Elastohydrodynamic Lubrication between Two Rollers
(Finite Width Analysis)

by Shigeaki KURODA ** and Kazuyoshi ARAI ***

The three dimensional EHL problem between two elastic rollers with finite width is analyzed numerically using finite element method and the Newton-Raphson method. The effects of side leakage flow on pressure distribution and fluid film profile are considered and the following results are obtained.
1. Near the center line of roller, pressure distribution and fluid film shapes are resemble to that of the infinite width case.
2. The maximum pressure appears near the side edge of roller.
3. The minimum film thickness is smaller than that of obtained by infinite width roller model.
The convergency is quite severe and the results obtained in this study are limited to several cases. Further numerical calculations are needed to analyze the finite width EHL problem.

key words: Lubrication, Elastohydrodynamic Lubrication, Finite Width, Elastic rollers, Pressure Spike, Constriction of Fluid Film.

1. Introduction

The basic theory of elastohydrodynamic lubrication (EHL) was established by Dowson and Higginson [1]. Since then many investigators have published a lot of papers concerning on the EHL problems between two elastic rollers but the effects of side leakage flow was neglected. In a lubricant fluid, however, fluid pressure decreases sharply toward the side edge of rollers and side flow occurs near the edge of rollers.

The purpose of this paper is to analyze the effects of side flow on the pressure distribution and fluid film profile in EHL problem between finite width rollers. The deformation of rollers is evaluated three dimensionally using finite element method. The Reynolds equation is also formulated by finite element method. These two equations are combined and solved using the Newton-Raphson method.

In this paper, the pressure spike and the constriction of film thickness are analyzed for finite width EHL problem, and their distribution in axial direction are presented. The infinite width EHL problem is also calculated and compared to the results of the finite width EHL problem.

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2. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>width of Hertzian contact zone</td>
</tr>
<tr>
<td>D</td>
<td>matrix related to pressure and displacement</td>
</tr>
<tr>
<td>E</td>
<td>equivalent elastic modulus</td>
</tr>
<tr>
<td>f</td>
<td>load per unit length of roller</td>
</tr>
<tr>
<td>G</td>
<td>material parameter G = a-p</td>
</tr>
<tr>
<td>h</td>
<td>lubricant film thickness</td>
</tr>
<tr>
<td>h0</td>
<td>film thickness on center line of roller with no deformation</td>
</tr>
<tr>
<td>H</td>
<td>dimensionless film thickness H = h/h0</td>
</tr>
<tr>
<td>L</td>
<td>width of roller</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
</tr>
<tr>
<td>P</td>
<td>dimensionless pressure P = p/E</td>
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<tr>
<td>R</td>
<td>effective radius of roller pair</td>
</tr>
<tr>
<td>R1</td>
<td>external radius of elastic roller</td>
</tr>
<tr>
<td>R2</td>
<td>internal radius of elastic roller</td>
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<tr>
<td>u</td>
<td>surface velocity of roller</td>
</tr>
<tr>
<td>U</td>
<td>speed parameter U = u/ER</td>
</tr>
<tr>
<td>v</td>
<td>displacement of lubricating surface</td>
</tr>
<tr>
<td>w</td>
<td>load</td>
</tr>
<tr>
<td>W</td>
<td>load parameter W = w/ER</td>
</tr>
<tr>
<td>x</td>
<td>coordinate</td>
</tr>
<tr>
<td>X</td>
<td>dimensionless coordinate X = x/b</td>
</tr>
<tr>
<td>z</td>
<td>axial coordinate</td>
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<tr>
<td>Z</td>
<td>dimensionless axial coordinate Z = z/b</td>
</tr>
<tr>
<td>μ</td>
<td>viscosity</td>
</tr>
<tr>
<td>μ0</td>
<td>viscosity at ambient pressure</td>
</tr>
<tr>
<td>a</td>
<td>pressure exponent of viscosity</td>
</tr>
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</table>
3. Numerical Procedure

3-1. Computational model

In this paper an EHL problem between two elastic rollers with finite width are investigated numerically. The Reynolds equation and elastic equation are formulated and solved using finite element method and the Newton–Raphson method. Fig. 1 shows the geometry and the surface velocity of elastic rollers. The following assumptions are made for numerical analysis.

