The Radial Flow of a Thin Liquid Film
(5th Report, Influence of Wall Roughness on Laminar-turbulent Transition)

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This paper describes the influence of wall roughness on the transition from laminar to turbulent flow occurring in the radial flow of a thin liquid film. The properties of liquid surface was visually observed using the following three types of distributed roughnesses; (A) saw-toothed roughness and (B) trapezoidal shaped roughness, both of which were regularly constructed in a concentric configuration; and (C) irregularly distributed sand roughness. It was found that the point of transition shifted upstream as the height of wall roughness increased under otherwise identical conditions, and that the transition occurred in each of the three regions, namely, (1) the stagnant region, (2) the region in which the laminar boundary layer grows, and (3) the region in which the whole flow is the laminar boundary layer. Furthermore, experimental results showed that the critical Reynolds number decreased as the height of wall roughness increased.

Key words: Boundary Layer, Laminar-Turbulent Transition, Liquid Film Flow, Radial Flow, Visual Observation

1. Introduction

When a water jet impinges against a flat plate, a liquid film flows radially (which is hereinafter called an impinging water jet). The impinging water jet is effective for cooling the flat plate, and several studies(1)–(4) have been reported on this flow. However, they mainly dealt with a circular hydraulic jump, and were not discussed the transition from laminar to turbulent flow.

Incidentally, when water is released from a narrow cylindrical opening into the atmosphere along the flat plate, a thin radial liquid film flow of high speed and large velocity gradient is obtained (which is called a radial liquid film flow, for short, hereinafter). The authors pointed out that a laminar-turbulent transition induced from the disturbance inside a liquid film, in a sense similar to a laminar-turbulent transition occurring in the boundary layer on a flat plate in a uniform flow, occurs in a radial liquid film flow. The process of the transition was clarified, and the critical discharge Reynolds number, the critical local Reynolds number, and the relationship between the transition radius and the discharge Reynolds number were determined(5). At the same time, measuring the liquid film thickness(6), velocity distribution(7) and wall surface pressure fluctuations(8), and referring to the stability of flow, the transition phenomenon was clarified in detail.

This report describes the influence of the wall roughness of flat plate on the transition from laminar to turbulent flow.

In the previous works, the transition of radial liquid film flow along a smooth flat plate into a turbulent flow was limited to the developed liquid film region in which the liquid film flowed as a laminar boundary layer. Using a rough flat plate, it was found that the laminar-turbulent transition occurs also in a region of stagnation flow opposite to the circular nozzle and in a laminar boundary layer developing region. Also as a result of observation of the mode of a flow around the turbulent wedge spreading widely downstream of a single roughness element, the nature of the D wave which appeared on the liquid surface preceding the laminar-turbulent transition was studied.

2. Experimental Apparatus

The experimental apparatus is the same as shown in Fig. 2 of the second report(6). The inside diameter of the nozzle was 25 mm, and the opening ranged from 0.5 to 2 mm.

Wall roughness profiles of the flat plate (diameter of 400 mm in diameter) are shown in Fig. 1, and values of roughness ε and pitch p are given in Table 1. The roughness can be classified into A, B, C types. The A type is a saw-toothed roughness and B type is a trapezoidal shaped

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roughness, both of which are regularly distributed in a concentric configuration. These disc plates made of brass were fabricated by using an NC lathe, and a diamond cutting tool (the tool shape: lateral cutting blade angle 45°, front cutting blade angle 45°, and nose radius 3 µm). On the contrary, the C type is an irregularly distributed sand roughness, and the plate was fabricated by the sand-blasting method using emery on a glass plate. The value shown in Table 1 was obtained by the method of mean roughness of ten measurements following the JIS.

The position of the point of the transition was determined mainly by visual observation of the liquid surface with the aid of a stroboscope, and where necessary, the visual observation was confirmed by photography.

