On the Effect of Trapped Air in a Liquid Conduit on the Transient Flow Rate*

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Trapped air in a liquid conduit has a great influence on the fluid transients. In this paper, especially its effect on the transient flow rate in a test pipeline is studied experimentally and analytically. Transient flow velocity, following quick opening of a valve attached at the downstream end, was measured. The experiments showed that the transient flow velocity often exceeds the final steady velocity. The numerical calculation by means of characteristics method showed a good agreement with the experiments. Maximum value of the transient flow velocity depends on the volume and the position of the trapped air, and the pressure loss of the pipeline and the valve, and the downstream pressure of the valve. Five dimensionless parameters, which represent above-mentioned factors respectively, are derived. And the relations between the maximum flow velocity and these five parameters are expressed graphically.

Key Words : Unsteady Flow, Fluid Transportation Systems, Trapped Air, Pipeline, Characteristics Method

1. Introduction

In a long pipe such as a liquid conduit or a pipeline, air which has entered the pipe or separated from the liquid often forms cavities. Such trapped air has a great influence on the fluid transients in the pipe, that is, it could cause not only an unusual high transient surge pressure but also a large transient flow velocity⁽¹⁾. In the previous papers, the authors have shown the influence of trapped air on the transient pressure in the pipe⁽²⁾⁽³⁾. In view of flow rate control, it is also important to clarify the influence of trapped air on the transient flow velocity. In this paper, such an influence is studied experimentally and analytically.

In the present experiments, we measured the transient flow velocity caused by quick valve opening at the downstream end of the line which contains trapped air. In the numerical analysis, the transient flow velocity is calculated by the characteristics method. It is shown that even a small volume of trapped air can cause a very large maximum transient flow velocity. And the relations between five parameters which describe the fluid transients in a fluid conduit containing trapped air and the maximum flow velocity are shown.

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Nomenclature

- a : velocity of pressure wave
- $B = a/g$
- C^+, C^- : names for characteristic lines
- c : orifice discharge coefficient
- D : pipe diameter
- d_n : diameter of opening of an orifice
- f : Darcy-Weisbach friction factor
- H : pressure head
- H_a : absolute pressure head of the atmosphere
- H_o : pressure head at the outlet of a valve at the downstream end of a pipe
- $H_o' = (H_o + H_a) / \{(a/g)V_\infty\}$
- H_s : pressure head of a constant pressure reservoir at the upstream end of a pipe
- $H_s' = (H_s + H_a) / \{(a/g)V_\infty\}$
- h_r : pressure head loss of a pipe per unit length
- L : pipe length
- N : number of reaches
- $R = fa/2gD$
- $R_f = \{fLV_\infty^2/(2gD)\} / \{(a/g)V_\infty\}$
- $R_o = \{(S/cS_n)(V_\infty^2/2g)\} / \{(a/g)V_\infty\}$
- S : sectional area of pipe
- S_n : area of opening of an orifice
- t : time
- $\Delta t = \Delta x/a$
- V : flow velocity
- V_∞ : final steady flow velocity
- $V' = V/V_\infty$
- $V_{\max}$: maximum flow velocity at the downstream end of a pipe
- $V'_{\max} = V_{\max}/V_\infty$
- W : trapped air volume
- W_s : initial trapped air volume
- x : position along pipe from the upstream end
- $\Delta x = L/N$
- x_o : position of the trapped air

$$x'_0 = x_0/L$$

α : ratio of initial trapped air volume to liquid volume in a pipe
 $= W_a/LS$

$$\beta = W_a / \{S(L/a)V_a\}$$

κ : ratio of specific heats

Subscripts

A, B, C, P: grid points

2. Numerical Analysis

2.1 Calculation model

In a long pipeline, even if it lies almost horizontally, there often exists a locally elevated portion, where air tends to be trapped. This paper treats the fluid transients caused by quick opening of the valve in a pipeline containing trapped air as shown in Fig.1. The valve opening time and the valve operating process have an influence on the fluid transients. In this paper, the fluid transients following a quick valve operation are studied, for the purpose of considering the influence of the existence of trapped air on the fluid transients.