1. The two elastic rollers have same configuration with width L and external radius R1. The rollers are fixed to rigid axis at internal radius R2. The ratio of R2/R1 is 0.6.
2. The two rollers revolute with surface velocity u under external load w.
3. The equivalent elastic modulus is E.
4. The lubricating fluid is incompressible Newtonian fluid and no inertia force is considered.
5. The phenomena are isothermal.
6. Fluid viscosity varies with fluid pressure.

3-2. Reynolds Equation

The lubricating fluid satisfies the following Reynolds equation.

\[
\frac{\partial}{\partial x} \left( \frac{1}{12 \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{12 \mu} \frac{\partial p}{\partial z} \right) = \frac{\partial}{\partial x} (\mu u) \tag{1}
\]

The second term of left hand side of equation (1) is related to side flow and this term is neglected in conventional analysis of infinite width model. The lubricated region is a rectangular as shown in Fig. 2.

![Fig.1 Configuration of Model](image)

![Fig.2 Lubricating Region](image)

The following four boundary conditions are introduced.

1. At the inlet line, \( p = 0 \).
2. At the outlet line which coincides with the Hertzian contact region, \( p = 0 \).
3. At the center line of roller, \( \partial p / \partial r = 0 \).
4. At the side line, \( p = 0 \).

The boundary condition considering cavitation which denotes that at the outlet region \( \partial p / \partial x = 0 \), should be introduced for accurate calculation. In this study, however, above mentioned simple condition is employed at the outlet line, for it is necessary to satisfy the convergency condition at hundreds of point in the lubricating region.

The lubricated region is divided by 63 nodes in X direction and 8 nodes in Z direction. The Reynolds equation is transformed into a set of simultaneous linear equations by using finite element method with 468 triangle elements [2].

The relation between fluid pressure and fluid viscosity is described as

\[
\mu = \mu_0 \exp(\alpha p) \tag{2}
\]
3-3. Film Thickness and elastic equation

The deformation of elastic rollers by fluid pressure are calculated three dimensionally by using finite element method. The length of lubricating region is only several times of Hertzian contact length, and within this quite narrow region several hundreds of nodal points of finite element method are located. The two types of three dimensional cubic element, that are ten-node-element and eight-node-element, are used as shown in Fig.3. By combining these two types of element, the rollers are devided quite finelly near the surface and relatively roughly near the center axis.

On analyzing EHL problem, only the deformation of lubricated surface are needed and the deformation of inside the elastic body is not needed [3]. Considering the axisymmetry of roller, the roller are devided by substructure shown in Fig.4. By composing the adjacent two substructure and eliminating internal nodes [4], the following relation between fluid pressure \( p \) and displacement \( v \) at each node in lubricating surface are obtained.

\[
[v] = [D] \cdot [p] \quad \quad \quad \quad \quad (3)
\]

The fluid film thickness at nodal point \( i \) in the lubricated region is described by

\[
h_i = h_0 + x_t/2R + 2v_i = h_0 + x_t/2R + 2 \sum_{j=1}^{n} D_{ij} p_j \quad \quad \quad \quad \quad (4)
\]

By substituting Eq.(4) into Eq.(1), fluid equation and elastic equation are combined.

3-4. External load

The external load \( w \) is applied to the two rollers. The applied load \( w \) can be be expressed as

\[
w = f \cdot L = \int p \cdot ds \quad \quad \quad \quad \quad \quad (5)
\]

where \( s \) is lubricating area.

3-5 Convergency calculation

Substituting Eq.(2) and Eq.(4) into finite element formulation of Eq.(1) and using Eq.(5), simultanious non-linear equations are obtained. The non-linear equations are solved by the Newton-Raphson method. The convergency condition in Newton Raphson method is \( E < 10^{-6} \).

4. Results and discussions

Before discussing the results of the finite width EHL problem, the results of two dimensional infinite width EHL problem which are obtained by present method are shown to compare with the existing results.

