A Study of High Temperature Low Cycle Fatigue Criterion in Biaxial Stress State*

by

Naomi HAMADA**, Masao SAKANE*** and Masateru OHNAMI****

This paper describes a new equivalent stress including the principal stress parallel to fatigue cracks based on COD concept and examines the applicability of that stress using a SUS 304 stainless steel at 823 K in air. Also the equivalent strain is derived from that equivalent stress using a continuum theory. Both the equivalent stress and the strain are proved to be good parameters to arrange the low cycle fatigue data in multiaxial stress state.

Key Words : Strength of Materials, Elevated Temperature, Biaxial Low-Cycle Fatigue

1. Introduction

Historically, there have been many studies on low-cycle fatigue in multiaxial stress state[1-18] and many parameters have been proposed to arrange the low-cycle fatigue data in that stress state. Among them typical parameters are equivalent strain of Mises' type[19], combination of maximum shear strain with normal strain acting on the maximum shear plane[4,7,9] and the parameter which relates to hydrostatic stress[8]. However there is no parameter which can arrange the low cycle fatigue data in a wide range of multiaxial stress states at elevated temperatures, especially for the case of alternating principal stress axes[12,17] and creep-fatigue conditions[6,18].

The authors[12,16,18] have studied the strain controlled biaxial low cycle fatigue at elevated temperatures and discovered the following points.

(i) Equivalent strain of Mises' type can be a good parameter to evaluate the low cycle fatigue life in proportional loading, but cannot be in non-proportional loading because of the larger cyclic strain hardening by alternating principal stress axes.

(ii) Mises' equivalent strain as well as the maximum principal strain cannot be a comparison parameter to estimate the biaxial low cycle fatigue life in creep-fatigue conditions because the stress biaxiality affects the relaxation behavior during the strain hold-time.

These results signify that only the strain criterion is not sufficient as a comparative parameter because the cyclic deformation behavior affects the biaxial low cycle fatigue life. So the applied stress also should be taken into consideration. Furthermore since most of the fatigue life is the crack propagation period, the criterion should be mainly based on the crack propagation consideration. Especially a stress component parallel to the crack increases the crack propagation rate in biaxial stress state[6,20], and therefore the criterion should include the effect of the parallel stress.

This paper develops a new equivalent stress that can estimate accurately the biaxial low cycle fatigue life considering the crack propagation rates in biaxial stress state. And to check its validity, the criterion is applied to the biaxial low cycle fatigue data of an austenitic stainless steel SUS 304 at 823 K in air. A new equivalent strain is also developed from the equivalent stress because most of low cycle fatigue tests are made in strain controlled mode.

2. Test Specimen and Test Procedure.

This study uses a hollow cylindrical specimen of a solution treated SUS 304 austenitic stainless steel at 1373 K whose chemical composition is listed in Table 1 and grain size is ASTM No.3. The specimen has a center notch hole of 1-mm-diameter for easy confirmation of the crack initiation, and the shape and dimensions are illustrated in Fig.1. An

* Received 14th September, 1983.

** Research Assistant, Faculty of Science and Engineering Ritsumeikan University, 56-1 Tojiin Kita-machi Kita-ku Kyoto, 603, Japan.

*** Assistant Professor, Faculty of Science and Engineering Ritsumeikan University, 56-1 Tojiin Kita-machi Kita-ku Kyoto, 603, Japan.

**** Professor, Faculty of Science and Engineering Ritsumeikan University, 56-1 Tojiin Kita-machi Kita-ku Kyoto, 603, Japan.

Table 1 Chemical compositions of the material.

							W _i (%)
Si	Mn	P	S	Ni	Cr	C	Fe
0.38	1.13	0.008	0.021	8.74	18.52	0.06	bal.

Fig.1 Shape and dimensions of the test specimen (mm).

(a)

(b)

Fig.2 Arrangement of the biaxial low cycle fatigue life by (a) maximum principal stress range and (b) equivalent Mises' type.

originally developed biaxial electric hydraulic servo machine, which can apply an axial load in combination with a torsional load, was employed for making maximum principal stress controlled tests at 0.1 Hz using triangular stress waves at 823 K in air. Stress ratio, λ , in the study is defined as $\lambda = \sigma_2/\sigma_1$, where σ_1 is the maximum principal stress while σ_2 is the minimum principal stress. Note that macro cracks initiated on the circular hole propagated normal to the maximum principal stress axis.

