Study on Three-Dimensional Flow and Heat Transfer in Miter-Bend

(2nd Report, Analysis of Heat Transfer in Laminar Region)*

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Three-dimensional flow and heat transfer in a rectangular miter-bend, with a 90° turn, were studied for the purpose of examining the effect of centrifugal force due to bending, in connection with a corrugated wall channel as a means of augmenting the forced-convection heat transfer with a single-phase flow in a heat exchanger.

Analysis of the results is made on the heat transfer in a laminar region for aspect ratio $\alpha=1$, Reynolds number $Re=300$ and Prandtl number $Pr=0.71$. Heat transfer characteristics under the constant wall heat flux condition in terms of distributions of temperatures, local Nusselt numbers and cross-sectional mean Nusselt numbers are shown with detailed examination of heat transfer mechanism.

Key Word: Convective Heat Transfer, Numerical Analysis, FDM, Miter-Bend, Three-Dimensional Flow, Laminar Flow, Heat Exchanger

1. Introduction

The authors (1)-(5) have systematically studied a two-dimensional corrugated wall channel as a means of augmenting the forced-convection heat transfer with a single-phase flow in a heat exchanger. As a first step of three-dimensional analysis, they have also reported in the previous paper (6) on the analysis of a three-dimensional flow with aspect ratio $\alpha=1$ and Reynolds number $Re=300$ in a miter-bend. This paper presents results of an analysis of heat transfer characteristics in a miter-bend.

There are few studies on heat transfer in a 90° miter-bend, but many studies about a bend with a smooth curvature as follows. Mori et al. (7), (8) studied the heat transfer in a bend by modelling the secondary flow as for a developed flow. As for the heat transfer in an entrance flow, many studies were made by Janssen et al. (9), and others. Yee et al. (10) analyzed a heat transfer in a laminar region by a perfect elliptic equation, showed the distributions of temperatures and Nusselt numbers, and pointed out that a parabolic or a semi-elliptic method was not adequate for a channel with large curvature. Chilukuri et al. (11) studied the heat transfer in

the case where the floating power acted, and showed the aspects of local Nusselt numbers at several flow sections. Akiyama (12) too made a study of the same kind. Mayle et al. (13) and Seki et al. (14), (15) showed the distribution of local Nusselt numbers of the flow direction both on the concave and the convex surfaces by experiments in a turbulent region, and indicated that Taylor-Görtler vortex on the concave surface augmented the heat transfer.

As mentioned above, the heat transfer in a bend with a smooth curvature has been well studied analytically and experimentally. In a miter-bend, a similar complex three-dimensional flow by centrifugal force due to bending seems to influence the heat transfer considerably. So in this paper, the heat transfer characteristics in a miter-bend were investigated on the basis of the results of the previous analysis of flow. The distributions of temperatures, local Nusselt numbers and cross-sectional mean Nusselt numbers for aspect ratio $\alpha=1$, Reynolds number $Re=300$ and Prandtl number $Pr=0.71$, were shown with detailed examination of heat transfer mechanism. The analysis was made numerically in the same way as in the previous paper, and an energy equation was solved by using Patankar's finite difference method (16) on the temperature equation for a steady incompressible viscous laminar flow.

Nomenclature

α : thermal diffusivity
 D_2, D_3 : channel widths
 d_e : characteristic diameter $=2D_2D_3$
 f : Fanning's friction coefficient
 h : heat transfer coefficient
 I, J, K : grid numbers

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- N_u : Nusselt number
 p : pressure
 Pr : Prandtl number
 q : heat flux
 Re : Reynolds number $= u_m d_e / \nu$
 t : time
 u : velocity
 x_1, x_2, x_3 : coordinates
 $\textcircled{1} \textcircled{2}$: divided zones in channel
 α : aspect ratio $= D_3 / D_2$
 θ : temperature
 λ : thermal conductivity
 ν : kinematic viscosity
 ρ : density
 Subscripts
 1 : in x_1 direction
 2 : in x_2 direction
 3 : in x_3 direction
 in : inlet
 m : mean
 max : maximum
 n : number of iterations
 s : starting point
 w : wall
 * : modified
 - : nondimensional
 - : integral mean

2. Calculation Method

Considering a steady incompressible viscous laminar flow in the channel and constant physical properties of fluid, the basic equations for three-dimensional flow and heat transfer are continuity, Navier-Stokes and energy equations as follows:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} \quad (2)$$

$$\frac{D\theta}{Dt} = \alpha \Delta \theta \quad (3)$$

The calculation method is similar to that in the previous paper (6). The channel is divided into two zones with overlapping part as shown in Fig.1, and three-dimensional Cartesian coordinates are taken in each zone; the origin is at the side end of the inner bending corner.

