Prediction of Outlet Flow Characteristics of Centrifugal Impellers

(1st Report, Consideration of Velocity Distortion)

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The analytical method for predicting the flow characteristics at the outlet of centrifugal impellers is presented taking into account the influence of velocity distortion in both the peripheral and the hub-to-shroud directions and its effectiveness was confirmed with the flow measurements around pump impellers.

The results show that the increase of wall friction due to a mixing of flow just after the impeller exit should be adequately considered for successful prediction, and the influence of velocity distortion in the hub-to-shroud direction should also be considered for the case of the parallel-walled diffuser width being larger than the impeller exit width, while it is negligible in the case of the diffuser width being equal to the latter.

It is pointed out that the peripheral velocity component of the outlet flow used for the determination of slip factor and the theoretical head should be defined by the mass-averaged value of the section.

Key Words : Fluid Machine, Centrifugal Impeller, Absolute Flow, Slip Factor, Parallel-Walled Diffuser, Velocity Distribution

1. Introduction

It is still difficult to predict the outlet flow characteristics from a centrifugal impeller for a wide range of flow-rates with sufficient accuracy, as the flow is periodic with a pitch of blade-to-blade velocity variation and the sectional velocity distribution is highly distorted in the hub-to-shroud direction.

As for the influence of velocity distortion in the peripheral direction due to the blade-to-blade flow and the blade wake flow, Dean and Senoo⁽¹⁾ have succeeded in analyzing the loss mechanism and predicting the total pressure loss. Ishida and Senoo⁽²⁾ have examined the mixing process and total pressure loss, and given reasonable interpretations to the mixing mechanism. Johnston and Dean⁽³⁾ have presented a simple prediction method of total loss due to a flow mixing.

On the other hand, the influence of hub-to-shroud velocity distortion at the impeller outlet has also been treated by Senoo and others⁽⁴⁾ and the velocity distortion in the downstream parallel-walled channel is analyzed.

Nevertheless, it is still unknown how the velocity distortion and the total pressure loss at the impeller outlet vary for a wide range of variations of flow-rates in connection with the impeller performances. This study is aimed at establishing an analytical method for predicting the outlet flow characteristics and the performances of a centrifugal impeller for

a wide range of flow-rates. In the first report, the prediction method of the outlet flow characteristics is presented with the influence of velocity distortion taken into consideration. From a viewpoint of practical application, the flow is treated as quasi-steady with time-averaged velocity.

Nomenclature

- b : channel width
- Cf : wall friction coefficient
- k : slip factor defined as $=(v'_{2m}-v'_{2})/u_2$
- r : radius, $R=r/r_2$
- u : peripheral velocity of impeller
- v : velocity of absolute flow from the impeller outlet
- z : coordinate in the transverse direction of the channel
- α : angle of absolute flow measured from peripheral direction
- β : blade angle measured from peripheral direction
- ψ : total pressure coefficient $=H/(u_2^2/g)$
- ϕ : flow-rate coeff. $=\overline{v_{m2}}/u_2=Q/(2\pi r_2 b_2 u_2)$
- η_v : volumetric efficiency
- ε : reduction-rate of channel width due to blade thickness
- τ : shearing stress of wall

Subscripts:

- 1: impeller inlet
- 2: impeller outlet (prime refers to just before impeller outlet)
- m: meridian component
- θ : peripheral component
- A: sectional average of A , $=\int_0^b A dz/b$
- $\overline{A}$: mass average of A , $=\int_0^b A v_m r dz / \int_0^b v_m r dz$

3. Theory

It is necessary to quantify the influence of velocity distortion in the

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blade-to-blade and hub-to-shroud direction in order to determine the impeller outlet flow characteristics for a wide range of flow-rates with satisfactory accuracy.