3. Results and Discussion

3.1 Radius of transition

Supposing the equivalent length of an opening to be \( \sqrt{CDH} \), where \( D \) is the nozzle diameter, \( H \) is the opening and \( C \) is the discharge coefficient of the opening, the discharge Reynolds number from the opening is defined as \( Re=Q/\sqrt{CDHv} \), where \( Q \) is the flow rate and \( v \) is the coefficient of kinematic viscosity of the liquid. Putting the dimensionless value of radius \( r \) as \( R_\text{t}=(r/\sqrt{CDH})Re^{-1/3} \), the dimensionless transition radius is expressed as \( r_\text{t}^* \). In the case of a radial liquid film flow over a smooth flat plate, a laminar-turbulent transition occurs when the discharge Reynolds number \( Re=7.4	imes10^5 \) (5), and the radius of transition is given as follows:

\[
r_\text{t}^*=870Re^{-0.4}\left(\frac{e}{\mu m}\right)\]

(see Fig. 12 in the first report).

The relation between \( r_\text{t}^* \) and \( Re \) for a rough flat plate with a type roughness is shown in Fig. 2 whereas \( e=24 \mu m \) and in Fig. 3 wherein \( e=70 \mu m \). In these diagrams, the solid line is expressed as eq. (1).

As mentioned in the first report, a laminar-turbulent transition occurred in the flow when the granular waves on the liquid surface (SL waves) were 50% or more in the circumferential direction. Therefore, the transition radius \( r_\text{t} \) was determined at an initiating position of SL waves. In Figs. 2 and 3, however, the result includes cases where SL waves were less than 50% (for example 10%).

Incidentally, the dimensionless radius for a laminar boundary layer to develop and reach the liquid surface is \( r_\text{t}^*=0.23 \) (according to the result of measurement of velocity distribution(7), \( r_\text{t}^*=0.21 \) to 0.23).

### Table 1. Surface roughness.

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>( e ) µm</th>
<th>( \rho ) µm</th>
<th>( e/\rho )</th>
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<td>2</td>
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<tr>
<td>5</td>
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<td>70</td>
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<tr>
<td>11</td>
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<tr>
<td>14</td>
<td>C</td>
<td>49</td>
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</tr>
</tbody>
</table>

![Fig. 1. Roughness curve.](image)

![Fig. 2. Transition radius (e=24 µm).](image)

![Fig. 3. Transition radius (e=70 µm).](image)
The thickness of liquid film reaches the minimum at $r^* = 0.45$, and its value is $h^* = (b/CDH)Re^{0.231}$ (while the flow is laminar, the roughness of the wall surface hardly affects the liquid film thickness (9)). The minimum value $h_{min}$ is about 0.15 mm for $Re = 10^3$ and about 0.19 mm for $Re = 5 \times 10^3$ when $H = 0.5$ mm, and about 0.29 mm for $Re = 10^4$ and about 0.36 mm for $Re = 5 \times 10^4$ when $H = 2$ mm.

In the B type roughness of $\varepsilon = 0 \mu m$ [hereafter abbreviated as $\varepsilon = 0 \mu m(B)$] and $\varepsilon = 13 \mu m(S)$, effects of wall roughness on the laminar-turbulent transition were not noticeable. And the effect of roughness was scarcely noted at $\varepsilon = 8 \mu m$, $p = 20 \mu m(A)$ although it is not reported here.

At $r = 22 \mu m(A)$, as shown in Fig. 2, the effect of wall roughness was obvious. Provided $Re$ is equal, unlike the case of a smooth flat plate, the transition position is shifted upstream, and the rate of the shift of the transition position is greater when the $H$ is smaller and $Re$ is larger. Furthermore, for larger values of $Re$, the transition position tends to move upstream farther at the turning point of $r^* = 0.3$ to 0.4, as $Re$ increases more. It is also noted that the critical discharge Reynolds number decreases as compared with that of a smooth plate. However, with this value of roughness, the appearance of a concentric disturbance wave (D waves) which is amplified to lattice-shaped IS waves and granular SL waves is the same as in the case of a smooth flat plate.

