Study on Three-Dimensional Flow and Heat Transfer in Miter-Bend

(3rd Report, Effects of Reynolds Number and Aspect Ratio)*

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Three-dimensional flow and heat transfer in a rectangular miter-bend, with a 90° turn, were studied for the purpose of examining the effect of centrifugal force due to bending, in connection with a corrugated wall channel as a means of augmenting a forced-convection heat transfer with a single-phase flow in a heat exchanger.

Analysis of the results is made for Reynolds number $Re=10$ to 600 and aspect ratio $\alpha=0, 0.5$ and 1. Aspects of the flow and heat transfer characteristics are shown in terms of velocity vectors, path lines, intensities of secondary flow, wall pressure, wall shear stresses, local Nusselt numbers and cross-sectional mean Nusselt numbers.

Key Word: Convective Heat Transfer, Numerical Analysis, FDM, Miter-Bend, Three-Dimensional Flow, Reynolds number, Aspect ratio, Heat Exchanger

1. Introduction

The authors(1)-(5) have studied a two-dimensional corrugated wall channel systematically as a means of augmenting a forced-convection heat transfer with a single-phase flow in a heat exchanger. In the previous papers(6), (7), they have also reported on the analyses of a three-dimensional flow and heat transfer in a miter-bend for a laminar region with aspect ratio $\alpha=1$ and Reynolds number $Re=300$. In this paper the effects of aspect ratio and Reynolds number on the flow and heat transfer are reported. The past studies on a miter-bend were reviewed in the previous papers(6), (7). There is no study which systematically examined the effects of aspect ratio and Reynolds number.

Aspect ratio α is taken as $\alpha=0$ for a two-dimensional channel and as $\alpha=0.5$ for the channel between this and the square-duct channel of $\alpha=1$ in the previous paper. On the other hand Reynolds number Re is taken as $Re=10$ to 600. It seems that for $Re=10$ there is a little secondary flow and the flow is without separation and reattachment, and for $Re=600$ a steady laminar flow can not be maintained in a miter-bend.

Analyses were made numerically in the same way as the previous papers(6), (7).

The effects of aspect ratio and Reynolds number for velocity vectors, path lines, intensities of secondary flow, wall pressures, wall shear stresses, local Nusselt numbers and cross-sectional mean Nusselt numbers are examined in detail, and the effect of centrifugal force due to bending is systematically made clear.

Nomenclature

A, B, C, D: positions of maximum
 α : thermal diffusivity
 D_2, D_3 : channel widths
 d_e : characteristic diameter
 f : Fanning's friction coefficient
 h : heat transfer coefficient
 I, J, K: grid numbers

$$Nu: \text{Nusselt number} \left[\frac{h d_e}{\lambda} = \frac{(q_w / \lambda) d_e}{\theta_w - \theta_b} \right]$$

p : pressure
 Pr : Prandtl number
 q : heat flux
 Re : Reynolds number
 t : time
 u : velocity
 x_1, x_2, x_3 : coordinates
 $\Pi, \square$: divided zones in channel
 α : aspect ratio
 θ : temperature
 λ : thermal conductivity
 ν : kinematic viscosity
 ρ : density

Subscripts

1: in x_1 direction
 2: in x_2 direction
 3: in x_3 direction
 b : bulk
 m : mean
 $\max$: maximum
 s : starting point
 w : wall
 $*$: modified
 $-$: integral mean

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2. Calculation Method

Analytical model is shown in Fig.1. We divide the channel into two zones I and II with an overlapping part, and take three-dimensional Cartesian coordinates in each zone; the origin is at the side end of the inner bending corner. x_1 is in the flow direction, x_2 and x_3 are in a cross section perpendicular to it. D_2 and D_3 are the channel widths in x_2 and x_3 directions respectively. Inlet and outlet of the channel are at $1.5D_3$ upstream and $11.84D_3$ downstream of the inner bending corner respectively. The calculation method is the same as that in the previous papers (6), (7). Using the control volume of rectangular face as shown in Fig.2, we define pressure p and temperature θ at the central point of each volume and normal velocity components u_1 , u_2 and u_3 at the middle of each face respectively. Considering a steady incompressible viscous laminar flow in the channel and constant physical properties of fluid, the basic equations for the three-dimensional flow and heat transfer are continuity, Navier-Stokes and energy equations as follows:

Fig.1 Analytical model

Fig.2 Differential grids of miter bend

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} \quad (2)$$

$$\frac{D\theta}{Dt} = \alpha \Delta \theta \quad (3)$$

We calculate these equations numerically by using Patankar's finite difference method (8) for various Reynolds numbers Re and aspect ratios α , where $Re \equiv u_m d_e / \nu$ and $\alpha \equiv D_3 / D_2$. And u_m is the mean velocity and $d_e \equiv 2D_2 D_3 / (D_2 + D_3)$. The boundary conditions are almost the same as those in the previous paper. But as for aspect ratio $\alpha=0$, that is, a two-dimensional parallel channel, it is assumed that variables are constant in x_2 direction, and so variables for only x_1 and x_3 directions are calculated. And the condition for the convergence criterion and the interpolation method are the same as those in the previous papers (6), (7).