As for the blade-to-blade velocity distortion, the measurements⁽⁵⁾ of a blade-to-blade channel flow in a rotating measuring system have revealed that the flow becomes uniform within the Δr -region between the impeller exit and $1.1r_2$. On the other hand, the instantaneous measurements of an impeller outlet flow have shown that the Δr -region is about 20 percent of impeller radius for blower blades⁽²⁾ and 15-20 percent for pump impeller⁽⁶⁾. The literature⁽²⁾ states that the blade-to-blade velocity distortion is made uniform mainly due to a reversible work exchange between the pressure and the velocity heads, and that the total loss created at the impeller outlet (inlet region of the parallel-walled diffuser) is mainly due to the wall friction of a non-uniform flow.

Based on the above-mentioned results, we propose the following flow model of a parallel-walled channel downstream of the impeller exit;

(1) The flow becomes immediately uniform in the blade-to-blade direction just after the impeller exit. Here the hub-to-shroud velocity distortion is also taken into consideration adequately as described later.

(2) Just after the impeller exit there exists a mixing region Δr in which the wall friction is much larger than that in the case of a uniform flow.

3.1 Impeller outlet flow and theoretical head

The theoretical head of a centrifugal impeller is determined by the balance of angular momentum as follows, if the flow has no pre-rotation at the impeller inlet:

$$H_{th} = \frac{u_2}{g} \int_0^{b_1} r_2 v_{m1} v_{s1} dz / \int_0^{b_1} r_2 v_{m1} dz = \frac{u_2 \bar{v}_{s1}}{g} \quad (1)$$

where the impeller exit radius is treated as constant, and $\bar{v}_{s1}$ is the mass-averaged peripheral velocity component at the impeller exit. The equation (1) shows that Euler head gives a theoretical head even for a highly distorted flow in the hub-to-shroud direction only when the mass-averaged value is used for the peripheral velocity and the impeller radius is constant over the whole impeller exit.

The peripheral velocity component v_s of an absolute flow just after the impeller exit becomes much smaller than that in the no-slip outlet condition due to the flow circulation around a blade. The slip velocity has been confirmed⁽⁵⁾ to be well calculated by the Busemann theory, which analyzes the impeller flow with logarithmic spiral blades of zero thickness.⁽⁷⁾ Here we assume that the tangential velocity of the outlet flow is given as follows using the Busemann factor and the meridian velocity component v_{m2} just before the impeller

outlet with the blade thickness taken into consideration;

$$\bar{v}_{s2} = (1-k)u_2 - \bar{v}_{m2} \cot \beta_2 \quad (2)$$

Then the theoretical head coefficient of the impeller is given as follows;

$$\psi_{th} = 1 - k - \phi \cot \beta_2 / \eta_0 \epsilon_2 \quad (3)$$

3.2 Flow in the parallel-walled diffuser

The angular momentum balance of Δr -region is described as follows;

$$\frac{d}{dr} (2\pi \rho \int_0^b r^2 v_m v_s dz) = -2(2\pi r^2 \tau_s) \quad (4)$$

In the centrifugal channel r is perpendicular to z and $rv_m = \text{const.}$ from the equation of continuity. Besides, the shearing stress component of the wall in the peripheral direction is expressed as $\tau_s = C_f \rho v_{max} v_{smax} / 2 \approx C_f \rho \bar{v}_{smax}^2 / 2$. Then equation (4) is simplified as follows.

$$\frac{d}{dr} (r \bar{v}_s) = -C_f r \bar{v}_{smax}^2 / b \bar{v}_m \quad (5)$$

where the wall friction coefficient C_f is known⁽²⁾ to be much larger in Δr -region due to the non-uniformity of the flow than that of a fully developed boundary layer flow, C_{f0} , in the downstream region. In Fig. 1, the measured results in the literature⁽²⁾ for the flow angle $\alpha_2 = 20^\circ$ are re-arranged and reproduced as the variation of the friction ratio $m = C_f / C_{f0}$ with the radius ratio $R = r / r_2$ together with the mean stream line distance ratio x / r_2 . As the ratio m changes, in general, with the mean stream angle α_2 of the absolute flow, the relation of m with x / r_2 in Fig. 1 is generalized for an arbitrary flow angle α_2 to give the following empirical formula.

