda \dots \dots \dots (44)$$

The position of the cutter axis is given by Eq.(43). The tooth surface of bevel gear is expressed with the two parameters (l, m) . To distinguish these parameters from those of the foregoing expressions in the section 3.2, let us put the superscript $(-)$ on them. When the value $\bar{l}$ is given, Eq.(1) gives us $\bar{\theta}$, and when $\bar{m}$ is given, Eqs.(6)-(9) give us the values of $\bar{\varphi}_c$, $\bar{\varphi}_g$, $\bar{r}$ and $\bar{n}$. Additionally, considering that the angle $\bar{\varphi}_{g\lambda}$ is constant, the value $\bar{\lambda}$ is derived from Eq.(44). Substituting

$\gamma=45^\circ, l_o=153 \text{ mm}, l_i=100 \text{ mm},$
 $M=2, N=3$

Fig.6 Tooth profile generated by rounded tip-corner of the cutter

$\gamma=45^\circ, l_o=153 \text{ mm}, l_i=100 \text{ mm},$
 $M=3, N=3$

Fig.7 Generating point located outside of the cutter profile

this value into Eqs.(39) and (40), we can get the tooth surface $\bar{T}_\lambda$ and its normal $\bar{n}_\lambda$. For the tooth surface to contact with the cutter surface at the point $\bar{T}_\lambda$, the following condition should be satisfied.

$$[\bar{n}_\lambda \cdot (\overrightarrow{OO_E} - \bar{r}_\lambda) \times \mathbf{k}] = 0 \quad (45)$$

We can find a point, i.e. the parameters $\bar{l}$ and $\bar{m}$ that satisfy Eq.(45), using the trial-and-error method. Let the vector $\bar{T}_{\lambda E}$ be the position $\bar{T}$ that satisfies Eq.(45). And put;

$$\bar{r}_k = \{(\overrightarrow{OO_E} - \bar{r}_{\lambda E}) \cdot \mathbf{k}\} \quad (46)$$

$$\bar{r}_n = [(\overrightarrow{OO_E} - \bar{r}_{\lambda E}) - \bar{r}_k \mathbf{k}] \quad (47)$$

$$\bar{\theta}_a = \tan^{-1}\{(\bar{r}_n - x_0)/\bar{r}_k\} \quad (48)$$

$$\bar{\theta}_b = \tan^{-1}\{\sqrt{(\bar{r}_n - x_0)^2 + \bar{r}_k^2} - R_b/R_0\} \quad (49)$$

$$\bar{\theta} = \alpha - \bar{\theta}_a - \bar{\theta}_b \quad (50)$$

Then, putting the value $\bar{\theta}$ into Eq.(21), we get the point $\bar{T}_E$ on the profile L_c of the cutter. By the way, the point $\bar{T}_{E\lambda}$ on the imaginary profile $L_{c\lambda}$ of the cutter can be given by the previous section. Then, the gap Δ between the profile L_c and the profile $L_{c\lambda}$ in the direction of the profile normal can be given by Eq.(51).

$$\Delta = R_0 \left\{ -\sum_{i=0}^n C_i \bar{\theta}^i + \tan \alpha - \bar{\theta} - \tan \bar{\theta}_b \right\} \quad (51)$$

3.6 The result of the numerical investigation

Table 1 shows the gear dimensions used in the numerical investigation. Figures 4 and 5 show the gap Δ that corresponds to the unremoved volume at a cone distance of $l = 150\text{mm}$ under the conditions $N=3$, $n=6$ and $M=2$. Figure 4 shows the value Δ at a cone distance of $l = 150\text{mm}$. This value of

$\gamma = 45^\circ$, $l_c = 153\text{ mm}$, $l = 100\text{ mm}$,
 $M = 1.5$, $N = 3$

Fig.8 Critically generated profile of the gear

$\gamma = 15^\circ$, $l_c = 153\text{ mm}$, $l = 100\text{ mm}$,
 $M = 3.5$, $N = 3$

Fig.9 Critically generated profile of the gear

l is the same as the cone distance of $l_c = 150\text{mm}$ where the axial profile of the cutter is defined after the equivalent spur gear. The tooth profile in the case of Fig.4 is interfered a little, because the profile modification $E(\theta)$ differs a little from the approximate value $\sum_{i=0}^n C_i \theta^i$. To avoid this interference, the value l_c is trially changed by a small amount. Figure 5 shows the cases of $l_c = 153\text{mm}$ and $l = 150\text{mm}$.

Figures 5 and 6 show the cases under the same conditions except the value l . And, these figures indicate that the cutter profiles overlap each other and envelope the tooth profile and that the overlapped area of the profiles decreases as the cone distance l decreases. So, we take note only of the inside of the bevel gear.

Figure 7 shows the case that the generating point does not exist on the cutter profile. And, Fig.8 shows that the tooth profile of gear is generated by the rounded tip-corner of the cutter. To prevent these troubles, Fig.8 has a smaller value of the exponent constant M .

The maximum value M_m when the cutter generates the tooth profile without the troubles is searched trially in the case of the pitch angles $\gamma = 15^\circ$ and $\gamma = 75^\circ$. Figures 9 and 10 show these cases respectively. The maximum value M_m for the pitch angle γ is as follows.

$$(\gamma = 15^\circ; M_m = 3.5), (45^\circ; 1.5), (75^\circ; 1.1)$$

This shows that the smaller pitch angle γ gives a larger value of M_m , resulting in a remarkable effect of the curved profile of the cutter.

3.7 Simplified cutter profile

In the method explained in the previous sections, some complicated calculations are necessary to derive and express the cutter profile that forms the correct tooth profile of the outside heel of the gear. Here, through with a smaller effect than that given by the proposed cutting method, we can get the simplicity of the cutter profile by using a nonmodified involute profile of the equivalent spur gear that is defined at a small distance from the outer-end of the gear teeth and also by allowing a small amount of the unremoved volume on the outside tooth surface heel of the gear.

$\gamma = 75^\circ$, $l_c = 153\text{ mm}$, $l = 100\text{ mm}$,
 $M = 1.1$, $N = 3$

Fig.10 Critically generated profile of the gear

4. Conclusions

This paper proposes a new cutting method in which the end milling cutter forms the tooth surface at the outside of the bevel gear and generates the tooth surface at the inner side. This is a way to avoid the intersectional interference between the cutter profile and the tooth profile and a way to control such that the generating point does not exceed the tip limit of the cutter.

The ratio of the unremoved volume on the outside heel of the theoretical tooth surface generated by the curved profile cutter to that by the straight profile cutter is estimated at about 1/20 for the pitch angle $\gamma = 15^\circ$ and about 1/7 for $\gamma = 75^\circ$. This shows that the effect of the curved profile cutter is remarkable in the case of the smaller pitch angle.

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