3. Calculation Results and Examination

The calculations in this paper are made for aspect ratio $\alpha=0, 0.5$ and 1 and Reynolds number $Re=10$ to 600 as shown in the upper part of Table 1. When aspect ratios are different, the characteristic diameters are different. And the ratio of maximum velocity u_{max} to mean velocity u_m is not constant even in a straight duct. Hence it is not naturally adequate to study for a constant Reynolds number Re on the flow in a miter-bend where the centrifugal force is strong. So for

Table 1 Calculation extents and results

Geometry										
n		0			0.5		1			
(k, D ₃)		2			4.3		1			
Re ⁻¹ (u _m k) ⁻¹		150	300	600	300	10	75	150	300	600
u _m D ₃		75	150	300	225	10	75	150	300	600
(u _{max} /D ₃) ⁻¹		113	225	450	448	21	157	314	629	1258
Straight Duct	u _{max} /u _m	1.5			1.992		2.096			
	f/Re	24.0			15.548		14.227			
	Nu	8.235			3.017		3.091			
$\left(\frac{p-p_s}{1/2\rho u_m^2}\right)_{max}$	A	0.56	0.77	0.86	3.13	7.19	1.59	2.53	2.97	3.20
	B				0.68	0.04	0.25	0.94	1.44	1.77
	C	2.17	1.96	1.86	1.86	—	3.16	2.63	2.27	1.97
	D									
f/Re _{max}	A	62.6	90.8	130.7	114.8	22.9	53.7	74.5	98.7	112.5
	B				123.6	21.7	40.8	61.9	84.8	101.4
	C	15.4	36.0	48.3	70.7	35.9	39.5	42.3	48.0	54.0
	D	4.5	10.6	21.1	51.2	—	6.9	11.7	25.6	44.5
(Nu*) _{max}	A	8.30	11.47	15.38	22.58	4.13	12.79	15.80	17.73	17.66
	B				7.57	34.21	5.00	7.63	9.77	11.16
	C	8.07	8.54	10.62	6.20	458.8	6.33	5.09	5.05	6.53
	D		4.69	7.76	10.00	3.05	3.25	3.77	5.79	7.05
u*/u _m	I 10	0.40	0.40	0.38	0.66	0.42	0.48	0.50	0.57	0.69
	I 30	0.51	0.59	0.65	0.86	0.36	0.85	0.90	1.04	1.20
	I 40	0.05	0.07	0.07	0.34	0.00	0.11	0.22	0.31	0.34
	I 59	0.00	0.00	0.00	0.06	0.00	0.00	0.01	0.04	0.06

reference, d_e/D_3 , $u_m D_3/\nu$ and $u_{max} D_3/\nu$ are shown in Table 1. As for a straight duct, u_{max}/u_m , fR_e and N_u are known for each aspect ratio. They are constant for various Reynolds numbers, if the flow is hydrodynamically and thermally fully-developed. They are shown in Table 1 for reference, too. Then fR_e is the product of Fanning's friction coefficient and Reynolds number, and N_u is Nusselt number. So the mean values of them are shown. Further in the figures, we show velocity vectors in such way that the mean velocity u_m becomes the same for all Reynolds numbers, and show the channels in such way that the width D_3 becomes the same for all aspect ratios.

3.1 Effects of Reynolds number

Results for aspect ratio $\alpha=1$ and Reynolds number $R_e=10, 75, 150$ and 600 are examined comparing with the results for $R_e=300$ in the previous papers (6), (7).

Velocity vectors for $R_e=10$ and 600 are shown in Fig.3 and Fig.4. Figures (a) to (f) correspond to the sections of $K=2$ and $K=10$ downstream of the bending, the section of $I=30$ perpendicular to the flow direction and the sections of $J=2, 4$ and 6 in the flow direction, respectively. When Reynolds number is as small as $R_e=10$, the stream from upstream is hardly pushed against the outer wall in the bending and it turns downstream smoothly, and then never flows back at the bending corners although the velocity drops. The effect of

bending doesn't persist so far downstream, and so the velocity profile becomes almost that of a straight duct at D_3 downstream of the bending. The velocity near the center of the channel section is large as a whole. There is a little secondary flow in the neighborhood of the bending, and the fluid flows almost straight without curling spirally. When Reynolds number is as large as $R_e=600$, the fast main flow from upstream runs against the outer wall of the bending because of a centrifugal force due to bending and then moves downstream. If Reynolds number is larger, this tendency increases much and the effect of bending remains far downstream. As shown in Fig.4(d), the velocity vectors are fairly large at the bending near the side wall and downstream of it. They have different aspects from those at the sections of $J=4$ and 6 , and a complicated three-dimensional flow with a large secondary flow occurs. This tendency is seen for $R_e=300$ in the previous paper, but it is more remarkable for $R_e=600$. When Reynolds number is larger, the extent of vortex zone of the outer bending corner increases and the separation point on the wall upstream of the outer bending corner shifts upstream. The extent of backflow zone downstream of the inner bending corner increases with Reynolds number, too. And a complicated three dimensional flow occurs there. Especially at the backflow zone downstream of the inner