2.2 Numerical analysis using the characteristics method

When the flow in a horizontal pipeline is one-dimensional and $V \ll a$, the equations of motion and continuity are as follows:

$$\frac{\partial V}{\partial t} + g \frac{\partial H}{\partial x} + gh_f = 0 \quad (1)$$

$$\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0 \quad (2)$$

These equations are transformed into four ordinary differential equations by the characteristics method. They are grouped and designated as C^+ and C^- equations⁽⁴⁾; i.e.,

$$C^+ \begin{cases} \frac{dH}{dt} + \frac{a}{g} \frac{dV}{dt} + ah_f = 0 & (3) \\ dx/dt = a & (4) \end{cases}$$

$$C^- \begin{cases} \frac{dH}{dt} - \frac{a}{g} \frac{dV}{dt} - ah_f = 0 & (5) \\ dx/dt = -a & (6) \end{cases}$$

where h_f denotes pressure head loss per unit length of the pipe.

For steady turbulent flow, h_f is

$$h_f = \frac{fV|V|}{2gD} \quad (7)$$

The line is divided into N equal reaches. The integration of Eq. (3) along the C^+ line between A and P, in Fig. 2, and a similar integration of Eq. (5) along the C^- line between B and P, yield

$$C^+ : H_P = C_P - BV_P \quad (8)$$

$$C^- : H_P = C_B + BV_P \quad (9)$$

where

$$C_P = H_A + BV_A - \Delta t RV_A |V_A|$$

$$C_B = H_B - BV_B + \Delta t RV_B |V_B|$$

H_P and V_P at each divided point in the pipe are found from Eqs. (8) and (9).

At the downstream end, the boundary condition with an orifice is

$$SV_P = cS_n \sqrt{2g(H_P - H_D)} \quad (t > 0) \quad (10)$$

Without an orifice, it is

$$H_P = H_D \quad (t > 0) \quad (11)$$

H_P and V_P at the downstream end are found from Eqs. (8) and (10) or (11).

Let us assume that the trapped air changes adiabatically. Then we have

$$(H_P + H_a)W^{\kappa} = (H_s + H_a)W_s^{\kappa} \quad (12)$$

Figure 3 shows a model of trapped air. The continuity equation at the junction is used.

$$dW/dt = -S(V_1 - V_2) \quad (13)$$

Subscripts 1 and 2 denote the left and right sides of grid intersection points, respectively. The integration of Eq. (13) between P and C yields

$$W_P - W_C = -(S\Delta t/2)(V_{1P} + V_{1C} - V_{2P} - V_{2C}) \quad (14)$$

Variables at the trapped air position are found from Eqs. (8) and (9) introducing subscripts 1 and 2, and Eqs. (12) and (14).

The boundary condition at the upstream end is

$$H_P = H_s \quad (15)$$

H_P and V_P at the upstream end are found from Eqs. (9) and (15). Under the final steady condition,

$$H_s = \frac{fLV_s^2}{2gD} + \left(\frac{S}{cS_n}\right)^2 \frac{V_s^2}{2g} + H_D \quad (16)$$

When there is no orifice at the downstream end, the second term of the right hand can be omitted. Because the liquid is at rest initially, the pressure head is H_s and the flow velocity is zero everywhere in the pipe. The friction factor of the pipe and the discharge coefficient of the orifice are found experimentally.