$$m = \begin{cases} 0.751(0.341(R-1)/\sin \alpha_2 + 0.0240)^{-1/3} & (R \leq 1 + 1.17 \sin \alpha_2) \\ 1.0 & (R \geq 1 + 1.17 \sin \alpha_2) \end{cases} \quad (6)$$

And for the friction coefficient of a fully developed parallel-walled channel flow it is convenient to use the following empirical formula⁽⁸⁾ obtained from the flow measurements of a rotating disk enclosed in a housing.

$$C_{f0} = 0.0925 / Re^{1/5}, \quad Re = v_{smax} r / \nu \quad (7)$$

Substituting Eqs. (6) and (7) into Eq. (5) leads to a differential equation for

Fig. 1 Increase of friction coefficient at the inlet region of parallel-walled diffuser due to a flow mixing⁽²⁾.

$\bar{v}_\theta$ and $v_{\theta\max}$. The ratio $\bar{v}_\theta / v_{\theta\max}$ or $\bar{v}_\theta / \bar{v}_s$ is a measure of the velocity distortion of the section, and from the present measurements described later are given the following empirical equations for wide variations of flow-rate, radius and impeller angle.

$$\bar{v}_\theta = 0.880 v_{\theta\max} \quad \dots\dots\dots (8)$$

$$K \equiv \frac{\bar{v}_\theta}{\bar{v}_s} = \begin{cases} 0.812\phi^{-0.114} (b=b_2) \\ 0.706\phi^{-0.214} (b=1.88b_2) \end{cases} \quad \dots\dots\dots (9)$$

From Eqs.(8) and (9) the velocity distortion rate $K' \equiv \bar{v}_\theta / v_{\theta\max} = 0.880K$ is described as a function of ϕ . Substituting Eqs.(6),(7),(8) and (9) into Eq.(5) and integrating it, the following simple relation is obtained;

$$\frac{r\bar{v}_\theta}{r_2\bar{v}_{\theta 2}} = \left(1 + \frac{4}{5} \frac{C_{f02}}{K'^2} \frac{r_2\bar{v}_{\theta 2}}{b\bar{v}_{m2}} A\right)^{-5/4} \quad \dots\dots\dots (10)$$

where $A \equiv \int_1^R m dR$ is given as follows

$$A = \begin{cases} 1.86 \sin \alpha_2 (1/m^2 - 0.148) & (R \leq 1 + 1.17 \sin \alpha_2) \\ 1.58 \sin \alpha_2 + R - 1 & (R > 1 + 1.17 \sin \alpha_2) \end{cases}$$

and C_{f02} is given by putting $r=r_2$ in Eq.(7).

3.3 Hydraulic losses

The hydraulic loss from the inlet of a centrifugal impeller to the parallel-walled diffuser consists of the deceleration losses at the impeller inlet and in the impeller channel, the friction losses in the impeller channel and in the parallel-walled diffuser and the mixing loss at the diffuser inlet. Among them the losses in the impeller should independently be discussed in another study, and here, for the sake of simplicity, we use the conventional one-dimensional calculation⁽⁹⁾ with a modification of the volumetric efficiency η_v .

As for the losses in the parallel-walled diffuser channel, the total pressure loss due to the mixing of flow is given, as described above, by the increment of the wall friction loss due to the non-uniformity of flow in Δr -region. Then the radial variations of the static pressure and the total pressure loss are to be determined by the momentum balance in the radial direction, but the results becomes very complicated and not practical, when the velocity distortion in the hub-to-shroud direction is taken into consideration. In pumps and blowers in practical use Δr -region is generally very narrow, as

a volute casing or diffuser vanes are mounted around the impeller. Accordingly, we make the following one-dimensional calculation of loss in the parallel-walled diffuser.