When the wall roughness increases, the effect of roughness on the transition becomes more evident. As a result, in the case of $H = 2$ mm as shown in Fig. 3, $r^*$ lies on three straight lines depending on the value of $Re$. Let us call the transitions in the ranges expressed by lines a, b, c types a, b, c, respectively. In the present experimental range, in the A type roughness of 30 mm or more and in 57 mm(B) and 49 mm(O), transition of type c appeared. In 24 mm(A) in Fig. 2, transitions of type a and type b appeared, not reaching the state of type c. Close examination of the result of the flow along a smooth flat plate as reported in Fig. 12 of the first report seems to show that there is the first sign of the transition of type b at $Re = 2 \times 10^4$, $r^* = 0.3$. The appearance of three types of transition is peculiar to the radial liquid film flow. When the flow is laminar, the radial liquid film flow terminates to a hydraulic jump through three regions, that is: (1) the region of stagnation flow opposed to the nozzle; (2) the region of development of laminar boundary layer after discharging from the opening; and (3) the developed liquid film region. The relation between the region of flow and the mode of transition is described below.

The appearance of the type c transition is shown in Fig. 4, which refers to $r = 54 \mu m(B)$, $H = 1.5 \mu m$, $Re = 1.5 \times 10^4$. The photograph was taken by inclining the camera about 30 degrees to the liquid surface, and the illumination of time duration 28 usec was given from a position opposite to the camera. In Fig. 4, the liquid surface immediately after discharging from the nozzle is relatively smooth, and appears white in the photograph due to the reflection of the light on the liquid surface. At $r = 18 \mu m$, relatively coarse granular waves began to appear on the liquid surface, and at $r = 23 \mu m$, very fine granular waves (SL waves) covered the entire surface. In this case, the SL waves began to appear uniformly in the circumferential direction, unlike type a or type b transition. Or if $Re$ changes, the position $r_*$ is found independent of the change of $Re$. Accordingly, the position of type c seems to be a laminar-turbulent transition occurring in the stagnation flow region (1), and it is assumed that SL waves emerge when the turbulent boundary layer is developed to reach the liquid surface, which is further discussed below.

When $Re$ is gradually decreased from the lower limit of type c transition, SL waves come to appear in the circumferential direction in a zigzag form as in the case of the type a transition, and the initiating position of SL waves soon shifts downstream. However, if the dimensionless radius $r_*$ at which SL waves appear is smaller than 0.21 to 0.22, they are not preceded by D waves. When $r_*$ moves farther downstream, D waves and LS waves appear upstream of SL waves. The smallest dimensionless radius at which D waves appear is 0.21 to 0.22. This value nearly corresponds to the position at which the laminar boundary layer develops to reach the liquid surface. Hence, the type b transition takes place in the region where a laminar boundary layer develops after water is discharged from the opening. Meanwhile, in Figs. 2 and 3, it is found that $r_*$ changes with respect to $Re$ in nearly the same gradient as that of the type b transition in a range of $r^* = 0.3$ to 0.4. This is due to the fact that the disturbance in the liquid film is considerably amplified in the boundary layer developing region and, in the developed liquid film region, while the local Reynolds number $Re = q/2\pi r$ decreases as the radius increases and the amplification.
factor of disturbance declines as \( r \) increases, the liquid film thickness decreases up to \( r^* = 0.45 \) and the effect of roughness increases relatively, which acts to prevent the decline of the amplification factor of disturbance. Therefore, as the wall roughness increases, it is considered, the region of the type \( b \) transition spreads up to a greater \( r^* \).

The \( a \) transition occurs mainly due to amplification of the disturbance in the developed liquid film region. It has been already made clear in the fourth report that transition is possible in the developed liquid film region in spite of the decline of the local Reynolds number along with \( r \).