Fig.1 Pipe line model used in the experiment

Fig.2 $x-t$ grid

Fig.3 trapped air

3. Experiment

Figure 4 shows our experimental setup. The pipe is a copper tube which is 10 mm in diameter and 100 m in length. The upstream end of the pipe is connected with a constant pressure reservoir. The valve set at the downstream end is a manual operating valve. Because the valve is almost equal to the pipe in diameter, the pressure loss at the valve ought to be zero. In order to create a pressure loss at the downstream end, an orifice is set just before the valve. The opening diameter of the orifice is 3.2 mm or 1.65 mm. In order to model trapped air, air-water vessels are used, which are placed at 20, 40, 60, 80 m from the upstream end of the pipe. Using an electro-magnetic flow meter, the transient flow velocity is measured at the downstream end of the pipe and recorded on an electro-magnetic oscillograph. And using piezo-resistance pressure transducers, the transient pressures at 0, 40, and 100 m from the upstream end are measured. The temperature of the water is $20 \pm 2^\circ\text{C}$ during the experiments.

In the experiments, the fluid transients are caused by a quick opening of the valve. The valve operating time is about 0.05 s, which is negligibly short compared with the round trip time of the pressure wave ($2L/a$). The pressure head is 62 m at the constant pressure reservoir. Because the valve outlet leads to the atmosphere, H_0 is zero.

4. Experimental Results

Figure 5 shows the experimental results without an orifice at the downstream end. As shown in Fig. 5(a), for the case without trapped air, the transient flow velocity at the downstream end ($x=100$ m) increases stepwise. The change is almost equal to the well known transient change in flow velocity caused by a quick opening of the valve without trapped air, so that the transient flow velocity can be measured accurately by the electro-magnetic flow meter. In these experiments as shown in Fig. 5, the flow velocity is 1.4 m/s under the final steady condition, and the Reynolds number is 14,000. During the fluid transients, the pressure at the downstream end ($x=100$ m) is almost equal

to the atmospheric pressure. On the other hand, the pressure at $x=40$ m oscillates and gradually dampens. Broken lines in these figures denote the calculated results obtained by the characteristics method. The calculated results agree well with the experimental ones. As shown in Fig. 5(b), when there is trapped air at $x_0=80$ m, the fluid transients are different from those in Fig. 5(a). In this case, the flow velocity gradually increases and fluctuates with several peaks. And the amplitude of the pressure fluctuation at the upstream side of the trapped air volume is smaller than that in Fig. 5(a). And the period of fluctuation is shorter than that without trapped air, because the pressure wave propagates between the upstream end and the trapped air volume. This tendency is more remarkable with a larger volume of trapped air as shown in Fig. 5(c) and (d). That is, the first peak of transient flow velocity at the downstream end is larger and far exceeds the final steady flow velocity. On the other hand, the pressure fluctuation at the middle point of the line is small and settles down to the final steady value smoothly.

Figure 6 shows experimental results with the air trapped at various points in the pipe. When the air is trapped near the downstream end, the transient flow velocity considerably exceeds the final steady

Fig.4 Experimental setup

Fig.5 Records of the fluid transients (without an orifice, $x_0=80$ m)

value. When air is trapped near the upstream end, the overshoot is small. Especially in Fig. 6(d), the transient flow velocity was quite similar to that without trapped air (see Fig. 5(a)).

Figure 7 illustrates the relation between the maximum flow velocity at the downstream end and the volume of trapped air. The dimensionless maximum flow velocity $V'_{\max}$ denotes the ratio of the maximum flow velocity to the final steady flow velocity. And the dimensionless volume of air α denotes the ratio of the initial trapped air volume to the volume in the pipe. The larger α becomes, the larger $V'_{\max}$ becomes. But the rate of increase of $V'_{\max}$ becomes smaller as α becomes larger. When air is trapped near the downstream end, $V'_{\max}$ is quite large.

The cause of the overshoot in transient flow velocity is considered as follows. Trapped air divides the water column into two sections. Since the water column between the trapped air and the downstream end is shorter than the total pipe length, the water column is more accelerated than in the case without trapped air and the flow velocity at the downstream end exceeds the final steady flow velocity. When air is trapped near the downstream end, the acceleration becomes larger. And when trapped air is large, the water column will be accelerated for a longer time. In both cases, the maximum transient flow velocity becomes larger.

