Natural Frequencies of Stator Core of Induction Motor* (2nd Report, Analysis of Radial Natural Frequencies)

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The method of calculating the natural frequencies of coupled vibrations between the radial vibration of a part of the ring and the tangential vibration of the teeth with the stator core of the induction motor is discussed. The difference between the calculated values of natural frequencies and the measured ones with three models of the stator core is within a few per cent. Therefore, it is possible to predict the natural frequencies of the stator core in the design of an induction motor. The effect of the shear deformation of the teeth on the natural frequencies, and the nature of the natural frequencies with omission of the terms for kinetic energy on the rotary motion of the teeth are also discussed.

Key Words : Vibration, Stator Core of Induction Motor, Natural Frequency, Radial Vibration, Vibration of Ring, Vibration of Tooth

1. Introduction

The stator core of an induction motor receives a magnetic exciting force from the rotor during operation. When an exciting frequency agrees with one of the natural frequencies of the stator core, resonance occurs in the stator core and undesirable vibration or noise arises. Many researchers have studied the exciting force due to the rotor and made a vibration analysis in which the stator core is approximately represented by a ring⁽¹⁾⁽²⁾⁽³⁾. But, it seems that the vibration analysis has not been done with regard to the coupled vibration between the ring part and the teeth part. To establish a prediction method for the natural frequencies of the stator core to avoid resonance, the natural frequencies are discussed with regard to the coupled vibration between the vibration of a ring in the radial direction and the in-plane bending vibration of teeth. In a previous paper⁽⁴⁾, the experimental and approximately calculated natural frequencies were discussed using three test models of stator core. In this paper the Rayleigh-Ritz method is used in the vibration analysis of the stator core, and the natural frequencies of the coupled vibration between the vibration of a ring in the radial direction and that of teeth in the direction of rotation are calculated. As the tooth length is considered to be under four times the width and it seems that the effect of shear deformation cannot be ignored, in this paper the analysis is done considering the effect of the shear deformation.

2. Calculating Method

2.1 Nomenclature

- A, A_R : areas of cross section of tooth and ring
- b, b_i, b_o : tooth widths
- $b_1 = 2\pi R_i / N, b_2 = 2\pi R_o / N$: Lengths of a ring corresponding to a tooth
- E : modulus of longitudinal elasticity
- G : modulus of transverse elasticity
- g : acceleration of gravity
- h : plate thickness
- I, I_R : moments of inertia of tooth and ring
- $1/k'$: form factor for cross section
- L, l_i : lengths
- l : length of tooth
- N : number of teeth around the ring
- n : deformation wave number to circumference of ring
- p : circular frequency
- $q_1 \sim q_4$: factors
- R, R_i, R_o : center radius, inner radius and outer radius of ring
- T : kinetic energy
- t : time
- u, v : displacements
- $\dot{u}, \dot{v}$: velocities
- V : strain energy
- x : length
- y, z : displacements
- α : angular displacement
- γ : specific weight
- θ : angular coordinate
- θ_i : angular coordinate at position of tooth, $i = 1 \sim N$

2.2 Analytical model and coordinate

Analytical model and coordinate used in calculation are shown in Fig. 1. Widths of the teeth are b_i and b_o , length is l . Center radius of the ring is represented by R , inner radius by R_i , and outer radius by R_o . Plate thickness of a core is h . Coordinates with the ring are represented by R and θ , and coordinate with tooth by x . Displacements of the ring are represented

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by u and v , angular displacement of the ring by α , and displacements of the teeth by y and z .

2.3 Terms considered in calculation

In this calculation, the following terms are considered:

- (1) Strain energy V
 - (i) Term V_R due to bending deformation of a ring
 - (ii) Term V_B due to bending deformation of teeth
 - (iii) Term V_S due to shear deformation of teeth
- (2) Kinetic energy T
 - (i) Term T_R due to motion of a ring
 - (ii) Term T_A due to motion of teeth in radial direction
 - (iii) Term T_B due to motion of teeth in tangential direction
 - (iv) Term T_r due to rotation of teeth

2.4 Deflection curve

Deflection curve of the ring is assumed as follows:

$$u = q_1 \cos n\theta \cos pt \quad \dots\dots\dots(1)$$

From the condition where elongation of the middle circle of a ring is zero, the next equation can be obtained⁽⁵⁾.