$$\gamma Q h_f = 4\pi \int_{r_2}^r \tau \bar{v} r dr \approx 2\pi \int_{r_2}^r m C_{f0} v_{\theta\max}^2 \bar{v}_\theta r dr \quad \dots\dots\dots (11)$$

4. Experimental Apparatus and Procedures

The test stand shown in Fig.2 consists of a vertical shaft centrifugal pump impeller and a parallel-walled diffuser channel. The absolute flow at the impeller outlet might be influenced mainly by the shape, dimension and surface roughness of an impeller and a diffuser channel, and in the present report the influences of the blade outlet angle β_2 , the diffuser width b and the surface roughness of the impeller are studied. In the case of the diffuser channel width being equal to that of the impeller (called "two-dimensional diffuser"), the velocity and the pressure distributions of the outlet flow from the impeller were measured with three different configurations of the impeller, that is, Impellers A, A' and B as shown in Table 1. Then for the case of the diffuser width being different from that of the impeller, that is $b=1.875b_2$ (called "three-dimensional diffuser"), the velocity and the pressure measurements were also performed. The asymmetry of pressure and flow angle at the mid-height of the channel around the impeller was about $\pm 1\%$ of the averaged values at most. The Reynolds number based on the impeller tip speed and diameter was $Re = 2u_2 r_2 / \nu = 1.8 \times 10^6$, the flow-rate Q ranging from 0.14 to 1.10 m^3/min .

The volumetric efficiency η_v of the impeller was calculated by considering the angular momentum balance at the back space of the impeller.⁽¹¹⁾ The results show that $\eta_v=0.95$ at $\phi=0.10$ and $\eta_v=0.89$ at $\phi=0.03$ for each of the three impellers. These values affected little the calculation of ψ_{th} in Eq.(3).

Fig. 2 Test stand of vertical shaft type⁽¹⁰⁾.

Table 1 Dimensions of impellers used.

Impeller	Ns	r_2	r_1	β_2	β_1	Z	b_2	b_1	surface
A	200	130	65	22.5°	14°	8	16.0	28.0	smooth
A'	220	130	65	22.5°	14°	8	16.0	30.0	40S
B	215	130	65	45°	18°	10	16.0	28.0	smooth

(mm)

Ns denotes the specific speed at b.e.p. [m, m^3/min , rpm], z is the blade number and "smooth" shows that the blade-to-blade channel is machined.

5. Measured Results and Comparison with Theory

5.1 Velocity distortion and slip factor

The measured velocity profiles of the diffuser channel are illustrated in Fig.3 at several radial positions, and for several flow-rates in Fig.4. Figure 3 shows that the velocity profile at low flow-rates is much different from that at high flow-rates and is characterized by a concave configuration and the development of a three-dimensional reverse flow near the wall at low flow-rates, while it is rather flat at high flow-rates and there is no reverse flow. In both cases, the velocity profile changes little in the radial direction except that the deviation of the maximum meridian velocity v_m to the upper wall increases with radius at high flow-rates.

Figure 4(a) shows that the velocity profile at $r=1.1r_2$ changes largely with the flow-rate and the change of the meridian velocity profile is especially remarkable.

Above mentioned results are for the case of a two-dimensional diffuser ($b=b_2$), but in the case of a three-dimensional one, as shown in Fig.4(b), a three-dimensional reverse flow near the wall is seen to become larger at all flow-rates. This is due to a sudden expansion of the channel at the impeller exit.

When defining the Euler head and the slip factor from the impeller outlet flow with such distorted velocity profile, it has not been clear which one should be used as the peripheral velocity component v_θ , the sectional averaged or the mass-averaged or the maximum value. But according to Eq.(1) it is seen that the mass-averaged value should be used in centrifugal impellers. The difference between the mass-averaged and the sectional averaged values is illustrated in Fig.5 for the case of impellers A and B. In both impellers the mass-averaged values $\bar{v}_\theta$ and $\bar{Pr}$ are 10-20 % larger than the sectional averaged values.