Figure 8 shows the relation between the maximum flow velocity $V'_{\max}$ and the total trapped air volume at four points ($x_s = 20, 40, 60$ and 80 m). $\alpha/4$ denotes the volume of trapped air at each point. The maximum flow velocity in Figure 8 is

Fig.6 Records of the fluid transients (without an orifice, $\alpha = 0.006$)

close to that in Figure 7, where the air is trapped near the downstream end ($x_s = 80$ m). That is, when air is trapped in several portions in a pipe, the transient flow velocity is considered to mainly depend upon the air trapped at the nearest portion to the downstream end.

Figure 9 shows the experimental results for the case with an orifice at the downstream end. The pressure just before the orifice once drops almost to zero and then recovers. So the water column between the trapped air and the downstream end is not accelerated very much. Consequently, the transient flow velocity does not overshoot very much. Under some condition,

Fig.7 Relation between the maximum flow velocity and the volume of trapped air (without an orifice)

Fig.8 Relation between the maximum flow velocity and the volume of trapped air (without an orifice, $x_s = 20, 40, 60, 80$ m)

Fig.9 Records of the fluid transients ($x_s = 60$ m)

the transient flow velocity changes stepwise. As shown in Figure 10 for the case with an orifice, the maximum flow velocity is smaller than that without an orifice. Figure 11 shows the relation between the maximum flow velocity and the trapped air position x_g . When air is trapped near the downstream end, the maximum flow velocity becomes very large. And when there is no orifice, the effect of trapped air is more remarkable. These results are summarized as follows. The maximum flow velocity caused by quick valve opening becomes large under the following conditions;

- * large initial volume of trapped air
- * small pressure loss at the downstream end and
- * existence of trapped air near the downstream end.

Since the calculated results agree well with the experimental ones, the model for calculation is considered appropriate.

5. Dimensionless Parameters

Using the following dimensionless variables,

$$V' = \frac{V}{V_m}, \quad H' = \frac{H + H_a}{(a/g)V_m}, \quad W' = \frac{W}{W_s} \\ t' = \frac{t}{L/a}, \quad x' = \frac{x}{L} \quad (17)$$

Equations (1) and (2) are transformed as follows.

Fig.10 Relation between the maximum flow velocity and the volume of trapped air ($x_g = 80$ m)

Fig.11 Relation between the maximum flow velocity and the trapped air position ($\alpha = 0.012$)

$$\frac{\partial V'}{\partial t'} + \frac{\partial H'}{\partial x'} + R_f V' |V'| = 0 \quad (18)$$

$$\frac{\partial H'}{\partial t'} + \frac{\partial V'}{\partial x'} = 0 \quad (19)$$

And boundary conditions are as follows.
Downstream end of the pipe:

$$H' - H'_0 = R_o V'^2 \quad (20)$$

Trapped air:

$$x' = x'_g \quad (21)$$

$$H' W'^2 = H'_s \quad (22)$$

$$\beta \frac{dW'}{dt'} = -(V'_1 - V'_2) \quad (23)$$

Upstream end of the pipe:

$$H' = H'_s = R_f + R_o + H'_0 \quad (24)$$

Now five dimensionless parameters are obtained.

$$R_f = \frac{fL V_m^2 / 2gD}{(a/g)V_m} = \frac{fL V_m}{2aD} \quad (25)$$

$$R_o = \frac{(S/cS_n)^2 (V_m^2 / 2g)}{(a/g)V_m} = \left(\frac{S}{cS_n}\right)^2 \frac{V_m}{2a} \quad (26)$$

$$H'_0 = \frac{H_0 + H_a}{(a/g)V_m} \quad (27)$$

$$\beta = \frac{W_s}{S(L/a)V_m} = \frac{a}{V_m} \alpha \quad (28)$$

$$x'_g = \frac{x_g}{L} \quad (29)$$

R_f denotes the ratio of the final steady pressure head loss due to the pipe friction to the pressure head $(a/g)V_m$ ($=H_m$), which is the surge pressure head corresponding to the final steady flow velocity V_m , while R_o denotes the ratio of the final steady pressure head loss across the orifice to H_m . H'_0 is the ratio of the absolute pressure head at the downstream end to H_m . β is the ratio of the initial volume of trapped air to the liquid volume which flows out at the final steady flow velocity, while pressure wave goes and returns through the pipe. x'_g is the dimensionless position of trapped air.

