On the Strength of Racks for Jack-up Units
2nd Report: Mainly on Elastic Behavior of a
Full Scale Rack with a Back-Plate and
Pinions during Static Loading Test*

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A static loading test is conducted for a rack of 80.9 mm in module with a back-plate for a jack-up unit by using a 10,000 ton press. The tips of two rack teeth are in mesh with the two pinion teeth. 124 strains, 20 displacements, and 4 loads are measured at various locations in the two pinions, the rack, and the apparatus under 32 different static loading conditions. An elastic stress analysis is also conducted by using the FEM under plane stress condition in order to compare strain results. It is shown that the computed strains are higher than those measured by using strain gages for the compressive fillets of the rack and vice versa for the tensile fillets. It is also shown that the tangential strains in the vicinity of the pinion sides are higher than those at the center of the face width for compressive fillets of pinions and vice versa for tensile fillets. Other aspects of strain distribution are also discussed.

Key Words : Strength of Materials, Ocean Engineering, Jack-up Unit, Rack, Pinion, Strain, Elastic Behavior, Static Loading Test

1. Introduction

Since jack-up units are subject to many kinds of load conditions due to weather variations, (1) the rack teeth are possibly fractured and/or locally deformed significantly through contact with the pinion teeth. On the other hand, this type of rack has been designed based on empirical strength design criteria, since the information, such as the endurable maximum jacking load and the relation between the jacking load and the contact deformation, has not been clarified until now.

DNV has recently been attempting to establish the strength design criteria for jack-up units according to the draft proposal of ISO for the strength design of gears (2)-(5) which are based on conventional elastic design criteria; however, these standards are not always considered appropriate for this application. Ishihama et al.(6) conducted a fracture test of the racks for jack-up units subject to the mesh with mating rack teeth. The results, however, do not conform to those for the service conditions, since the racks are in mesh with pinions in service. In this study, the racks and pinions designed and manufactured under the same conditions as those

for a service unit are used for a static fracture test with the meshes of rack and pinion teeth. A total of 148 displacement, strain and load values are obtained for 32 static load cases. A finite element elastic stress analysis is also conducted under plane stress condition for stress comparison. This report mainly discusses the elastic behavior of a rack and pinions.

2. Experimental method

2.1 Apparatus

Figure 1 shows a schematic diagram of the apparatus used. Spline teeth of 6mm in module and 60 in number are machined on the shafts of Pinions A and B which are in mesh respectively with Rack Teeth A and B; consequently, in this condition, pinion shafts cannot rotate because of the meshing with spline teeth machined on the sleeves. The horizontal

Figure 1 Schematic diagram of rack fracture testing apparatus

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reaction forces generated during the process of loading the flange of a rack specimen by the use of a 10,000 ton press are supported by a pair of side-frames. Shims are used to adjust the position of the A-side sleeve so that the parallelism between the flange face and the upper face of the bed plates is within the acceptable range when the contact areas between Pinion Tooth A1 and Rack Tooth A and between Pinion Tooth B1 and Rack Tooth B are fairly large and the initial meshing locations for the respective pairs coincide with each other. Shims are also placed on the upper face of the flange of a rack specimen to uniformly load the specimen, since the parallelism between the lower face of the loading head of the 10,000 ton press and the upper face of the specimen flange is not satisfactory when the apparatus is placed in the 10,000 ton press. The distance between pinion shaft centers can be adjusted to within 1472mm by inserting plates between sleeves and the side frames.

The applied load was calibrated by using 4 load cells placed under the side frame. These load cells have 4 biaxial strain gages at the equidistant circumferential locations and the load is thereby calibrated by measuring axial and circumferential strains. The load cell calibration was conducted by using a 1,000 ton Maekawa MS compression testing machine. The linearity of the four load cells was found to be satisfactory and the load-strain constants for these load cells coincided with each other.

The length of the rack specimen was set at 7 pitches based on Oda's results(7) and Saint-Venant's Principle. Rack Teeth A and B which are to mesh with Pinion Teeth A1 and B1 respectively are located almost in the middle of the specimen and the rack teeth immediately above these teeth are cut such that single contact can be realized until a fracture of the rack teeth occurs. Figure 2 shows the details of the meshing portion and a cross-sectional view of the apparatus. It can be seen from this figure that the face width of the pinions is considerably greater than that of the rack. Thicknesses of a back-plate and a chord, and the outer diameter of the chord are also shown in Figure 2.

Table 1 shows the chemical analysis and the condition of heat treatment for each HT80 steel of the rack specimen components. The pinion material is nickel-chrome-molybdenum steel. Both the pinions and rack were torch-cut and then the pinions were quenched and tempered, while the rack was lonealled at a temperature of $560^{\circ}\text{C} \pm 20^{\circ}\text{C}$ for 2 hours.

Figure 2 Front view and cross-section of meshing part

Table 2 Mechanical properties of rack materials

Component	Material	Yield Point	Tensile Strength	Reduction in Area	Elongation
Rack	HT 80	606 MPa	708 MPa	69%	24%
Chord		755 MPa	814 MPa	—	23%
Back Plate		735 MPa	804 MPa	—	24%

Table 3 Specifications of rack and pinion

Item	Rack	Pinion
Module	8.0851 mm	
Pressure Angle	25°	
Number of Teeth	∞	7
Reference Pitch Diameter	∞	565.955 mm
Face Width	A B 129.3 mm 128.1 mm	—

Table 2 shows the mechanical properties of each rack component material. The yield and tensile strengths of pinion material are considerably higher than those of rack materials. Table 3 shows the specifications of the pinions and rack used.