3. Experimental Results and Discussion.

3.1. Maximum principal stress range and Mises' equivalent stress range.

Figs.2 (a) and (b) show the arrangement of biaxial low cycle fatigue data with (a) maximum principal stress range $\Delta\sigma_1$ and (b) Mises' equivalent stress range $\Delta\sigma_{eq}$. The fatigue data in reversed torsion show smaller fatigue strength by the arrangement of maximum principal stress while Mises' equivalent stress range reverses the arrangement where the push-pull data exhibit smaller fatigue strength. Both the parameters cannot arrange the biaxial fatigue data in a factor of two band.

Fig.3 shows the change in the ratio of crack initiation cycle, N_c , to failure cycle, N_f , with maximum principal stress range. The crack initiation cycle in this paper is defined as $N_c = (N_1 + N_2)/2$, where N_1 is a cycle where we can confirm the crack at the notch root first and N_2 the monitoring cycle just before N_1 . Average crack length at N_c is 0.1 mm which corresponds to one grain in length. Values of the ratio are from 20 % to 40 % so that the role of the crack propagation is dominant but the role of the crack initiation is small. Cracks are monitored at predetermined intervals by an automatic camera and crack length is measured on negative films.

Fig.4 is a plotting of crack

Fig.3 Variation of the ratio of crack initiation cycle to failure cycle with maximum principal stress range.

propagation rate against the crack length. Crack propagation rate in the reversed torsion is larger than that in the push-pull at the same crack length. As the tests were made in principal stress controlled condition, where the values of applied stress normal to the crack were the same in both the push-pull and reversed torsion, the larger crack propagation rate in reversed torsion was a result of the application of the minimum stress parallel to the crack. This effect of the parallel stress in low cycle fatigue was confirmed by the experiments and FEM analysis[6,20].

Now we discuss the correlation between the results in Fig.2 and Fig.4 from the crack behavior. Most of the fatigue life is the crack propagation period as is shown in Fig.3 so we assume that the failure life corresponds to the crack propagation life in the following.

The arrangement of the biaxial fatigue data with the principal stress range, $\Delta\sigma_1$, only considers the stress normal to the crack and it does not include the stress parallel to the crack. But the experimental data in Fig.4 show the stress parallel to the crack has an effect to increase the crack propagation rate. This is the reason why the arrangement of the biaxial fatigue data using $\Delta\sigma_1$ fails. On the contrary, equivalent stress of Mises' type includes the effect of the parallel stress excessively because the stress is equated as $\Delta\sigma_{eq} = (\Delta\sigma_1^2 + \Delta\sigma_2^2 - \Delta\sigma_1\Delta\sigma_2)^{1/2}$. The equation shows the parallel stress having the same effect as the principal stress normal to the crack. However the parallel stress does not have the same effect on crack propagation rate as the normal stress, which is cause of the discrepancy of the arrangement of the results in Fig.2 (b).

Fig.5 is an arrangement of the crack propagation rate with crack opening displacement, COD, defined as the displacement at the notch root. The figure says that the correlation of the data is so good that if we can formulate COD by a simple equation using the applied

stresses, the equation will provide a good comparison parameter to arrange the biaxial fatigue data. In the following we formulate the equation by FEM analysis.

3.2. Equivalent stress, $\Delta\sigma^*$, which includes the effect of parallel stress to the cracks.

Fig.6 is a result of the elastic-plastic FEM analysis for a center cracked plate in static tension, showing the relation between the crack opening displacement at the center of the crack and $(2-\lambda)$. The width of the FEM model is 16.5 mm, the length 16.5 mm and the crack length 2 mm. The ordinate of the figure is COD/σ_1 and the abscissa is the principal stress ratio ($\lambda = \sigma_2/\sigma_1$). The constitutive equation of the analysis in plane stress is $\epsilon_{eq} = (\sigma_{eq}/E) + C(\sigma_{eq}/\sigma_{eqv})^n$, where σ_{eq} and ϵ_{eq} are equivalent stress and strain of Mises' type respectively. The material constants for the analysis are $E=150000$ MPa, $C=180 \times 10^{-4}$, $\sigma_{eqv}=100$ MPa and $n=8$. The figure shows that the following equation holds,