Fig.12 Relation between the transient flow velocity and dimensionless parameters

The fluid transients depend upon these five parameters. Figure 12(a) shows variations of the transient flow velocity with β . When no air is trapped there ($\beta=0$), the transient flow velocity increases stepwise. But when air is trapped there ($\beta=20$), it exceeds the final steady flow velocity very much. When trapped air volume is relatively small, the transient flow velocity does not always take maximum value at the first peak, but takes it at the second peak or one of the following peaks. Figure 12(b) shows variations of the transient flow velocity with R_f . The larger the pipe friction loss is (that is, smaller diameter and larger length), the larger the overshoot becomes. When R_f is small, the flow velocity takes maximum value at the second peak or one of the following peaks. Figure 12(c) shows variations with R_v . The larger the fluid resistance at the downstream end is, the smaller the overshoot is. And as shown in the experimental results, under some value of R_v , the change of the transient flow velocity is stepwise. Figure 12(d) shows variations with H'_0 . When H'_0 is large, because the trapped air does not expand too much, the overshoot is small.

Figure 13 shows the relation between the maximum flow velocity $V'_{\max}$ and dimensionless parameters. The broken lines in Figures 13(a) and (b) indicate the

region, where the flow velocities have maximum value at the second peak or one of the following peaks. In Figures 13(a) and (b), $V'_{\max}$ becomes large when β or R_f is large, or when R_v is small. Especially, when R_v is small, $V'_{\max}$ becomes very large. As shown in Figure 13(d), the larger H'_0 is, the smaller $V'_{\max}$ is. And as shown in Figure 13(e), the larger x'_g is, the larger $V'_{\max}$ is. The reason is that the acceleration of the water column is in inverse proportion to its length.

Figure 14 shows the relation between $V'_{\max}$ and dimensionless parameters graphically. Figure 14(a) shows the relation among $V'_{\max}$, R_f and R_v . Figure 14(b) shows the relation among $V'_{\max}$, x'_g and β . In the region between the broken line and R_v axis in Figure 14(a) and that between the broken line and β axis in Figure 14(b), the flow velocity takes maximum value at the second peak or one of the following peaks. As shown in Figure 14(a), the larger β is the smaller this region becomes. As shown in Figure 14(b), when x'_g is small, the flow velocity takes maximum value at the second peak or one of the following peaks, and the maximum flow velocity is not so large. But when x'_g is close to unity, that is, when air is trapped near the downstream end, the flow velocity takes maximum value at the first peak, and the maximum value is very large.

Fig.13 Relation between the maximum flow velocity and dimensionless parameters

Fig.14 Graphical representations of relation between the maximum flow velocity and dimensionless parameters

6. Conclusions

In this paper, the influence of trapped air on the transient flow velocity in a pipeline is investigated. Using a model pipeline, the fluid transients caused by quick valve opening are studied experimentally and analytically.

When trapped air exists in a liquid conduit, the transient flow velocity often exceeds the final steady flow velocity extremely. Such fluid transients depend upon five dimensionless parameters R_f , R_v , H'_0 , β , and x'_0 . Each parameter denotes:

- R_f : pipe friction loss
- R_v : fluid resistance at the downstream end
- H'_0 : pressure at the outlet of the downstream end
- β : volume of the trapped air
- x'_0 : position of the trapped air

The relations between the maximum flow velocity and these parameters are clarified and presented graphically.

It is concluded that the maximum value of the transient flow velocity tends to increase when R_f , β , and x'_0 become large, and when R_v and H'_0 become small.

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