Fig.3 Velocity vectors ($R_e=10$)

Fig.4 Velocity vectors ($R_e=600$)

bending corner and downstream of it, the flow is complicated and unstable as shown in Fig.4(f). When Reynolds number is very large, it seems that the instability increases and the flow becomes unsteady at these zones. Figures 5(a) and (b) show an oblique projection and an orthogonal projection of path lines of $J=2$ in the neighborhood of the side wall at the inlet for $Re=600$. They show such complicated aspects that the fluid flows downstream while backflowing many times at the separation zone downstream of the inner bending corner.

Next we examine the effects of Reynolds number on the wall pressure $(p-p_s)/(pu_m^2/2)$, the wall shear stress fR_e and the modified local Nusselt number N_u^* . Heat transfer characteristics are shown under the constant heat flux condition for Prandtl number $P_r=0.71$ such as in the previous paper (7). N_u^* is a modified Nusselt number as defined in the previous paper (7). It can be used for the three-dimensional flow and its mean value agrees with the common one for the fully-developed flow in a straight duct. They have maximums as pointed out in the previous papers (6), (7) at the positions of symbols A, B, C and D in Fig.6, that is, near the impinging point, on the side wall near the outer wall, on the inner wall near the side wall and upstream of the outer bending corner. These values are

(a) Oblique projection

(b) Orthogonal projection

Fig.5 Path lines ($\alpha=1$, $Re=600$, $J=2$)

shown in the middle part of Table 1. When Re is larger, $(fR_e)_{\max}$ is larger at each position. $(N_u^*)_{\max}$ has almost the same tendency as a whole, but some values are abnormal. The effects of Reynolds number on the distributions of the flow directions of $(p-p_s)/(pu_m^2/2)$, fR_e and N_u^* at the middle of the inner and the outer wall are shown in Figs.7 to 9, respectively. The horizontal axes are x_1/D_3 in the flow direction which makes the values on the outer wall continuous. When Reynolds number is large, those distributions in the flow direction have great changes and the effect of bending remains fairly far downstream. And the positions of maximum in the neighborhood of the impinging point shift upstream with the wall pressure and the local Nusselt number, and downstream with the wall shear stress. The effects of Reynolds number on the distribution of the cross-sectional mean Nusselt numbers N_{um} in the flow direction are shown in Fig.10. N_{um} is defined by using the integral mean wall temperature $\bar{\theta}_w$ at sections perpendicular to the flow direction as mentioned in the previous paper (7). And it doesn't have such abnormal values as N_u^* has. For $Re \geq 150$, N_{um} has a maximum at D_3 downstream of the bending and the effect of bending remains fairly far downstream. On the other hand for $Re \geq 75$, it has no maximums. N_{um} is small as a whole and shows the value in a straight duct at $5D_3$ downstream of the bending.

Lastly the effects of Reynolds number on intensities of secondary flow are examined. We use $u_{\max}'/u_m$ to express intensities of secondary flow qualitatively, where $u_{\max}'$ is the maximum

Fig.6 Positions of maximums

Fig.7 Wall pressure distribution

of velocity vectors at the section perpendicular to the flow direction, that is, $u_{\max} = \max |(u_2^2 + u_3^2)^{1/2}|$. The values of them for $I=10$ upstream, $I=30, 40$ and 59 downstream of the bending, are shown in the lower part of Table 1. When Reynolds number is large, the intensities of secondary flow downstream of the bending are large and the effect of secondary flow remains far downstream. On the other hand, the intensities of secondary flow at the section of $I=1$ in the immediate upstream of the bending are almost constant and considerably large for various Reynolds numbers.

Fig.8 Wall shear stress distribution

Fig.9 Modified local Nusselt number distribution

Fig.10 Cross-sectional mean Nusselt number distribution

3.2 Effects of aspect ratio

Results for aspect ratio $\alpha=0$ and 0.5 are examined comparing with those for $\alpha=1$. We calculate for $\alpha=0$ and $Re=150$ to 600 and for $\alpha=0.5$ and $Re=300$.