$$u = \frac{\partial v}{\partial \theta}$$

Using this equation and Eq. (1), we obtain:

$$v = \frac{1}{n} q_1 \sin n\theta \cos pt \quad \dots\dots\dots(2)$$

Deflection curves of a tooth are assumed as follows:

$$z = u = q_1 \cos n\theta \cos pt \quad \dots\dots\dots(3)$$

$$y = y_1 \sin n\theta \cos pt \quad \dots\dots\dots(4)$$

(a) Dimensions

(b) Ring displacement

(c) Tooth displacement

Fig. 1 Analytical model and coordinate

where

$$y_1 = Y_a q_1 + Y_b q_2 + Y_c q_3 + Y_d q_4$$

$Y_a q_1$: term due to the effect of vibration of a ring

$Y_b q_2, Y_c q_3$: terms due to bending deformation of a tooth

$Y_d q_4$: term due to shear deformation of a tooth

Y_a, Y_b, Y_c and Y_d in Eq. (4) are determined by the connecting condition between the ring and the teeth and by the boundary condition of the teeth, the results being as follows:

(1) Y_a

From the connecting condition between the ring and the tooth, the next equation can be obtained.

$$Y_a q_1 \sin n\theta \cos pt = v - \alpha x \quad \dots\dots\dots(5)$$

From Eqs. (2) and (5) and $\alpha = \left(\frac{1}{R}\right) \times \left(\frac{\partial v}{\partial \theta}\right)$, the next equation can be obtained.

$$Y_a = \frac{1}{n} + \frac{n}{R} x \quad \dots\dots\dots(6)$$

(2) Y_b

It is assumed that Y_b can be represented by the next equation of the fourth degree with x .

$$Y_b = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + a_5 x^4$$

There exist the following boundary conditions with Y_b :

$$Y_b = 0, \quad dY_b/dx = 0 \quad \text{on } x=0$$

$$\frac{d^2 Y_b}{dx^2} = \frac{d^3 Y_b}{dx^3} = 0 \quad \text{on } x=L$$

The following equation was used as an admissible function with Y_b :

$$Y_b = x^2 - \frac{2}{3L} x^3 + \frac{1}{6L^2} x^4 \quad \dots\dots\dots(7)$$

(3) Y_c

It is assumed that Y_c can be represented by an equation of the fifth degree with x . Y_c represents a term concerning the difference of the deflection curves between the tooth and the ring, and Y_c is defined only on the tooth (that is, a range of $l_1 \leq x \leq L$), and $Y_c = 0$ on the other part. The following boundary conditions exist concerning Y_c :

$$Y_c = 0, \quad dY_c/dx = 0 \quad \text{on } x=l_1$$

$$\frac{d^2 Y_c}{dx^2} = \frac{d^3 Y_c}{dx^3} = 0 \quad \text{on } x=L$$

The following equation was used as an admissible function with Y_c :

$$Y_c = (x - l_1)^2 - \frac{1}{2l} (x - l_1)^3 + \frac{1}{20l^3} (x - l_1)^5 \quad \dots\dots\dots(8)$$

(4) Y_d

There exist the following boundary conditions with Y_d :

$$Y_d = 0 \quad \text{on } x=0$$

$$\frac{dY_d}{dx} = 0 \quad \text{on } x=L$$

The following equation was used as an admissible function with Y_d :

$$Y_d = x - \frac{1}{2L} x^2 \dots\dots\dots(9)$$

2.5 Strain energy

2.5.1 Term V_R due to bending deformation of a ring

Strain energy V with bending deformation of a ring becomes as follows (5):

$$V = \frac{EI_R}{2R^4} \int_0^{2\pi} \left(\frac{\partial^2 u}{\partial \theta^2} + u \right)^2 R d\theta$$

$$= \frac{EI_R}{2R^3} (1 - n^2)^2 \pi q_1^2 \cos^2 pt$$

Maximum value $V_{\max}$ of strain energy becomes as follows:

$$V_{\max} = V_R = \frac{EI_R}{2R^3} (1 - n^2)^2 \pi q_1^2 \dots\dots\dots(10)$$

2.5.2 Term V_B due to bending deformation of teeth

We consider both the bending deformation of the ring corresponding to a tooth and that of the teeth.