Thus, types \( a \), \( b \), \( c \) transitions are promoted by the wall roughness, but the cause seems to lie in the peculiarity of the radial liquid film flow. Therefore, if the discharge Reynolds number \( Re \) is larger, it is supposed, type \( b \) or \( c \) transition appears even on a smooth flat plate.

In the result with \( H = 0.5\), \( 0.7 \), \( 1 \) mm in Fig. 3, the type \( c \) transition can exist, but type \( a \) or \( b \) transition cannot. A similar tendency was noted at \( H = 0.5\), \( 0.7 \) mm for \( \varepsilon = 64 \mu m \). In these cases, concentric D waves appearing in the type \( a \) and \( b \) transition were no longer seen. A mode of the transition at \( \varepsilon = 70 \mu m \), \( H = 1 \) mm, \( Re = 4.3 \times 10^4 \) shown in Fig. 5. This diagram reveals the existence of D waves in a reticulate pattern composed by intersection of two sets of arc curve groups. The D waves distorted by the wall roughness began to appear at \( r = 25 \) mm \( (r^* = 0.18) \) in the conditions in Fig. 5, and the flow changed to a turbulent flow at \( r = 55 \) mm \( (r^* = 0.38) \). In this case, the maximum value of boundary layer thickness \( \delta \) in the range of existence of D waves is about 0.34 mm. On the other hand, according to the stability calculation (8), the critical layer in the laminar boundary layer developing region and in the developed liquid film region was about 0.2 to 0.3 times the boundary layer thickness. Therefore, the disturbance in the liquid film travels at the height of 0.07 to 0.1 mm from the wall surface. Accordingly, it is possible that the wall roughness of \( \varepsilon = 70 \mu m \), \( p = 162 \mu m \) directory stimulate the disturbance. Meanwhile, when \( Re \) is smaller than the value in Fig. 5, reticulate D waves gradually change into concentric D waves.

3.2 Critical discharge Reynolds number

Effects of wall roughness on the critical discharge Reynolds number \( Re_{cr} \) are shown in Fig. 6 which refers to A type and B type, and in Fig. 7 which represents C type roughness. The critical discharge Reynolds number \( Re_{cr} \) is defined as the \( Re \) when SL waves are 50% in the circumferential direction. However, since it is very difficult to generate the state where SL waves are 50%, actually the value when SL waves are present to an extent of 20 to 80% was used. Incidentally, the value was \( Re_{cr} = 7.4 \times 10^4 \) in the case of a smooth flat plate.

It is found from Fig. 6 that \( Re_{cr} \) at \( \varepsilon = 6 \mu m \)(B) was similar to that of a smooth flat plate, and a very slight decrease is noted for roughness of \( \varepsilon = 13 \mu m \)(B). As the roughness increases, \( Re_{cr} \) decreases, but the effect of B type roughness on the critical discharge Reynolds number is found to be smaller than that of A type. In the case of \( \varepsilon = 70 \mu m \)(A), when \( H = 0.5 \) mm, \( Re_{cr} \) was 3.1 \times 10^5, decreasing to about 40% of the case of smooth plate.

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Fig. 5. Deformation of D waves.

![Distorted D Wave and SL Wave](image)

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Fig. 6. Critical discharge Reynolds number (A type, B type).

![Fig. 6](image)

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Fig. 7. Critical discharge Reynolds number (C type).

![Fig. 7](image)
The effect of C type roughness on the critical local Reynolds number may be said to be similar to that of A type, considering the average of four discs.

3.3 Critical local Reynolds number

Effects of wall roughness on the critical local Reynolds number $Rer_{cr}$ are shown in Fig. 8 which represents C type roughness. In the case of a smooth flat plate, the value was $Rer_{cr} = 470$. From Figs. 8 and 9, it is known that in the case of A type and C type roughnesses $Rer_{cr}$ can be expressed almost by a single curve in terms of $\varepsilon/H$ regardless of the value of $\varepsilon$. It is considered that the type roughness is more influential to $Rer_{cr}$ than C type. In the case of B type, the effect on $Rer_{cr}$ was smaller than that of A type.