Fig.1 Analytical model

x_1 is in the flow direction, x_2 and x_3 are in the cross section perpendicular to it. D_2 and D_3 are the channel widths in x_2 and x_3 directions respectively. Inlet and outlet of the channel locate at $1.5D_3$ upstream and $11.84D_3$ downstream of the inner bending corner respectively. The differential grids of the channel are shown in Fig.2. Using the control volume of rectangular faces as shown in Fig.3, we define pressure p and temperature θ at its central point and normal velocity components u_1, u_2 and u_3 at the middle of each face respectively. So we calculate numerically by using Patankar's finite difference method (16). In this paper we use the values of velocity and pressure which were calculated in the previous paper, and therefore we calculate only temperatures.

The boundary conditions for the temperature equation are as follows. It is assumed that the wall heat flux is constant, and all the walls are heated. Because this condition is practical case and makes it easy to compare with experimental results. We use a developed temperature profile in a straight duct for the same thermal condition at the inlet, which is calculated in the same way. At the outlet the temperature gradient is made constant, as the temperature distribution seems to be fully-developed there. The values at the boundaries of two zones are given by the values at the corresponding inner points in overlapping parts, and so the two zones are continuously connected.

Fig.2 Differential grids of miter-bend

Fig.3 Control volume

In calculations, we apply an upwind-difference method, and take 1.0 as a value for the relaxation parameter. We set $\max |(\theta_{i,j,k} - \theta_{i,j,k}^*)| / (q_w d_e / \lambda) < 10^{-6}$ for convergence criterion, where n is the number of iterations and $\theta_{i,j,k}$ is the temperature at each grid point. Interpolation method is the same as in the previous paper⁽⁶⁾, that is, Lagrange's second-ordered interpolation formula⁽¹⁷⁾.

3. Calculation Results and Examination

In this paper, heat transfer characteristics under the constant wall heat flux condition for aspect ratio $\alpha=1$, Reynolds number $Re=300$ and Prandtl number $Pr=0.71$ are reported, and the distributions of temperatures and Nusselt numbers are shown with detailed examination of heat transfer mechanism.

3.1 Temperature distributions

The temperature contours at each section of the channel are shown in Fig.4 and Fig.5. Figure 4 corresponds to the section of $J=6$. Figs.5 (a) to (d) correspond to the sections perpendicular to the flow direction. The position of $I=10$ locates upstream of the bending and those of $I=30, 40$ and 59 downstream of the bending, respectively. The numerals

Fig.4 Temperature contours ($J=6$)

Fig.5 Temperature contours

written alongside the curved lines indicate the nondimensional temperatures $\theta' \equiv (\theta - \theta_s) / (q_w d_e / \lambda)$, where θ_s is the reference temperature at a position which is at the center of the cross section at the inlet, and then q_w and λ are wall heat flux and thermal conductivity respectively.

As shown in Fig.4, the temperature contours upstream of the bending in a miter-bend are almost similar to those of a straight duct, and lines are parallel to walls. But they are complicated at the bending. That is, the cold main flow of the middle of channel goes against the outer wall downstream of the bending, and the temperature is low there. The temperatures in the neighborhood of the separation point upstream of the bending and of the wall of the separation zone downstream of the bending are high because of the slow flow.

The temperature contours at the sections perpendicular to the flow direction are shown in Fig.5. The contours at $I=10$ just upstream of the bending are concentric circular as at the inlet, but the temperatures near the inner wall are low as compared with those near the outer wall, because the flow is accelerated by inclining towards the inner wall. The temperature contours at the section of $I=30$ downstream of the bending are no longer concentric circular, and the temperature is lowest on the outer wall and highest on the inner wall, because of the cold main flow running against it. The temperature near the side wall is slightly low due to a secondary flow. At the section of $I=40$, the temperature is lowest in the neighborhood of both side walls near the outer wall, and is low at the horseshoe zone near the side wall and the outer wall because of the effect of the secondary flow. At the outlet section of $I=59$, the temperature contours become almost concentric circular the same as those at the inlet, although the temperature is not lowest at the center of section but near the outer wall.