Strain energy V with bending deformation of teeth becomes as follows:

$$V = \sum_{i=1}^N \left[\frac{1}{2} \int_0^L EI \left(\sin n\theta_i \cos pt \frac{d^2}{dx^2} (Y_b q_2 + Y_c q_3) \right)^2 dx \right]$$

$$= \frac{E}{2} \left(\sum_{i=1}^N \sin^2 n\theta_i \right) \cos^2 pt \left\{ \int_0^L I \times \left(\frac{d^2 Y_b}{dx^2} q_2 + \frac{d^2 Y_c}{dx^2} q_3 \right)^2 dx \right\}$$

Maximum value $V_{\max}$ of strain energy becomes as follows:

$$V_{\max} = V_B = \frac{E}{2} \left(\sum_{i=1}^N \sin^2 n\theta_i \right) \times (Q_1 q_2^2 + Q_2 q_2 q_3 + Q_3 q_3^2) \dots\dots\dots(11)$$

where

$$\left. \begin{aligned} Q_1 &= \int_0^L I \left(\frac{d^2 Y_b}{dx^2} \right)^2 dx \\ Q_2 &= 2 \int_0^L I \left(\frac{d^2 Y_b}{dx^2} \right) \left(\frac{d^2 Y_c}{dx^2} \right) dx \\ Q_3 &= \int_0^L I \left(\frac{d^2 Y_c}{dx^2} \right)^2 dx \end{aligned} \right\} \dots\dots\dots(12)$$

If the tooth dimensions are known, Q_1 , Q_2 and Q_3 in Eq. (11) can be calculated by Eqs. (12).

2.5.3 Term V_S due to shear deformation of teeth

Strain energy V with shear deformation of teeth becomes as follows:

$$V = \sum_{i=1}^N \left[\frac{1}{2} \int_0^L k' AG \left(\sin n\theta_i \cos pt \frac{d}{dx} (Y_d q_4) \right)^2 dx \right]$$

$$= \frac{1}{2} k' G \left(\sum_{i=1}^N \sin^2 n\theta_i \right) \cos^2 pt \left\{ \int_0^L A \left(\frac{dY_d}{dx} q_4 \right)^2 dx \right\}$$

Maximum value $V_{\max}$ of strain energy becomes as follows:

$$V_{\max} = V_S = \frac{1}{2} k' G \left(\sum_{i=1}^N \sin^2 n\theta_i \right) Q_4 q_4^2 \dots\dots\dots(13)$$

where

$$Q_4 = \int_0^L A \left(\frac{dY_d}{dx} \right)^2 dx$$

2.6 Kinetic energy

2.6.1 Term T_R due to motion of the ring

Kinetic energy T with a ring becomes as follows:

$$T = \frac{A_R \gamma}{2g} \int_0^{2\pi} (v^2 + v^2) R d\theta$$

$$= \frac{A_R \gamma}{2g} R p^2 \left(1 + \frac{1}{n^2} \right) \pi q_1^2 \sin^2 pt$$

Maximum value $T_{\max}$ of kinetic energy becomes as follows:

$$T_{\max} = T_R = \frac{A_R \gamma}{2g} R p^2 \left(1 + \frac{1}{n^2} \right) \pi q_1^2 \dots\dots\dots(14)$$

2.6.2 Term T_A due to motion of teeth in radial direction

Kinetic energy T due to motion of teeth in radial direction becomes as follows:

$$T = \sum_{i=1}^N \left[\frac{\gamma}{2g} \int_{x_1}^L A \dot{Z}^2 dx \right]$$

$$= \frac{\gamma}{2g} \left(\sum_{i=1}^N \cos^2 n\theta_i \right) p^2 \left(\int_{x_1}^L A dx \right) q_1^2 \sin^2 pt$$