$$\left. \begin{aligned} \frac{COD}{\sigma_1} &= A(2-\lambda)^m, \\ \text{or} \\ \frac{COD}{A'} &= \alpha\sigma_1(2-\lambda)^m, \end{aligned} \right\} \quad (1)$$

where $A = \alpha A'$ and α is a non-dimensional constant. The value of m is 0.075 in the range $1 \leq (2-\lambda) < 2$ and 0.5 in the range $2 \leq (2-\lambda) \leq 3$. As the left of equation (1)₂ is the crack opening displacement in biaxial stress field, we now define it as a new

Fig.4 Crack propagation rate vs. crack length plotting.

Fig.5 Crack propagation rate vs. crack opening displacement plotting.

Fig.6 Analytical relation between COD/σ_1 and $(2-\lambda)$ by FEM.

equivalent stress i.e., $\Delta\sigma^* = a\Delta\sigma_1(2-\lambda)^m$ after changing σ^* and σ_1 to their ranges. $\Delta\sigma^*$ is the equivalent stress in biaxial stress and we can calculate it easily by simply knowing the applied principal stresses $\Delta\sigma_1$ and $\Delta\sigma_2$. Value of a is $1/\sqrt{2}$ because $\Delta\sigma^*$ should agree with σ_1 in uniaxial tests. Note that the new equivalent stress in this paper corresponds with the crack opening displacement and we can replace it by COD. But we let it have the dimension of stress because the stress is more widely acceptable compared with the COD in practice.

Fig.7 (a) is a rearrangement of Fig.2 using the new equivalent stress $\Delta\sigma^*$. The arrangement of the data is fairly improved and all the data fall into the scatter band of a factor of two. Thus it is concluded that the equivalent stress is applicable to the biaxial low cycle fatigue data in principal stress controlled tests. Also Figs.7 (b), (c) and (d) are arrangements of the results in principal strain controlled tests of the authors [12,16,18] at (b) 823 K, (c) 923 K and (d) 823 K in air. All the data in Figs.7 (b) and (c) are the results of no alternation tests while some of the data in Fig.7(d) are the results of the alternation test, where the shape of the specimen is the same as that in this paper. In the symmetrical and unsymmetrical waveforms of Fig.7(d), solid lines denote the push-pull straining and dotted lines the reversed torsion straining. Principal stresses at half life are used for calculating the equivalent stress $\Delta\sigma^*$, especially, in the case of Fig.7(d) the principal stress amplitude in tension side σ_1^* is used. Figs.7 (b), (c) and (d) show that $\Delta\sigma^*$ or σ_1^* is also applicable to the strain controlled biaxial data with and without the alternation of the principal stress axes. Stress range in the alternation

* The combined solid and dotted line shows the test of changing principal stress axes. The detailed description on the test method was reported previously [12,16].

Fig.7 Arrangement of the biaxial low cycle fatigue data by equivalent stress on COD criterion.

tests increased about 50 % compared with the no-alternation tests and equivalent strain of Mises' type could not arrange the biaxial low cycle fatigue data in the alternation tests[12,16]. Therefore, it can be concluded that the equivalent stress is a better comparison parameter because it is applicable to the alternation tests.

3.3. A new equivalent strain derived from the equivalent stress.

Most of the biaxial low cycle fatigue tests are carried out in strain controlled mode so that the parameter to arrange the fatigue lives should be developed on strain basis. We now derive the equivalent strain corresponding with $\Delta\sigma^*$ in the following.