The nondimensional temperatures in the flow direction are shown in Fig.6, where the horizontal axis is x_1/D_3 in the flow direction which describes the temperatures on the outer wall continuously. The solid lines correspond to the central temperatures θ_w' of the

Fig.6 Temperature distribution in the flow direction

inner wall, the outer wall and the side wall, and the dashed line corresponds to the integral mean temperature θ_w' for the walls around the sections perpendicular to the flow direction. Furthermore, the chain line indicates the nondimensional bulk temperature θ_b' , which is calculated taking the mixing mean temperature at the inlet as the reference value and assuming that the fluid flows regularly without separation and holds all the heat quantity between the heat starting point and the considered point, as in the previous paper. Therefore, θ_b' increases straightly in the flow direction, and the gradient at $x_1/D_3=0 \sim 0$ of the bending is $1/2$, since the heat quantity is $1/2$ of that at other points. This bulk temperature can be thought to correspond approximately to the mixing mean temperature at the section perpendicular to the flow direction. As the walls are heated under the constant heat flux condition, the wall temperatures generally should be higher than the corresponding bulk temperatures. But the central temperature at the outer wall is lower than the bulk temperature as shown in Fig.6. Furthermore the central temperatures on all the walls should be the same at the place where the flow is hydrodynamically and thermally fully-developed as at the inlet, and should increase straightly in the flow direction with a certain constant value from the bulk temperature, but the development is too slow at the outlet of this calculation region to become as mentioned above. The integral mean temperature is mostly higher than the central one as it is high at the corner of the section, and is higher than the bulk temperature.

Figure 7 corresponds to the nondimensional temperature distribution on the walls around the sections perpendicular to the flow direction, where the horizontal axis is shown to connect that wall to the side wall to the center. The curved lines correspond to the local wall temperatures θ_w' , and the straight solid lines and the straight dashed lines correspond to the integral mean wall temperatures θ_w' and the bulk temperatures θ_b' , respectively. In Fig.7, the temperatures on the side wall near the

Fig.7 Temperature distribution around walls

outer wall and most of the outer walls are found to be lower than the bulk temperatures θ_b' . The temperature on the inner wall is lower than on the outer wall upstream of the bending, but higher downstream of the bending, as a whole. Then the temperature distribution at $I=59$ of the outlet is not perfectly equal to that in a straight duct, and has a similar tendency. On the inner wall of $I=40$ downstream of the inner bending corner, the temperature has a maximum at the center of the wall, which is not seen at any other place.

3.2 Nusselt number distribution

The local Nusselt number N_u is generally defined with the bulk temperature θ_b shown in Section 3.1 as follows:

$$N_u = \frac{hd_e}{\lambda} = \frac{(q_w/\lambda)d_e}{\theta_w - \theta_b} \quad (4)$$

In the case of a three-dimensional flow, the corresponding value of θ_b should be used for N_u of the walls at the sections perpendicular to the flow direction. The cross-sectional mean Nusselt number is defined as a mean value on the wall of the section perpendicular to the flow direction as follows:

$$N_{um} = \frac{(q_w/\lambda)d_e}{\theta_w - \theta_b} \quad (5)$$

As shown in Fig.6 and Fig.7, the wall temperature θ_w may be lower than the bulk temperature θ_b , and then its N_u is negative. This will be very inconvenient to discuss the goodness of heat transfer. This doesn't happen in a two-dimensional channel(1), and it is necessary to modify the definition of the local Nusselt number for the heat transfer in a three-dimensional flow. Consequently in this paper, we define the modified Nusselt number for convenience sake as below. At first we consider the temperature θ_c instead of θ_b , which is uniformly lower than θ_b at each place of the flow direction and agrees with the central temperature at the inlet, that is, the minimum at the place where the flow is thermally fully-developed. Using this θ_c , we define the local Nusselt number N_{uc} and the cross-sectional mean Nusselt number N_{ucm} in the same way as Eqs. (4) and (5).

