On the Force and Vortex Shedding on a Circular Cylinder from Subcritical up to Transcritical Reynolds Numbers*

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Pressure distributions and vortex shedding were measured in a cryogenic wind tunnel from subcritical up to transcritical Reynolds numbers ($10^5 \leq Re \leq 10^7$) and Mach numbers up to 0.3 without changing experimental arrangement. Drag coefficients were calculated using pressure distributions. Pressure distributions and drag coefficients show characteristic changes for subcritical, lower transition, critical, upper transition and transcritical Reynolds numbers respectively. Strouhal number is constant in the range of Reynolds numbers $10^5 \leq Re \leq 3.2 \times 10^5$. Narrow band vortex shedding could not be found in the critical Reynolds numbers range ($3.2 \times 10^5 \leq Re \leq 6.9 \times 10^5$), but it was found again in the upper transition and transcritical Reynolds number ranges. The pressure distributions and spectra of velocity fluctuations are presented for several characteristic values of Reynolds numbers. Results are also compared with those of other authors.

Key Words : Flow Measurement, Fluid Vibration, High Reynolds Number Flow, Flow Around a Cylinder, Drag Coefficient and Strouhal Number

1. Introduction

In the phenomena of flow around large structures i.e. wind loading, building motions and vibrations due to wind, wind generated noise, unstable vibrations arising on take-off and landing of aircraft etc., Reynolds numbers of the flows become large. In order to do wind tunnel measurements about bodies in a stream using a geometrically similar model, Reynolds numbers must have the same value for the model and for the prototype. In this case it is assumed that the fluid can be considered incompressible; further, that no free surfaces exist and hence the action of gravity is eliminated by statical buoyancy. But there are scarcely any wind tunnels to generate a high Reynolds number flow. A cryogenic wind tunnel was built at the University of Tsukuba in 1980 and 1981. Using this wind tunnel we can get a high Reynolds and low Mach number flow around a model. The acquired maximum Reynolds number is $Re = 10^7$ for the model with representative length of 50 mm.

Many investigations have been made about the flow around a cylinder, i.e., drag acting on it and the vortex shedding. Nowadays the range of Reynolds numbers has come to include higher ones, but the coincidence of the measured results are not so good, especially in the range $Re \geq 10^6$.

There are also scarcely any experimental results which were done in the range $Re \geq 5 \times 10^6$ (1) ~ (10). In this investigation, using a cylindrical model with diameter of 50 mm and relative surface roughness of $(k_s/d) = 10^{-5}$ in the test section of the newly built cryogenic wind tunnel, measurements of static pressure distributions, calculations of the drag acting on the cylinder and vortex shedding frequency were carried out in the range of Reynolds numbers $10^5 \leq Re \leq 10^7$. These results were compared with those of the other authors.

Nomenclature

Re	: Reynolds number = Ud/ν
St	: Strouhal number = nd/U
C_p	: pressure coefficient = $-(P_\infty - P_\theta)/(1/2)\rho U^2$
C_D	: drag coefficient = $\int_0^{2\pi} C_p \cos\theta d\theta$
U	: velocity
d	: diameter
ν	: kinematic viscosity
P	: pressure
T	: temperature
μ	: viscosity coefficient
R	: gas constant
n	: vortex shedding frequency
θ	: angular distance
k_s/d	: relative roughness

Subscripts

b	: corresponds to 180°
θ	: refers to angle θ
s	: corresponds to separation point
∞	: corresponds to undisturbed stream
min	: minimum value
crit	: critical value

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2. Wind Tunnel and Experimental Procedure

We define Reynolds number as follows:

$$Re = \frac{Ud}{\nu} = \frac{\rho Ud}{\mu} = \frac{PUd}{\mu RT} \propto \frac{PUd}{RT^{1.4}} \quad (1)$$

High Reynolds number flow can be obtained in the wind tunnel using a gas with large molecular weight M (1), using a large experimental model d (2), increasing the flow velocity U (3), increasing the pressure P in the wind tunnel (4), and decreasing the temperature T in the wind tunnel (5). Almost all experiments in the past were done by using the method (2) or (3), but not so high Reynolds number can be obtained by using these methods. In our cryogenic wind tunnel high Reynolds number flow can be obtained by using (3), (4) and (5) simultaneously. Reynolds number and Mach number can be changed independently.

The specifications of our wind tunnel used in our experiments are given in Table 1. Fig. 1 shows its block diagram. Liquid nitrogen stored in a tank (1) is pressurized and conveyed by a pump (2) into the wind tunnel (3). Another piping (6) leads through evaporator (4) and heater (5) to the wind tunnel. It serves to seal bearings of blower (7). Pressure in the tank is controlled by adjusting the pressure regulating valve (8). Then the liquid nitrogen is conveyed through flow control valve (9) and nozzle control valves (10) and sprayed through four spray nozzles (11) into the wind tunnel. The temperature in the wind tunnel is controlled by adjusting the volume of conveyed liquid nitrogen through by-pass piping (12). The nitrogen gas in the wind

tunnel is circulated by a blower and a motor (13). The nitrogen gas is discharged into the atmosphere through two exhaust ports (14) and pressure regulating valves (15). Pressure in the wind tunnel is controlled by these valves. A safety valve with a suction blower is installed. The outer surface of wind tunnel is made of mild steel. For thermal insulation, wood, urethane, glasswool and, in the inner most part, S-form are used. The material of corner vane and blower is SUS 304.