Equivalent stress developed and equivalent strain to be derived are the parameters to relate with the stress and strain intensities at the crack tip, so that we can assume the following equation on the analogy of the relation, $K_t = K_t'^{1/n}$, in nonlinear fracture mechanics[21];

$$\epsilon^* = \sigma^{*1/n}, \quad (2)$$

where K_σ is the inelastic stress concentration factor, K_ϵ the inelastic strain concentration factor and n the strain hardening exponent in the constitutive equation on Mises' basis. Meanwhile the following equation holds between Mises' equivalent stress and the maximum principal stress in plane stress condition;

$$\sigma_{eq} = \sigma_1(\lambda^2 - \lambda + 1)^{1/2}, \quad (\lambda = \frac{\sigma_2}{\sigma_1}). \quad (3)$$

Now we employ the following power law type constitutive equation;

$$\sigma_{eq} = B\epsilon_{eq}^n. \quad (4)$$

Combining equations (1)-(4) yields;

$$\begin{aligned} \epsilon^* &= [aB\epsilon_{eq}^n(\lambda^2 - \lambda + 1)^{-1/2}(2 - \lambda)^m]^{1/n} \\ &= C'\epsilon_{eq}[(\lambda^2 - \lambda + 1)^{-1/2}(2 - \lambda)^m]^{1/n}, \end{aligned} \quad (5)$$

where $C' = (aB)^{1/n}$.

As the ratio of the principal stress in plane stress is $\epsilon_1 : \epsilon_2 : \epsilon_3 = 1 : \varphi : -\nu/(1-\nu)(\varphi+1)$ from no volume change in plastic deformation, Mises' equivalent strain can be equated as,

$$\epsilon_{eq} = \frac{2}{\sqrt{3}}\epsilon_1(\varphi^2 + \varphi + 1)^{1/2}, \quad (6)$$

assuming that Poisson's ratio is 0.5 in plastic range. φ is the ratio of principal strains and $\varphi = \epsilon_2/\epsilon_1$. Substituting Eq.(6) into Eq.(5) yields;

$$\begin{aligned} \Delta\epsilon^* &= C\Delta\epsilon_1(\varphi^2 + \varphi + 1)^{1/2} \\ &\quad \times [(\lambda^2 - \lambda + 1)^{-1/2}(2 - \lambda)^m]^{1/n}, \end{aligned}$$

$$\text{and} \quad C = \frac{2}{\sqrt{3}}C', \quad (7)$$

after changing the stress and strain amplitudes to their ranges. We can replace the stress ratio λ by strain ratio φ using the flow rule of Mises' type, i.e., $\lambda = (2\varphi + 1)/(\varphi + 2)$. Thus the new equivalent strain is finally written as,

$$\begin{aligned} \Delta\epsilon^* &= C\Delta\epsilon_1(\varphi^2 + \varphi + 1)^{1/2} \left[(\varphi + 2) \right. \\ &\quad \left. / (3(\varphi^2 + \varphi + 1))^{1/2} \left(\frac{3}{\varphi + 2} \right)^m \right]^{1/n}. \end{aligned} \quad (8)$$

The constant C is determined such that $\Delta\epsilon^*$ reduces to $\Delta\epsilon_1$ in push-pull tests. For example, the value of C is 0.55 in the case of $m=0.5$ and $n=0.47$.

Figs.8 (a) and (b) are arrangements of the strain controlled biaxial low cycle

Fig.8 Arrangement of the biaxial low cycle fatigue data by equivalent strain on COD criterion.

Fig.9 Arrangement of the biaxial low cycle fatigue data by equivalent strain on COD criterion [5].

fatigue data of the same type specimen as this paper at 823 K[12] and 923 K[18] in air. The material is type 304 stainless steel and test frequency is 0.1 Hz. Values of the constants m , n and C are shown in the figure. The figure says that $\Delta\epsilon^*$ can arrange the biaxial fatigue data with a wide range of ϕ in a factor of two band.

Fig.9 is also an arrangement of the low cycle fatigue data of Blass and Zamrik [5] by the equivalent strain, $\Delta\epsilon^*$, at (a)

Fig.10 Arrangement of the biaxial low cycle fatigue data by equivalent strain on COD criterion [4].

922 K, (b) 811 K and (c) room temperature. The specimen is a smooth hollow cylinder of type 304 austenitic stainless steel. Almost the data all into a factor of two band but at 922 K. The scatter of the data at 922 K in the original paper is large and the authors of that paper mentioned that the experiment at the temperature was not good which may reflect the scatter of the arrangement using $\Delta\epsilon^*$. Thus the equivalent strain is applicable not only to the notched specimens but also to the smooth specimens.