By the way, at $\varepsilon = 70 \mu m(A)$ and $H=0.5 mm$, $Rer_{cr}$ was 280. This value is a minimum in the range where B waves could be observed over a smooth glass plate. Furthermore, as mentioned in the fourth report, the stability limit in the liquid film developing region is $Re_{cr}=U/V = 450$ ($U$: liquid surface velocity), which is converted to $Rer_{cr}=276$. Therefore, in the case of $\varepsilon = 70 \mu m(A)$, $H=0.5 mm$, the laminar-turbulent transition occurs at $Rer$ corresponding to the stability limit in the case of a flow along a smooth flat plate. At $\varepsilon = 70 \mu m(A)$, the smallest value of $Rer$ where B waves could be observed was 210. Accordingly, if the termination of B waves and stability limit are correlated, the wall roughness will decrease the stability limit. The larger the wall roughness, the smaller will be the difference between $Rer_{cr}$ and stability limit.

3.4 Effects of wall roughness on the laminar-turbulent transition in the laminar boundary layer developing region

As mentioned above, if a laminar-turbulent transition occurs in the laminar boundary layer developing region, the turbulent boundary layer develops to reach the liquid surface in the circumferential direction. If the transition occurs near the nozzle, there is a distance between the actual transition position and that where the BL waves initiate, but when the transition occurs immediately before the laminar boundary layer reaches the liquid surface, BL waves will appear immediately on the liquid surface. Accordingly, only about the result of $r_s = 0.19$ to 0.21, the effect of wall roughness was studied. The dimensionless boundary layer thickness $\delta^+ = (U/V) Re^{-1/2}$ is given by $\delta^+ = 5.9% r_s^{3/4}$. The relation between $Re_{cr}$ defined by using and the main stream velocity (discharge velocity) $U_0$ and $U_0V/U$ is shown in Fig. 10. The relation between the transition local Reynolds number $Rer$ and the dimensionless transition radius $r_s^{-1}$ in the developed liquid film region in the case of a smooth flat plate is given as follows. See Fig. 13 of the first report.

$$Rer_{cr} = 175 r_s^{-2}$$

For $r_s = 0.243$, $Rer_{cr}$ is obtained as 2963, which is converted to $Re_{cr}$ as $Re_{cr} = 4233$.

From Fig. 10, it is evident also in the laminar boundary layer developing region that the roughness of A type is most strongly influences the laminar-turbulent transition, and that the transition Reynolds number at $U_0V/U = 300$ is lowered to 30-35% of that of a smooth flat plate. From experimental results for 54 $\mu m(A)$, 63 $\mu m(A)$, and 70 $\mu m(A)$, it is limited to 25-30% of that of a smooth flat plate.
plate.

Incidentally, comparing the results reported by Peindt (10) who studied the influence of the surface roughness on the position of point of transition using a convergent and divergent channel of circular cross-section with a cylinder covered with sand placed axially in them with the results of C type at $U_d/\nu=300$, it is known that they agree very well with each other although experimental methods are completely different.

3.5 Turbulent boundary layer developing from an opening

The type c transition shown in Fig. 3 was interpreted as a phenomenon of appearance of SL waves due to the development of a turbulent boundary layer to reach the liquid surface resulting from the occurrence of a laminar-turbulent transition in the region of a stagnation flow opposed to the nozzle. In order to confirm this, the position at which the turbulent boundary layer reaches the liquid surface was analytically obtained, and was compared with the experimental result. Assuming the same flow model as the one in the second report (6), the Blasius frictional formula $\tau_f=0.0225\nu/\delta(U_d/\nu)^{1/3}$ was used as the turbulent shearing stress on a wall surface, and the velocity distribution within a turbulent boundary layer was approximated by $u/U=(z/\delta)^{-1}$. The turbulent boundary layer develops from the stagnation point, and the turbulent boundary layer reaches the liquid surface at $r_{ex}^*=rac{1}{1.606}$, which is the dimensionless value $r^*$.