The test section of wind tunnel has a square cross section of 500×500 mm. A cylinder with diameter 50 mm is installed in this part. The relative roughness of the cylinder is $(k_s/d) = 10^{-5}$. Pressure holes (0.3 mm) and thermocouples are installed on the surface. Pressure and thermocouple-outputs are led out through a tapping plate installed on the wind tunnel wall. A turbulence in the free gas stream in the test section without a cylinder is shown in Table 1. This is measured by a hot wire anemometer. The probe of the anemometer is I-type with diameter $5 \mu\text{m}$ for the measurement of a flow with atmospheric temperature and x-type with diameter $10 \mu\text{m}$ for the measurement of a cryogenic flow. To measure the velocity fluctuation in the wake of the cylinder, a hot wire probe with diameter $25 \mu\text{m}$ (I-type) is used.

3. Pressure Distribution, Drag Coefficient and Strouhal Number

3.1 Pressure distribution and drag coefficient

Figs. 3-a~f show pressure distributions for various values of Reynolds numbers. Fig. 3-a shows a typical pressure distribution at a subcritical Reynolds number. The minimum value of pressure coefficient takes $C_{p\min} = -1.2$, the angle corresponding to the separation point is $\theta_s = 80^\circ$, and the base pressure coefficient, C_{pb} , i.e. the pressure coefficient on the back of the cylinder, is $C_{pb} = -1.2$. Fig. 3-b shows the value in lower transition range, i. e. $C_{p\min} = -2.3$ and $C_{pb} = -0.6$. The patterns of pressure distribution in this Reynolds number range have two characters, on one side the character of subcritical Reynolds number range and on the other side that of the critical one. Fig. 3-c shows the value in the critical Reynolds number range. There is evidence of a laminar separation "bubble" at $\theta = 90 \sim 100^\circ$. It characterises the transition range from the laminar boundary layer to the turbulent boundary layer. The values in this range are as follows; $\theta_s = 130^\circ$, $C_{pb} = -0.3$ and $C_{pb\min} = -2.5$. The pressure distribution in this Reynolds

Table 1 Specifications and main dimensions of cryogenic wind tunnel

Type of Wind Tunnel	Göttingen Type Inside Insulated
Pressure	0.8 MPa
Temperature	112 K
Max. Velocity	65 m/s
Max. Reynolds Number	10^7 (Model Size 50 mm)
Max. Mach Number	0.3
Velocity Distribution	Lower than $\pm 1\%$
Turbulence	0.26 % (Atmospheric Temp., 0.1 MPa, 34 m/s) 0.37 % (110 K, 0.2 MPa, 38 m/s)

Fig. 1 Block diagram of cryogenic wind tunnel

Fig. 2 Surface roughness of cylinder tested

number range seems to be unstable; it sometimes changes remarkably with a slight change in the Reynolds numbers. The critical Reynolds number range seems to reach its upper limit at Reynolds number $Re = 6 \times 10^5$. Fig. 3-d shows pressure distribution in this Reynolds number range. Separation bubbles disappear in this Reynolds number range. Besides the disappearance of separation bubbles, concerning the point of separation and the base pressure coefficient, the characters of pressure distribution in the critical Reynolds number range are preserved. Fig. 3-e shows pressure distribution in the upper transition range, i.e. $C_{p\min} = -1.9$ and $\theta_s = 110^\circ$; the separation point comes again to the front side. The base pressure coefficient $C_{pb} = -0.4$ and the character of critical Reynolds number ranges are preserved. Fig. 3-f shows pressure distribution in the transcritical Reynolds number range. It shows the values $C_{p\min} = -1.9$ and $\theta_s = 110^\circ$ and $C_{pb} = -0.7$. There are no changes in the pattern of the pressure distributions with a change in the Reynolds number.

Fig. 4 and Fig. 5 show the dependence of separation point θ_s and base pressure coefficient C_{pb} on the Reynolds number respectively. From Fig. 4 the general tendency of the variation in the separation point θ_s with Reynolds numbers is made clear, i.e. a slight movement of θ_s

towards upper stream in the subcritical Reynolds number range, the movement of θ_s towards the downstream from the lower transition to the supercritical Reynolds number range and a stabilization of the separation point in the transcritical Reynolds number range.