Maximum value $T_{\max}$ of kinetic energy becomes as follows:

$$T_{\max} = T_A = \frac{\gamma}{2g} \left(\sum_{i=1}^N \cos^2 n\theta_i \right) p^2 B_A q_1^2 \dots\dots\dots(15)$$

where

$$B_A = \int_{x_1}^L A dx$$

2.6.3 Term T_B due to motion of teeth in tangential direction

Kinetic energy T due to motion of teeth in tangential direction becomes as follows:

$$T = \sum_{i=1}^N \left[\frac{\gamma}{2g} \int_0^L A \dot{y}^2 dx \right] = \frac{\gamma}{2g} \left(\sum_{i=1}^N \sin^2 n\theta_i \right) p^2 B_B \sin^2 pt$$

$$B_B = \int_0^L A (Y_a q_1 + Y_b q_2 + Y_c q_3 + Y_d q_4)^2 dx$$

Maximum value $T_{\max}$ of kinetic energy becomes as follows:

$$T_{\max} = T_B = \frac{\gamma}{2g} \left(\sum_{i=1}^N \sin^2 n\theta_i \right) p^2 B_B \dots\dots\dots(16)$$

$$\left. \begin{aligned} B_B &= \int_0^L A (Y_a q_1 + Y_b q_2 + Y_c q_3 + Y_d q_4)^2 dx = B_1 q_1^2 + B_2 q_1 q_2 \\ &+ B_3 q_1 q_3 + B_4 q_1 q_4 + B_5 q_2^2 \\ &+ B_6 q_2 q_3 + B_7 q_2 q_4 + B_8 q_3^2 \\ &+ B_9 q_3 q_4 + B_{10} q_4^2 \\ B_1 &= \int_0^L A Y_a^2 dx, \quad B_2 = 2 \int_0^L A Y_a Y_b dx \\ B_3 &= 2 \int_0^L A Y_a Y_c dx \\ B_4 &= 2 \int_0^L A Y_a Y_d dx, \quad B_5 = \int_0^L A Y_b^2 dx \\ B_6 &= 2 \int_0^L A Y_b Y_c dx \\ B_7 &= 2 \int_0^L A Y_b Y_d dx \\ B_8 &= \int_0^L A Y_c^2 dx, \quad B_9 = 2 \int_0^L A Y_c Y_d dx \\ B_{10} &= \int_0^L A Y_d^2 dx \end{aligned} \right\} \dots\dots\dots(17)$$

2.6.4 Term T_J due to rotation of teeth

Kinetic energy T due to rotation of teeth becomes as follows:

$$T = \sum_{i=1}^N \left[\frac{I}{2g} \int_0^L A x^2 a^2 dx \right] + \frac{I}{2g} \left(\sum_{i=1}^N \sin^2 n \theta_i \right) p^2 \left\{ \int_0^L I \left(\frac{dY_a}{dx} q_1 + \frac{dY_b}{dx} q_2 + \frac{dY_c}{dx} q_3 \right)^2 dx \right\} \sin^2 p t$$

$$= \frac{I}{2g} \left(\sum_{i=1}^N \sin^2 n \theta_i \right) p^2 \left(\frac{n}{R} \right)^2 \left(\int_0^L A x^2 dx \right) q_1^2 \sin^2 p t + \frac{I}{2g} \left(\sum_{i=1}^N \sin^2 n \theta_i \right) p^2 \left\{ \int_0^L I \left(\frac{dY_a}{dx} q_1 + \frac{dY_b}{dx} q_2 + \frac{dY_c}{dx} q_3 \right)^2 dx \right\} \sin^2 p t$$

Maximum value $T_{\max}$ of kinetic energy becomes as follows:

$$T_{\max} = T_J = \frac{I}{2g} \left(\sum_{i=1}^N \sin^2 n \theta_i \right) p^2 \left(\frac{n}{R} \right)^2 B_J q_1^2 + \frac{I}{2g} \left(\sum_{i=1}^N \sin^2 n \theta_i \right) \times p^2 (B_{11} q_1^2 + B_{12} q_1 q_2 + B_{13} q_1 q_3 + B_{14} q_2^2 + B_{15} q_2 q_3 + B_{16} q_3^2)$$

$$\left. \begin{aligned} B_J &= \int_0^L A x^2 dx, \quad B_{11} = \int_0^L I \left(\frac{dY_a}{dx} \right)^2 dx, \quad B_{12} = 2 \int_0^L I \left(\frac{dY_a}{dx} \right) \left(\frac{dY_b}{dx} \right) dx \\ B_{13} &= 2 \int_0^L I \left(\frac{dY_a}{dx} \right) \left(\frac{dY_c}{dx} \right) dx, \quad B_{14} = \int_0^L I \left(\frac{dY_b}{dx} \right)^2 dx \\ B_{15} &= 2 \int_0^L I \left(\frac{dY_b}{dx} \right) \left(\frac{dY_c}{dx} \right) dx, \quad B_{16} = \int_0^L I \left(\frac{dY_c}{dx} \right)^2 dx \end{aligned} \right\} \dots \dots \dots (18)$$