$$N_{uc} = \frac{(q_w/\lambda)d_e}{\theta_w - \theta_c} \quad (6)$$

$$N_{ucm} = \frac{(q_w/\lambda)d_e}{\theta_w - \theta_c} \quad (7)$$

Next we consider the ratio of the previous N_{um} to N_{ucm} . We multiply Eq.(6) and Eq.(7) by this ratio, and obtain the modified local Nusselt number N_u^* and the modified cross-sectional mean Nusselt number N_{um}^* as follows:

$$N_{u}^{*} \equiv N_{uc} \times \left(\frac{N_{um}}{N_{ucm}} \right)_{in} = \frac{(q_w/\lambda) d_e}{\theta_w - \theta_c} \left(\frac{\bar{\theta}_w - \theta_c}{\bar{\theta}_w - \theta_b} \right)_{in} \quad (8)$$

$$N_{u}^{*} \equiv N_{ucm} \times \left(\frac{N_{um}}{N_{ucm}} \right)_{in} = \frac{(q_w/\lambda) d_e}{\theta_w - \theta_c} \left(\frac{\bar{\theta}_w - \theta_c}{\bar{\theta}_w - \theta_b} \right)_{in} \quad (9)$$

Therefore at the inlet, N_{um}^{*} agrees with N_{um} and corresponds to the value for the fully-developed flow in a straight duct. Such modified local Nusselt number contours on each wall are shown in Fig.8. Figure 8(a) corresponds to the side wall of $J=1$, Figs.8 (b) and (c) to the inner walls of $K=1$ upstream and downstream of the bending, and Figs.8 (d) and (e) to the outer walls of $K=11$ upstream and downstream of the bending, respectively. As shown in these figures, the modified local Nusselt numbers give complicated contours in the neighborhood of the bending. They have a very large maximum value downstream of the impinging point and on the side walls near the outer wall, and also have a maximum on the inner wall near the side walls downstream of the inner bending corner and on the outer wall upstream of the outer bending corner. They increase a little on the inner wall upstream of the inner bending corner where the flow is accelerated by inclining towards the inner wall. Furthermore, they have a minimum on the outer wall upstream of the separation zone at the outer bending corner and on the inner wall downstream of the inner bending corner.

Fig.8 Modified local Nusselt number contours

These aspects, as a whole, almost agree with the wall shear stress contours in the previous paper (6). But the positions of the maximum and the minimum are slightly different and N_u^{*} has no minimum at the impinging point as shown in the wall shear stress contours.

The local Nusselt numbers and the cross-sectional mean Nusselt numbers at each wall are shown in Figs.9 to 11. Figure 9 corresponds to the modified local Nusselt number distribution in the flow direction at the middle of each wall. On the outer wall, N_u^{*} decreases while approaching the bending, and has a minimum at the bending. Then it increases and has a maximum immediately upstream of the outer bending corner. Downstream of the outer bending corner, it has a very large maximum value at the end of bending, and smoothly decreases towards the outlet. At the inner wall, N_u^{*} gradually increases while approaching the bending and has a maximum at the inner bending corner. It has a minimum immediately downstream of the inner bending corner and then it smoothly increases. On the side wall it shows an aspect similar to that on the outer wall, but the positions of its maximum and minimum are slightly different, that is, the maximum and the minimum on the side wall locate downstream and upstream of those on the outer wall, respectively.

Figure 10 shows the Nusselt number

Fig.9 Modified local Nusselt number distribution in the flow direction

Fig.10 Nusselt number distribution around walls

distribution on the walls of the sections perpendicular to the flow direction, where the solid lines and the dashed lines correspond to the modified local Nusselt numbers and the cross-sectional mean Nusselt numbers, respectively. N_u^* is large on the inner wall and small on the outer wall upstream of the bending, but this is reversed downstream of the bending. At $I=30$ downstream of $0.24D_3$ from the bending, N_u^* has maximums on the side wall near the outer wall and at the middle of the outer wall. N_u^* is generally smallest at the corner of section and is maximum at the middle, but it has two maximums on the inner wall near the side wall at $I=40$ downstream of $2.34D_3$ from the bending. The heat transfer augments on the outer wall downstream of the bending, and N_u^* is much larger than N_{um} at the middle of the outer wall.