Merely by observing the SL waves, it is not known whether the turbulent boundary layer develops from the stagnation point or not. But it is estimated that, at the lower limit $Re$ of the type c transition, the laminar-turbulent transition occurs immediately before discharge from the opening. Accordingly, instead of eq. (3), supposing that the laminar-turbulent transition occurs at $r=Re$ ($R$: nozzle radius), the position of the turbulent boundary layer to reach the liquid surface is given as follows.

$$ r_{ex}^*=2.90-54.5R^{1.11}R^{-0.31}r^{0.13} $$

Comparison between the experimental result and eq. (4) is made in Fig. 11, in which $\delta$ is the thickness of laminar boundary layer at $R$, and $r_{ex}^*$ is the position of the radius at which SL waves at the lower limit $Re$ of type c transition have appeared.

The experimental result in Fig. 11 shows a tendency of an elevation toward the left side on the whole. The reason is probably because the frictional loss increases due to the wall roughness and $r_{ex}^*$ decreases as compared with that of a smooth flat plate. So, estimating the value of $r/\delta=0$ from the two broken lines in the figure, the theoretical value turns out $10^\%$ greater. However, considering that, even in a laminar flow, the position of the boundary layer reaching the liquid surface was smaller in the measurement of the velocity distribution than in the analytical result, the analytical result and experimental result shown in Fig. 11 agree with each other fairly well. Therefore, the type c transition changes to a turbulent flow in the region of stagnation flow opposed to the nozzle while the turbulent boundary layer reaches the liquid surface, thereby causing the entire liquid film to be disturbed.

3.6 Disturbance waves (D waves)

The concentric disturbance waves (D waves) appearing on the smooth liquid surface before the laminar-turbulent transition are not fully elucidated in spite of their extremely important role in the turbulent transition of a radial liquid film flow. In this paragraph a discussion is made about the D waves appearing in the vicinity of a turbulent wedge occurring due to the single roughness element and in the vicinity of small lumps of turbulence. In either case, the conditions are $D=29\text{ mm}$, $H=1\text{ mm}$.

Figure 12 shows a turbulent wedge occurring due to a hemispherical protrusion of nearly $0.1\text{ mm}$ in diameter existing in the laminar boundary layer developing region. The discharge Reynolds number is $Re=1.4\times10^4$, the protrusion radius position is at $r=29\text{ mm}$ ($r^*=0.138$), and the thickness of boundary layer at protrusion position is $0.17\text{ mm}$. Or in the portions completely free from the effect of protrusion, D waves with wavelength of $0.9\text{ mm}$ appear at $r=64\text{ mm}$, LS waves at $r=77\text{ mm}$, and SL waves at $r=81\text{ mm}$. At $r=34\text{ mm}$ downstream of the protrusion, SL waves appear in a wedge form with an apex angle of $18^\circ$ although it is hard to discriminate in the photograph. Along the side of a wedge, very fine linear creases are found to be present at small intervals of about $0.2\text{ mm}$. The creases occur due to a shearing layer in the boundary between the laminar...
flow and the turbulent flow. Linear D waves having an angle of 60° to the side of a wedge begin to appear from around r=45 mm (r*=0.215). At this appearing point, the length of D waves is already about 2 mm. These D waves are amplified and extended in the length as they move downstream. From this amplification of D waves, it is judged that the transition occurred at r=65 mm ahead of others.