Integrating the measured pressure distribution, drag coefficients C_D were obtained. The blockage ratio d/h in this case is 10 % so, to correct them for the wall interference effects; we made use of the formulas of Allen & Vincenti (1944), which relate the measured values of V' and C_D' to the corrected values of V and C_D , i.e.

$$\frac{V}{V'} = 1 + \frac{1}{4} C_D' \left(\frac{d}{h} \right) + 0.82 \left(\frac{d}{h} \right)^2 \quad (2)$$

$$\frac{C_D}{C_D'} = 1 - \frac{1}{2} C_D' \left(\frac{d}{h} \right) - 2.5 \left(\frac{d}{h} \right)^2 \quad (3)$$

For the calculation of Strouhal numbers the above correction was not done. Fig. 6 shows values of the drag coefficient C_D for various Reynolds numbers. The values of the drag coefficient C_D for the subcritical Reynolds number ranges become $C_D \approx 1.2$. This value coincides with those by Okajima⁽⁶⁾ and Achenbach et al⁽¹²⁾. With an increase in the Reynolds number C_D decreases suddenly at about $Re = 2.1 \times 10^5$ and the value of C_D becomes $C_{D\min} = 0.38$ at $Re = 3.2 \times 10^5$. From this fact it seems that the critical Reynolds number is about $Re_{crit} \approx 2.6 \times 10^5$. This value is slightly lower than those of other authors. The Reynolds number range $3.2 \times 10^5 \leq Re \leq 6.9 \times 10^5$ corresponds to the supercritical

Fig. 3 Pressure distributions for various values of Reynolds number Re

Fig. 4 Dependence of separation point θ_s on Reynolds number Re

Fig. 5 Dependence of base pressure coefficient C_{pb} on Reynolds number Re

Fig. 6 Variation of drag coefficient C_D with Reynolds number Re

Reynolds number range and the values of the drag coefficient C_D in this range are $C_D \approx 0.5$. The values of C_D increase gradually for the Reynolds number range $Re \geq 6.9 \times 10^5$, and this tendency continues until $Re = 1.4 \times 10^6$. This range corresponds to upper transition and the values of drag coefficient C_D increase from 0.5 to 0.7. The values of drag coefficient C_D are constant at $C_D \approx 0.7$ for $Re \geq 1.4 \times 10^6$ and they continue until the measured maximum Reynolds number of 10^7 . For $Re \geq 3.2 \times 10^6$ the values of C_D coincide with the results of (2), (10), but are larger than those of (4), (7).

3.2 Strouhal number

The variation of Strouhal number St with Reynolds number is shown in Fig. 7 and the spectra of velocity fluctuation for typical Reynolds numbers are shown in Figs. 8-a-d. Strouhal number is $St \approx 0.2$ for the Reynolds number range $10^5 \leq Re \leq 2.8 \times 10^5$. There can be seen a monotonous velocity fluctuation in the velocity fluctuation spectrum. Strouhal number increases suddenly at about $Re = 3.6 \times 10^5$. This Reynolds number is the beginning of critical Reynolds number range and corresponds to a sudden reduction of the wake width. In the range $3.6 \times 10^5 \leq Re \leq 5 \times 10^5$, corresponding to the critical Reynolds number range, the Strouhal numbers deviate and vary from 0.08 to 0.33. In this range, as shown in Fig. 8-b, the periodicity of vortex shedding is not clear, and there can scarcely be seen any monotonous velocity fluctuations. The cause of deviations of the Strouhal numbers from experimental values by other authors seems to be in the lack of periodicity of velocity fluctuations. With an increase in the Reynolds number the deviation of the Strouhal numbers decreases for $Re \geq 5 \times 10^5$. A monotonous vortex shedding can be seen in the fluctuating velocity spectra for $Re = 5.7 \times 10^5$. With an increase in the Reynolds number, a monotonous velocity fluctuation can be recognized in the spectra of velocity fluctuation, especially clearly for $Re \geq 3.4 \times 10^6$. $St = 0.26$ was acquired for $Re \geq 5.7 \times 10^5$.

4. Conclusions

Using the same experimental arrangement, the surface static pressure distributions, the drag coefficients and the Strouhal numbers of a cylindrical model placed in a uniform stream in the range of $10^5 \leq Re \leq 10^7$ were measured and the following conclusions were obtained.

Fig. 7 Variation of Strouhal number St with Reynolds number Re

Fig. 8 Spectra of velocity fluctuation for typical Reynolds numbers Re

- (1) Typical features are obtained in the surface static pressure distributions for subcritical, lower transition, critical, upper transition and transcritical Reynolds number ranges respectively.
- (2) The calculated drag coefficient C_D also shows typical characteristics for each Reynolds number range. It is constant in the transcritical Reynolds number range.
- (3) The Strouhal number is constant in subcritical and lower transition Reynolds number range. But it is not constant in supercritical Reynolds number range. In Reynolds number range $Re \geq 5.7 \times 10^5$, it is constant again ($St \approx 0.26$).

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