Figure 11 shows the cross-sectional mean Nusselt number N_{um} and the modified cross-sectional mean Nusselt number N_{um}^* . Note that N_{um} is never negative since θ_w is always larger than θ_b as shown in Fig. 6. It is seen in Fig. 11 that N_{um} is qualitatively similar to N_{um}^* although the values downstream of the bending are considerably different. The cross-sectional mean Nusselt number at the inlet where the flow is assumed to be thermally fully-developed is the corresponding value in a straight duct, 2.980. It is in good agreement with its theoretical value⁽¹⁸⁾. The cross-sectional mean Nusselt number little changes upstream of the bending, but has a very small value of maximum immediately upstream of the bending. On the other hand, it shows a great change and a large value downstream of the bending. It has a minimum immediately downstream of the inner bending corner, has a large value of maximum at $0.66D_3$ downstream of the bending, and then smoothly decreases. It is proved that the effect of bending remains fairly far downstream, and the cross-sectional Nusselt numbers are large over a wide region downstream.

3.3 Comparison between wall pressure distribution and wall shear stress distribution

The distributions of wall pressures

Fig. 11 Cross-sectional mean Nusselt number distribution in the flow direction

and wall shear stresses are shown in Fig. 12 and Fig. 13. Figure 12(a) and Fig. 13(a) correspond to the flow direction, and Fig. 12(b) and Fig. 13(b) correspond to the sections perpendicular to the flow direction. The values on the vertical axis are the pressure coefficients $(p-p_s)/(pu_m^2/2)$ where p_s is the reference pressure at the center position of the inlet in Fig. 12, and the product fR_e of Fanning's friction coefficient f and Reynolds number R_e in Fig. 13. These distributions were not shown in the previous paper⁽⁶⁾, and so here the Nusselt numbers will be compared with them.

The wall pressure has a complicated distribution at the bending. But at $5D_3$ downstream of the bending, it is almost universal at a section and has the same pressure gradient as in a straight duct. The pressure on the wall round the section perpendicular to the flow direction is universal too anywhere except at the bending. On the other hand the wall shear stress has a large change in the neighborhood of the bending. This change remains far downstream of the bending and its distribution hardly becomes the same one in a straight duct. And there are maximums at the separation point, the impinging point and the reattachment point. They locate upstream of the outer bending corner, downstream of the outer bending corner and downstream of the inner bending corner, respectively. At the section perpendicular to the flow

(a) Distribution in the flow direction

(b) Distribution around walls

Fig. 12 Wall pressure distribution

(a) Distribution in the flow direction

(b) Distribution around walls

Fig.13 Wall shear stress distribution

direction the wall shear stress is very small at the corner of section, and generally it has a maximum at the middle of wall. But the position of its maximum shifts to the corner at the bending where the effect of centrifugal force due to bending is remarkable. Such aspects are the same as the local Nusselt number distribution in Section 3.2.

Here we try to examine the positions of maximum of the local Nusselt number, the wall pressure and the wall shear stress. The position of maximum of wall shear stress situates at the immediate downstream of that of wall pressure as pointed out in the previous paper (6). That of local Nusselt number comes between these two positions. Then it is proved that the heat transfer augmentation is large at the places where the impinging flow is accelerated and the wall shear stress has a maximum.

4. Conclusions

With respect to the three-dimensional flow and heat transfer in a rectangular miter-bend, with 90° turn, an analysis of the heat transfer in a steady laminar region is made numerically by finite difference method. The heat transfer characteristics under the constant wall heat flux condition are shown for aspect ratio $\alpha=1$, Reynolds number $Re=300$ and Prandtl number $Pr=0.71$. The results are summarized as follows:

- (1) The channel is divided into two zones with overlapping parts for a miter-bend, and the three-dimensional calculation is made for the heat transfer in the whole channel numerically by applying Patankar's finite difference method to each zone and linking the values in the overlapping part.
- (2) The effect of centrifugal force due to bending on the heat transfer is made clear by showing the temperatures, the local Nusselt numbers and the cross-sectional mean Nusselt numbers.
- (3) The complicated aspects of the heat transfer at the bending are made clear which are generated by the impinging

flow against the opposite wall and the secondary flow at the section perpendicular to the flow direction.

(4) It is pointed out that the definition of local Nusselt number has to be modified for such a three-dimensional flow as in a miter-bend, and the complicated distributions of the modified local Nusselt numbers and the cross-sectional mean Nusselt numbers are made clear.

(5) The effect of bending remains far downstream, and the cross-sectional mean Nusselt numbers are large over a wide region. Then considerable heat transfer augmentations are expected.

(6) The local Nusselt number distribution is almost the same as the wall shear stress distribution. It is proved that the heat transfer augmentations are large at the places where the impinging flow is accelerated and the wall shear stress has a maximum.

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