First the effects of aspect ratio on path lines are examined. The path lines for aspect ratio $\alpha=0$ and Reynolds number $Re=300$ and 600 are shown in Fig.11 and Fig.12. In the case of $\alpha=0$, the flow is two-dimensionally restricted and there can not be any secondary flow which goes round to the side direction. Hence for any Reynolds number, the tendency of the flow going against the opposite wall is less conspicuous comparing with the case for $\alpha=1$ and $Re=300$. And two separation zones in the neighborhood of the bending are isolated, and the flow just circulates there without the fluid upstream flowing therein. Furthermore the circulation in the separation zones should produce a cluster of closed curves, but because of a small numerical error, the circulation of the outer bending corner shows that path lines are curled with some width and the circulation of the inner bending corner shows that one path line continuously curls many times from the center of it. When Reynolds number becomes larger, the extents of these separation zone increase.

The orthogonal projections to each wall of path lines for $\alpha=0.5$ and $Re=300$ are shown in Fig.13. Figure 13(a), (b) and (c) show the path lines of $J=2, 4$ and 6 .

Fig.11 Path lines ($\alpha=0, Re=300$) Fig.12 Path lines ($\alpha=0, Re=600$)

As shown in these figures, the flow in the channel of $\alpha=0.5$ is similar to that of $\alpha=1$ and is a three-dimensional one which has a fairly large secondary flow. But comparing with the flow of $\alpha=1$ and $Re=300$ in the previous paper (6), the flow near the

(a) J=2

(b) J=4

(c) J=6

Fig.13 Orthogonal projections of path lines ($\alpha=0.5$, $Re=300$)

middle section has some two-dimensional aspects as seen in Fig.13(c).

Next the effects of aspect ratio on wall pressure, wall shear stresses and modified local Nusselt numbers are examined. These values of maximum for $\alpha=0$ and 0.5 are shown in the middle part of Table 1. As for the two-dimensional channel of aspect ratio $\alpha=0$, there can not be the position B on the side wall in Fig.6 and the change for x_2 direction, as a matter of course. In the case of $\alpha=0$, the values of maximum of wall pressure and modified local Nusselt number at the position A near the impinging point are not much larger than those in a straight duct. Especially the modified Nusselt numbers near the outer bending corner are much smaller than those in a straight

Fig.14 Wall pressure distribution

Fig.15 Wall shear stress distribution

Fig.16 Modified local Nusselt number distribution

duct. The values of maximum for $\alpha=0.5$ are almost similar to those for $\alpha=1$ and $Re=600$. So it is proved that the flow of $\alpha=0.5$ is fairly three-dimensional.

The effects of aspect ratio on the distributions of $(p-p_s)/(\rho u_m^2/2)$, fR_e and N_u^* in the flow direction at the middle of the inner and the outer wall for $Re=300$, are shown in Fig.14 to 16. The values of fR_e and N_u^* at the inlet are the same at both the inner and the outer walls, but they are different from the mean values shown in the upper part of Table 1 except for $\alpha=0$. Aspects of $\alpha=0.5$ are almost the same as those of $\alpha=1$ as a whole. So it is seen that a three-dimensional analysis is needed for a miter-bend with aspect ratio such as $\alpha=0.5$.

Lastly the effects of aspect ratio on intensities of secondary flow are examined. The values of u_{max}'/u_m are shown in the lower part of Table 1. In the case of $\alpha=0$, of course there is no secondary flow which circulates in a section. But the velocity component u_3 comes about because of an inclining of flow at the bending, and so u_{max}'/u_m is not zero. It is smaller than that of $\alpha=1$ and does not remain so far downstream. In the case of $\alpha=0.5$, the values of u_{max}'/u_m are not much larger than those of $\alpha=1$ at the bending. But they are not very small even at the place far downstream, and so it is seen that the secondary flow remains fairly far downstream.

4. Conclusions

The analyses of the effects of Reynolds number Re and aspect ratio α on the three-dimensional flow and heat transfer in a rectangular miter-bend with 90° turn are made numerically by finite difference method. The results are summarized as follows:

(1) Velocity vectors, path lines, intensities of secondary flow, wall pressures, wall shear stresses, local Nusselt numbers and cross-sectional mean Nusselt numbers are shown for aspect ratios $\alpha=0, 0.5$ and 1 and Reynolds numbers $Re=10$ to 600 . And the effects of Reynolds

number and aspect ratio on the flow and heat transfer are made clear.

(2) When Reynolds number is large, aspects of the flow and heat transfer are complicated and the effect of bending remains far downstream because of an increased effect of centrifugal force due to bending.

(3) In the case of aspect ratio $\alpha=0$, the flow doesn't incline to the opposite wall so much and two separation zones in the neighborhood of the bending are isolated. In the case of $\alpha=0.5$, the flow has some two-dimensional aspects comparing with $\alpha=1$, but it is fairly three-dimensional with a large secondary flow as a whole.

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