To make it clearer, the velocity distortion ratios $\bar{v}_\theta/\bar{v}_s$ and $\bar{v}_\theta/v_{\theta\max}$ are plotted against the flow coefficient in Fig.6. The notation A'-3 in the figure denotes the combination of A'-impeller and three-dimensional diffuser. It is noticed that the velocity distortion ratio $\bar{v}_\theta/\bar{v}_s$ or $\bar{v}_\theta/v_{\theta\max}$ depends on the flow-coefficient and the expansion rate of the channel width b/b_2 at the impeller outlet, is independent of radius ratio r/r_2 and impeller outlet angle β_2 , and is expressed as the empirical equation (9). The ratio $\bar{v}_\theta/v_{\theta\max}$ is almost constant and independent of r , ϕ , β_2 , b/b_2 and wall roughness, and is given by Eq.(8). The influence of b/b_2 will be further examined in the succeeding report.

(a) At low flow-rates
Fig. 3 Velocity distribution in parallel-walled diffuser

Fig. 5 Comparison of sectional averaged and mass-averaged values of v_θ and Pr

(a) Two-dim. diffuser
Fig. 4 Velocity distribution in the parallel-walled diffuser and change of its profile with flow-rate.

Fig. 6 Distortion rate of velocity profile and its variation with flow-rate.

Fig. 7 Comparison of slip factor calculated from sectional averaged velocity with that from mass-averaged velocity.

As for the slip factor $k \equiv (\bar{v}_{\theta 2\infty} - \bar{v}_{\theta 2})/u_2$ in Eq.(2), Wiesner⁽¹²⁾ has presented an empirical equation which gives a closer value to the Busemann factor. In order to examine the slip factor from the measured velocity at the impeller outlet, it is necessary to consider the sudden change in the flow characteristics and the velocity distortion at the impeller outlet. As equations (4) and (5) are valid even for the distorted velocity profile, it is possible to estimate the mean flow just before the impeller outlet by Eq.(10), which includes the influences of hub-to-shroud and blade-to-blade velocity distortions. The calculated results of the slip factors k and $\mu = \bar{v}_{\theta 2}/\bar{v}_{\theta 2\infty}$ using the measured velocity distributions at $R=1.1$ are shown in Fig. 7, in which the straight lines are the result using the Wiesner formula. It is seen that the slip factor k calculated using the mass-averaged value $\bar{v}_\theta$ gives good correlation to the Wiesner formula, and the slip factor calculated using the sectional averaged value $\bar{v}_\theta$ is only 70-85 % of the value obtained from the mass-averaged value $\bar{v}_\theta$.

The variation of velocity vectors just before the impeller outlet for the variation of flow-rates is illustrated in Fig. 8, in which the chain dotted lines

Fig. 8 Velocity vector of the absolute flow at the impeller exit.

neglect the influence of blade thickness at the impeller outlet. For the impeller of $\beta_2=22.5^\circ$ the peripheral velocity component varies largely with the flow-rate, but it varies little for $\beta_2=45^\circ$.

5.2 Flow in the parallel-walled diffuser and total pressure

The comparison of the calculated and the measured results of the flow angle $\alpha = \tan^{-1}(\bar{v}_m/\bar{v}_\theta)$ and the mass-averaged peripheral velocity is shown in Figs. 9, 10 and 11.

At low flow-rates the velocity distortion in the section is very large with a three-dimensional reverse flow near the wall and a large meridian velocity in the middle, as shown in Fig. 3(a), which results in a large variation of the flow angle in the section. But the flow angle calculated using the measured mass-averaged velocity is in good agreement with the theory as shown in Fig. 9, and increases largely with an increase of flow coefficient ϕ . The flow angle also varies in the radial direction due to the wall friction, but its rate is rather small compared with the variation with ϕ . This suggests that the mean streamline of Δ -region is well approximated by the logarithmic spiral if Δ -region is not too large and the flow-rate is not extremely high or low.