2.7 Calculation of natural frequency

From the condition that total maximum value ($V_R + V_B + V_S$) of strain energy equals total maximum value ($T_R + T_A + T_B + T_J$) of kinetic energy, the following equations can be obtained:

$$p^2 = \frac{D}{F}, \quad D = D_1 q_1^2 + D_2 q_2^2 + D_3 q_2 q_3 + D_4 q_3^2 + D_5 q_4^2$$

$$F = F_1 q_1^2 + F_2 q_1 q_2 + F_3 q_1 q_3 + F_4 q_1 q_4 + F_5 q_2^2 + F_6 q_2 q_3 + F_7 q_2 q_4 + F_8 q_3^2 + F_9 q_3 q_4 + F_{10} q_4^2 \quad \dots \dots \dots (19)$$

where

$$D_1 = \frac{EI_R}{2R^3} (1 - n^2)^2 \pi, \quad S = \sum_{i=1}^N \sin^2 n \theta_i, \quad C = \sum_{i=1}^N \cos^2 n \theta_i, \quad D_2 = \frac{E}{2} S Q_1$$

$$D_3 = ES Q_2 / 2, \quad D_4 = ES Q_3 / 2, \quad D_5 = k' GS Q_4 / 2$$

$$F_1 = \frac{A_R I_R}{2g} \left(1 + \frac{1}{n^2} \right) \pi + \frac{I}{2g} (CB_A + SB_1 + S \left(\frac{n}{R} \right)^2 B_J + SB_{11})$$

$$F_2 = \gamma S (B_2 + B_{12}) / 2g, \quad F_3 = \gamma S (B_3 + B_{13}) / 2g, \quad F_4 = \gamma S B_4 / 2g$$

$$F_5 = \gamma S (B_5 + B_{14}) / 2g, \quad F_6 = \gamma S (B_6 + B_{15}) / 2g, \quad F_7 = \gamma S B_7 / 2g$$

$$F_8 = \gamma S (B_8 + B_{16}) / 2g, \quad F_9 = \gamma S B_9 / 2g, \quad F_{10} = \gamma S B_{10} / 2g$$

From the condition that p in Eq. (19) becomes minimum, the following relationship can be obtained:

$$\partial p^2 / \partial q_1 = 0 \quad \dots \dots \dots (20)$$

From $p^2 = D/F$, the relationship $\partial (D/F) / \partial q_1 = 0$ is obtained. From this result, the following equation can be obtained:

$$\frac{\partial D}{\partial q_1} - p^2 \frac{\partial F}{\partial q_1} = 0 \quad \dots \dots \dots (21)$$

Substituting Eq. (19) into D and F in Eq. (21), the following equation can be obtained:

$$2D_1 q_1 - p^2 (2F_1 q_1 + F_2 q_2 + F_3 q_3 + F_4 q_4) = 0$$

Using partial differential by q_2 , q_3 and q_4 , the following equation can be obtained:

$$\begin{bmatrix} 2D_1 - 2p^2 F_1 & -p^2 F_2 & -p^2 F_3 & -p^2 F_4 \\ -p^2 F_2 & 2D_2 - 2p^2 F_5 & D_3 - p^2 F_6 & -p^2 F_7 \\ -p^2 F_3 & D_3 - p^2 F_6 & 2D_4 - 2p^2 F_8 & -p^2 F_9 \\ -p^2 F_4 & -p^2 F_7 & -p^2 F_9 & 2D_5 - 2p^2 F_{10} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = [H] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \quad \dots \dots \dots (22)$$

The following relationship holds with regard to natural frequency:

$$|H| = 0 \quad (23)$$

Solving Eq. (23) by numerical calculation, the natural frequency can be obtained.

3. Numerical results

3.1 Values used in calculations

Calculation is done with three test models of which natural frequency was measured and reported in the previous paper⁽⁴⁾. Dimensions of stator core are shown in Table 1. Other data are as follows: $E = 205.8 \text{ GPa}$ ($2.1 \times 10^4 \text{ kgf/mm}^2$), $G = 82.32 \text{ GPa}$ ($0.84 \times 10^4 \text{ kgf/mm}^2$), $g = 9.8 \text{ m/s}^2$, density is 7850 kg/m^3 , $1/k' = 1.2$.