Figure 13 shows a turbulent wedge occurring due to a semispherical protrusion of slightly more than 0.1 mm in diameter existing in the developed liquid film region. The conditions are Re=9.3x10⁴, protrusion radius position r=63 mm (r*=0.344), and liquid film thickness of protrusion position is 0.22 mm. Linear D waves appear from the position of about 10 mm downstream of the protrusion, and the length of waves is about 0.5 mm at the beginning of appearance.

The D waves appearing on the liquid surface are closely related with the disturbance in the liquid film. In Figs. 12 and 13, therefore, the D waves around the turbulent wedge are estimated to have appeared on the liquid surface as a result of amplification of the disturbance in the liquid film in the laminar region around the wedge due to the existence of a turbulent wedge. In Fig. 13, furthermore, D waves began to appear about 10 mm downstream of the position of protrusion, suggesting that D waves appear on the liquid surface when the disturbance in the liquid film is amplified to a considerable extent.

Comparing Fig. 12 and 13, the following may be concluded. In Fig. 12 when Re is greater and the protrusion is in the boundary layer developing region, the amplification factor of disturbance inside the liquid film is greater than in Fig. 13. In spite of this, the distance from the protrusion position to the position where D waves appear is longer in Fig. 12, and the position of appearance of D waves in Fig. 12 is assumed to be in the developed liquid film region. At the first appearance, the length of D waves is 2 mm, being already extended. Summing up these facts, it can be said that D waves are ones which appear only in the developed liquid film region.

Figure 14 shows the D waves caused by small lumps of turbulence. At the position of r=21 mm, small lumps of turbulence were intermittently formed by very fine fibers of 2 mm in length depositing on the wall surface. In Fig. 14, the small lumps flowing away downstream were momentarily captured, when Re is 7.4x10⁴, radius of laminar boundary layer to reach the liquid surface is r=41 mm, and the radius position of the rear end of small lumps is r=60 mm. In this figure, D waves are found to emerge upstream of small lumps of turbulence as if left over within the small lumps. It is found here that the phase velocity of D waves is considerably smaller than the moving velocity of the small lumps. The wavelength of the D waves induced by small lumps of turbulence flow is the same as that of the D waves appearing in a flow free from the effect of small lumps. Accordingly, it is known that the wavelength and phase velocity of D waves are the same, being specific to the flow of a developed liquid film, whether D waves appear locally and intermittently or wholly and continuously.

Although the wavelength of D waves differs with the nozzle diameter D and opening distance H, it is hardly changed by the discharge Reynolds number Re when D waves are amplified and the flow reaches a transition, and the phase velocity of D waves is about 0.05 to 0.08 times the
liquid surface velocity (considerably smaller as compared with the phase velocity of disturbance in the liquid film), as disclosed in the first report (5).

4. Conclusions

Using discs having saw-toothed roughness, trapezoidal shaped roughness and sand roughness, radial liquid film flows were tested and the following results were obtained.

(1) A laminar-turbulent transition occurs in the stagnation flow region, laminar boundary layer developing region, and developed liquid film region, and the process of transition occurring on each liquid surface was clarified.

(2) The critical local Reynolds number in the developed liquid film region decreases as the wall roughness increases, and in discs with saw-toothed roughness and sand roughness, the critical local Reynolds number can be expressed by one curve using the ratio of roughness to opening distance $c/H$. It is also found that the saw-toothed roughness has the strongest effect on the transition.

(3) The transition local Reynolds number in the laminar boundary layer developing region decreases as the wall roughness increases, but there is a lower limit, whose value is found to be about 30% of that of the case of a smooth flat plate.

(4) The position where the turbulent boundary layer developing from the stagnation flow region to reaches the liquid surface was analytically determined and compared with the experimental result, and both results agreed with each other fairly well.

Also observing the liquid surface around the turbulent wedge caused by protrusion and around the small lumps of turbulence flow generated intermittently, the nature of the disturbance waves (D waves) appearing on the liquid surface ahead of the laminar-turbulent transition was considerably clarified.

References