Figure 10 shows the variation of mass-averaged peripheral velocity $\bar{v}_\theta$ for wide variations of flow-rates. The theoretical

Fig. 9 Flow angle and its variation with flow-rate and radius.

Fig. 10 Mass-averaged peripheral velocity component and its variation with flow-rate and radius.

(a) Influence of impeller outlet angle.

(b) Influence of channel expansion rate.

Fig. 11 Influence of impeller outlet angle and expansion rate of the channel upon $\bar{v}_\theta - \phi$ relation.

results are seen to be in good agreement with the measured ones for $r/r_2=1.1$. But if the increase of the wall friction due to the non-uniform flow is neglected ($m=1$), the agreement becomes worse, as shown by the chain-dotted lines.

At large radii and low flow-rates the flow-angle α is comparatively small as shown in Fig.9, and accordingly the influence of wall friction becomes relatively large. In these regions the pre-sent theory gives less satisfactory results, which suggests that Eq.(7) of the wall friction based on one-seventh power law velocity assumption should be re-examined.

However, at small radii the present theory gives satisfactory results for different impeller angles and different expansion rates b/b_2 of the channel, as shown in Fig.11. In the case of a two-dimensional diffuser the present theory gives satisfactory results even if the distortion rate K' is approximated to unity, because the value of K' is nearly unity for all flow-rate range. But in the case of a three-dimensional diffuser, the theoretical results with the approximation of $K'=1$ deviate from the measured results,

as shown by the dotted line in Fig.11(b), which suggests that it is necessary to consider the velocity distortion rate K' when the diffuser width is larger than the impeller width.

Lastly, the performance curves of each impeller are shown and theoretical results about head coefficient are compared with the measurements in Fig.12. In Figs.12(a) and (b) each component of hydraulic loss is also shown, and the friction loss of Δr -region is seen to occupy large part of hydraulic loss. Moreover, the influence of velocity distortion rate is seen to be very small except for very low flow-rates in a two-dimensional diffuser. In a three-dimensional one the consideration of velocity distortion gives unexpectedly less satisfactory results, as shown by the dotted line in Fig.12(c). The reason is not yet clear.

In addition, it should be noted that a small difference in blade angle β_2 has noticeable influence on the calculated value of ψ_{th} at high flow-rates. Considering that the cutting of the blade outlet edge of the pressure side changes the pump performances⁽¹³⁾, the outlet angle of the

(a) Impeller B ($\beta_2 = 45^\circ$) (b) Impeller A ($\beta_2 = 22.5^\circ$)Fig. 12 Head coefficient measured at $R=1.1$ and its variation with flow-rate. (Efficiency η and shaft power coefficient v are measured data.)

Fig. 12(c) Head coefficient -- Impeller A'.

pressure side should be chosen as the blade angle in the calculation. In the present study the blade angle calculated from the measured blade thickness at the impeller outlet was used.

6. Conclusions

An analytical method for predicting the outlet flow characteristics from a centrifugal impeller is proposed with the hub-to-shroud and blade-to-blade velocity distortion taken into consideration, and its usefulness is confirmed by the measurements with pump impellers. The conclusions obtained are as follows;

- (1) In calculating the theoretical head coefficient and the slip factor from the measured velocity of the absolute flow at the impeller outlet, the mass-averaged velocity of the section should be used. The ratio of the mass-averaged velocity to the sectional one or to the maximum one of the section expresses the distortion rate of the velocity profile and is given by Eqs.(8) and (9).
- (2) It is necessary to consider the increment of the wall shearing stress near the inlet of the parallel-walled diffuser channel due to the non-uniform flow, in order to get satisfactory prediction of the

outlet flow characteristics.

(3) In the case of parallel-walled channel width being equal to the impeller outlet width, the influence of the velocity distortion in the blade-to-blade direction is small, but in the case of the former being larger than the latter its influence becomes remarkable.

(4) The rate of the wall friction loss in the mixing region just after the impeller outlet to the hydraulic loss is relatively large.

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