3.2 Numerical results

Calculated and measured results of natural frequencies with model A are shown in Fig. 2, with model B in Fig. 3, and with model C in Fig. 4. The difference between calculated natural frequencies and measured ones is read from Fig. 2 ~ Fig. 4, the results being as follows: with regard to model A it is 5.2 % for deformation wave number $n=8$, and 4.1 % for $n=9$, with regard to model B it is 5.2 % for $n=6$, and 6.8 % for $n=9$, with regard to model C it is 6.9 % for $n=7$, and 7.4 % for $n=9$. In the case of other deformation wave numbers, the differences between the calculated results and measured ones are values of about this order. That is, it is found that the calculated values and measured ones of natural frequencies agree within a few per cent error. Therefore, hereafter the natural frequency of the stator core can be estimated with a few per cent of error by calculation in the design stage.

Mean values of the difference between calculated and measured results are as follows: it is 3.9 % for model A, 5.7 % for model B, 6.9 % for model C, 5.4 % for the total. In the case of small deformation wave numbers at the lower natural frequencies, the differences between calculated results and measured ones tend to be slightly larger with regard to the natural frequency on which the effect of the ring is large. From these results, it is considered that calculated results are lower than measured ones because of the effect of rigidity of the teeth being neglected in the calculation of strain energy with bending deformation of the ring.

The calculated results of the mode shape for model A are shown in Fig. 5. This is the mode shape of a tooth in tangential direction, that is, the value of y_1 . At the position of maximum amplitude of the ring in the radial direction, the amplitude of the tooth becomes zero in the tangential direction. And, at the position where the amplitude of the ring becomes zero in the radial direction, the amplitude of the tooth becomes maximum in the tangential direction. The lower natural frequency of the two natural frequencies for the same value of nodal diameter is called the lower order natural frequency,

and the higher ones the higher order natural frequency. As the value of n increases from two to fourteen at the lower order natural frequency, the amplitude of the tip of the tooth has a tendency to increase. The amplitudes between the ring and the tooth are in phase at the lower order natural frequency, and the amplitudes of the ring and the tip of the tooth are out of phase at the higher order natural frequency. Corresponding to the increase of n at the higher order natural frequency, the amplitude of the tip of the tooth decreases. In the case of $n=14$, the amplitude between the root of the tooth and the tip of the tooth is out of phase, and its values become the same order. Ratio Q of the amplitude of the tip of the tooth to the maximum amplitude of the ring in the radial direction is as follows: at the lower order natural frequencies, $Q=1$ at $n=2$, $Q=3$ at $n=6$, $Q=19$ at $n=10$ and $Q=60$ at $n=14$, and at higher order natural frequencies, $Q=11$ at $n=2$, $Q=6$ at $n=6$, $Q=2.2$ at $n=10$ and $Q=2.5$ at $n=14$.

The calculated results of the effect of shear deformation of the teeth on the natural frequencies with model A, are shown in Fig. 6. The equation without the effect of shear deformation was obtained by omission of the term q_1 in Eq. (4). In this case the determinant of Eq. (23) becomes an equation consisting of three rows and three columns. From the condition that the determinant of this equation is zero, the natural frequency was decided. The effect of the shear deformation is observed only in the case of a large deformation wave number at the lower order natural frequencies and of a small deformation wave number at the higher order natural frequencies. By under the effect of the shear deformation, the natural frequency increases by 3.6 % at the lower order natural frequency of deformation wave number $n=14$, and 3.3 % at the higher order natural frequency of deformation wave number $n=2$. The results for models B and C are consistent with those for model A. The natural frequency of model B increases by 4.1 % at the lower order natural frequency of $n=14$, and 3.8 % at the higher order natural frequency of $n=2$. The natural frequency of model C increases by 4.9 % at the lower order natural frequency of $n=14$, and 4.6 % at the higher order natural frequency of $n=2$. Therefore, the effect of shear deformation is 3 ~ 5 %.