Fig.5 and Fig.6 show the pressure distribution and fluid film profile in the case of infinite width elastic roller. The pressure spikes and the constriction of fluid film are observed at the outlet region. As the speed parameter \( U \) decreases, the location of pressure spike moves backward and film thickness becomes thinner with the apparent constriction of fluid film at the outlet region. These results are fairly coincide with the results shown in

![Fig.4 Sub-Structurer](image)

![Fig.5 Pressure Distribution (Infinite Width Model)](image)

![Fig.6 Film Profile (Infinite Width Model)](image)
the reference [1]. The slight differences between the present results and the results shown in the reference [1] may be based on the fact that in the reference [1], the elastic deformation is calculated for infinite half plate while in this study the deformation is calculated by using finite element method for elastic roller with rigid axis as shown in Fig.1.

Fig.7, Fig.8 and Fig.9 illustrate the pressure distribution in the case of finite width elastic rollers. In Fig.7 where material parameter G = 1000, no pressure spike is appeared and only slight sign of spike is observed. As G increases, the pressure spike becomes apparent, as shown in Fig.8 and Fig.9. In the axial direction, the pressure is constant near the center line of the roller. At slightly inside the side edge the pressure has its peak value with sharp pressure gradient to the side edge of the roller. This tendency is observed also for the pressure spike distribution in the axial direction.

Fig.10 shows the fluid film profile corresponding to the pressure distribution shown in Fig.8. The fluid film profile in the X direction near the center line of rollers are resemble to that of the conventional two dimensional results. The film thickness is nearly constant at the center region and in the outlet region the constriction of film thickness is observed with the appearance of pressure spike. The constriction of film thickness is obvious in center region but is not observed near the side edge of roller. The film thickness does not change both in the X and Z direction in the center region and gradually decreases near the side edge line.

Fig.11 and Fig.12 illustrate the distribution of pressure and film thickness in the axial direction at five different cross sections. Fig.11 corresponds to Fig.9 and Fig.10 with material parameter G = 2000. In both cases the distributions of pressure and film thickness are resemble. It can be found from the figure that at every cross section the pressure is nearly constant in center region and has its peak value near the side edge of roller. The location of the maximum pressure at each cross section is found most outward at X=0 and shifts to inward near outlet region. The remarkable pressure increase in the axial direction is found at the cross section where pressure spike is found.

The fluid film thickness in axial direction is constant in center region and gradually decreases near the edge of roller. The ratio of fluid film thickness at the center line and at the edge line differs with X coordinate. The location of peak pressure in axial direction at X = 0.75 is coincide with the point where the fluid film thickness begins to decrease.
Fig. 11 Pressure and Fluid Film Thickness in Axial Direction (G = 2000)

1: Infinite Width
2: Finite Width

Fig. 12 Pressure and Fluid Film Thickness in Axial Direction (G = 2300)

1: Infinite Width
2: Finite Width

Fig. 13 Pressure distribution

Fig. 14 Fluid Film Thickness
The pressure peak in axial direction becomes remarkable as \( G \) increases. The convergency is quite severe and no solution is obtained for \( G = 2400 \). Further numerical analysis is needed to discuss this pressure peak in axial direction. Fig.13 and Fig.16 show the pressure distribution and film shape of finite width EHL problem and infinite width EHL problem for same condition. In the finite width case the pressure and film shape at the center line are shown. As the pressure at the edge line of finite width roller is \( p = 0 \), the pressure at the center line of finite roller is higher than the pressure of infinite case to bear the same load capacity. The film thickness of finite width case is smaller than that of the infinite width case.

5. Conclusions

The EHL problem between two finite width elastic rollers is investigated numerically. The convergency is severe and the obtained results are limited to several cases. Further numerical analysis must be made but the following results are obtained in this study.

1) The pressure profile and film shape in the X direction near the center line of elastic roller where the effects of side leakage are insignificant, is resemble to that of infinite width case. The pressure spike, the constriction of the film thickness in the outlet region and the constant film thickness region are observed. Higher pressure and thinner film shape are observed, compared with infinite width case, owing to the pressure drop in the axial direction.

2) Near the edge of roller, sharp pressure drop is observed in the axial direction. The pressure distribution in the axial direction shows its peak value near the edge of roller, just inside the pressure drops begins. The peak pressure in axial cross section increases as \( G \) increases.

3) The film thickness at the side edge of roller is smaller than that of at the center line of roller. The minimum film thickness is smaller than that of obtained by conventional infinite width theory.

6. Reference