Furthermore Fig.10 is an arrangement of the biaxial low cycle fatigue data by the equivalent strain $\Delta\epsilon^*$ for hollow cylindrical smooth specimens of AISI 316 stainless steel by Brown and Miller [4]. Total strain controlled tests were made at the shear strain rate of 1.5×10^{-3} /s at 673 K in air. The data come inside the factor-of-two band, and which means the equivalent strain $\Delta\epsilon^*$ is also applicable for this case.

4. Conclusions.

- (1) Maximum principal stress and Mises' equivalent stress cannot properly arrange the biaxial low cycle fatigue data under principal stress controlled conditions at elevated temperature.
- (2) The cause of poor arrangement is that the principal stress criterion does not include an effect of the parallel stress to the crack which has the effect to increase the crack propagation rate while Mises' equivalent stress does include the effect of the parallel stress too much.
- (3) A new equivalent stress criterion, based on the COD concept and developed by FEM analysis, can arrange the biaxial low cycle fatigue data properly.
- (4) A new equivalent strain is also derived from the equivalent stress. The equivalent stress is also applicable to the strain controlled biaxial low cycle fatigue data at room and elevated temperatures.

Acknowledgment

The authors wish to thank Professor Y. Awaya for the preparation of the specimen and also thank to Mr. T. Matsumoto, Mr. M. Yamagami and Mr. I. Yoshida for carrying out the experiments.

References

- [1] Ellison, E. G. and Andrews, J. M., J. Strain Analysis, Vol.8, No.3 (1973), 209.
- [2] Brown, M. W. and Miller, K. J., Proc. Inst. Mech. Eng., Vol.187 (1973), 745.
- [3] Krempl, E., ASTM STP, 549 (1974).
- [4] Miller, K. J., Conf. Mech. and Physics of Fracture, Institute of Physics, (1975), 287.
- [5] Blass, J. J. and Zamrik, S. Y., Symp. on Creep Fatigue Interaction, ASME, (1976), 129.
- [6] Ohnami, M. and Sakane, M., Bull. JSME, Vol.43, No.375 (1977), 3955.
- [7] Kanazawa, K. et al., Trans. ASME, J. Eng. Mater. Tech., Vol.99, No.3 (1977), 222.
- [8] Manson, S. S. and Halford, G. R., Trans. ASME, Ser. H, Vol.99 (1977), 283.
- [9] Brown, M. W. and Miller, K. J., Fatigue Engng. Mater. Struct., Vol.2, No.1 (1980), 19.
- [10] Lohr, R. D. and Ellison, G. R., Fatigue of Engng. Mater. Struct., Vol.3, No.1 (1970), 1.
- [11] Lohr, R. D. and Ellison, G. R., Fatigue of Engng. Mater. Structures, Vol.3, No.1 (1970), 19.
- [12] Ohnami, M. and Hamada, N., Proc. 25th Japan Congr. Mater. Res. (1982), 93.
- [13] Lefebvre, D., et al., Trans. ASME, J. Eng. Mater. Technol., Vol.103, No.1 (1981), 165.
- [14] Ellyin, F. and Valaire, B., Trans. ASME, J. Eng. Mater. Technol., Vol.103, No.3 (1981), 165.
- [15] Brown, M. W. and Miller, K. J., ASTM STP 770 (1982), 482.
- [16] Ohnami, M. et al., ASTM STP 853 (1985), 622.
- [17] Int. Symp. on Biaxial/Multi-axial Fatigue, ASTM (1982).
- [18] Hamada, N., Sakane, M. and Ohnami, M., Preprint of the 31th Annual Meeting of the Society of Mater. Sci., Japan (1982), 273 (in Japanese).
- [19] Cases of ASME Boiler & Pressure Vessel Code Case N-47, Class 1, Section III, Division 1, ASME
- [20] Hopper, C. D. and Miller, K. J., J. Strain Analysis, Vol.12, No.1 (1977), 23.
- [21] Hutchinson, J. W., J. Mech. Phys. Solid, Vol.16 (1968), 13.