Table 1 Stator core dimensions

Dimensions	Stator core		
	A	B	C
Outer diameter mm	750	670	475
width of ring mm	56	52	39.5
Length of tooth mm	59	42.5	38
Width at the tooth tip mm	11.93	9.99	9.12
Width at the tooth root mm	17.81	13.69	13.54
Number of teeth	63	72	54
Plate thickness mm	0.5	0.5	0.5

Shear deformation has an effect on strain energy and kinetic energy. If shear deformation has a large effect on only strain energy and a small effect on kinetic energy, natural frequency without shear deformation can be calculated with use of Eq. (22) and the value of the form factor for the cross section used in the calculation decreases. It is assumed that the form factor $1/k'$ for the cross section with models A ~ C is 0.01, and natural frequencies are calculated by Eq. (22). According to the results, the error of natural frequency without shear deformation is about 1 ~ 2 Hz at $n=14$ with regard to the lower order natural frequency. Thus, natural frequency without shear deformation may be calculated by such procedure that the new equation is not derived and that the value of the form factor for the cross section using in the calculation is decreased.

In the case of a large deformation wave number at the lower order natural frequencies and a small one of the higher order natural frequencies, natural frequencies appear by the effect of the natural frequency of teeth alone. Therefore, ignoring the effect of the ring, that is, the term $Y_{\theta} q$ in Eq. (4), from the condition that the center of the ring is fixed, the natural frequency of a tooth was calculated. The calculated natural frequency of a tooth alone is as follows: it is 3567 Hz for model A, 4792 Hz for model B, and 6117 Hz for model C. Natural frequencies of a tooth alone without shear deformation increase by 3.8 % for model A, 4.4 % for model B and 5.2 % for model C. This effect of shear deformation on natural frequency is of the same order as the above-mentioned case, that is, a large deformation wave number at the lower order natural frequencies and a small one at the higher

order natural frequencies.

The calculated natural frequencies of model A without the term for kinetic energy (T_j) due to rotation of the tooth are shown in Fig. 7. In the case of a small deformation wave number at the lower and higher order natural frequencies, even with omission of the term T_j , the natural frequencies do not increase so much, but in the case of a large deformation wave number at the higher order natural frequencies, fairly large errors (for example, 32 % for 10 deformation waves) occur. In the case of model B and model C, the results

Fig. 2 Natural frequency of model A

Fig. 3 Natural frequency of model B
• Experimental
△ Calculated

Fig. 4 Natural frequency of model C

turned out about the same as in the case of model A.

4. Conclusions

The natural frequency of the stator core of an induction motor was analyzed and calculated for the vibration in the radial

(b) The higher order natural frequency

Fig.5 Mode shapes of model A

direction. The results are as follows:

(1) The vibration is analyzed by the Rayleigh-Ritz method. Considering the shear deformation of the tooth, an equation is obtained with regard to the natural frequency of coupled vibrations between the vibration of a ring and that of teeth.

(2) Natural frequencies of three stator cores were calculated. The errors between the calculated and measured natural frequencies are within a few per cent. Therefore, hereafter the natural frequency of the stator core can be estimated with a few per cent of error in the design stage.

(3) Two natural frequencies of different mode shapes appear for the same deformation wave number. The lower one of them, that is, the lower order natural frequency, has such characteristic that the amplitude of the tooth tip increases with an increase of the deformation wave number n from 2 to 4 ... 14. At the higher order natural frequency, the amplitude of the tooth tip decreases with an increase of n , and at $n=4$, the amplitudes of root and tip of tooth are out of phase and becomes the same order of magnitude. At the lower order natural frequency, the amplitudes of the ring and the tooth tip are in phase, but at the higher order they come out of phase.

(4) The effects of shear deformation of tooth on natural frequency appear for a

- With the shear deformation
- ▲ Without the shear deformation

Fig. 6 The effect of shear deformation on natural frequency with model A

Fig. 7 Natural frequency of model A without term T_j

large deformation wave number at the lower order natural frequencies and a small one at the higher order natural frequencies. If shear deformation is ignored, the natural frequencies increase and their values are 3 ~ 5 per cent when the effect is large.

(5) When the term for kinetic energy due to rotation of the tooth is omitted, the natural frequency does not increase so much in the case of a small deformation wave number at the lower and higher order natural frequencies, but fairly large errors occur in the case of a large deformation wave number at the higher order natural frequencies.

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