2.2 Experimental method

2.2.1 Loading

The 10,000 ton press used was made by the Siempelkamp Company, West Germany, and is capable of detecting the displacements of the loading head at two points on Sides (F) and (B) shown in Figure 2 to the nearest 0.1mm. The loading was conducted gradually by lowering the loading head of the press by steps of 0.1mm and the stop position of the loading head was determined by watching the output of the four load cells. The strains and displacements were then measured and the meshing parts were photographed at each

Table 1 Chemical analysis and heat treatment condition of rack materials

Component	Material	Chemical Composition %													Heat Treatment
		C	Si	Mn	P	S	Cu	Ni	Cr	Mo	V	B	Ce		
		x 100	x 100	x 100	x 100	x 100	x 100	x 100	x 100	x 100	x 100	x 100	x 100		
Rack	HT 80	14	26	96	10	3	35	26	116	35	5	64	Quenching : 910 °C x 35min. Tempering : 600 °C x 90min.		
Chord		12	27	92	7	1	23	76	48	22	4	12	46	Quenching : 900 °C x 50min. Tempering : 640 °C x 85min.	
Back Plate		12	27	92	7	1	23	76	48	22	4	12	46	Quenching : 900 °C x 35min. Tempering : 645 °C x 65min.	

stop position of the loading head. Two video sets were also used to record the continuous meshing conditions from Sides (F) and (B). When the whole rack was in elastic condition, the loading was stopped at the interval of 0.4 through 1mm. After a portion of the rack teeth went into the plastic condition, the interval of loading was increased gradually until it finally reached 20mm. The loading head was stopped 31 times in total and after the 19th stop it was lowered by steps of 1mm. Figure 3 shows the relation between Measurement Numbers (which show the stop positions of the stroke) and the loads for Sides A, B, (F), and (B). It can be seen that all side loads agree well with each other. Figure 4 shows the relation between the stroke displacement and the total load. It can be seen that the total load becomes maximal at a stroke displacement of approximately 100mm.

2.2.2 Strain measurement

Strains were measured by using strain gages. Figure 5 shows the locations of the strain gages applied on Rack Tooth A. Strain Gages YCS-5, manufactured by Tokyo Sokki Kenkyusho, (the maximum measurable tensile and compressive strains are 10-20% and 5% respectively according to their catalogue) were applied to both Sides (F) and (B). Strain Gages YL-3 (the maximum measurable strains are the same as those for YCS-5)

were applied on the line on the fillet surfaces, and Strain Gages KFC-1-Cl-11, manufactured by Kyowadengyo Company, (the maximum measurable tensile and compressive strains are 2.8% according to their catalogue) were applied to the locations 5mm and 7mm apart from the above mentioned line at the center of the face width. Gage lengths for KFC, YCS and YL are 1mm, 3mm and 3mm, respectively. Strain gages were also applied to Rack Tooth B in the same method as that for Rack Tooth A. Strain Gages YCS-5 were applied to Sides (F) and (B) of the chord as shown in Figure 6. Figure 7 shows the locations of strain gages applied to Pinion Tooth A1. Biaxial Strain Gages KFC-1-DL6-11, manufactured by Kyowadengyo Company, were applied to the center of the face width and KFC-1-Cl1-11 were applied to other locations. The numbers

Figure 3 Relation between load and Measurement Number

Figure 4 Relation between total load and stroke displacement

Figure 5 Locations of strain gages applied on Rack Tooth A

Figure 6 Locations of strain gages applied on chord

Total Number of Elements=1202
Total Number of Joints =941

Figure 11 Finite element model of the rack

Figure 12 Mesh pattern of rack tooth fillets

Figure 13 Tangential strain distribution across face width for Pinion Tooth A1

Figure 14 Tangential strain distribution across face width for Pinion Tooth B1

respectively.

4. Experimental and analytical results and discussions

4.1 Strain distribution along fillets of pinion teeth

Figures 13 and 14 show the tensile and compressive tangential strain distributions across face widths of Pinion Teeth A1 and B1, respectively. The strains at both edges show those obtained from gages applied on the side surfaces. The jacking loads shown in these figures indicate the sum of two loads obtained from the respective two load cells on Sides A and B. From these figures, it can be seen that the compressive fillet strains are greater than the tensile fillet strains for the same jacking loads. This result was found to be due to the fact that the normal load on the tooth surface has a component in the upward tooth height direction according to the photographs of pinion and rack meshings. The nondimensionalized tensile and compressive stresses σ_{tn} and σ_{cn} , obtained by using Aida and Terauchi's Formulae, (9), (10) for the most critically stressed section determined by Hofer's method (11) are formulated as follows:

$$\begin{aligned}\sigma_{tn} &= \sigma_t / (P/bm), \text{ and} \\ \sigma_{cn} &= \sigma_c / (P/bm),\end{aligned}\quad (1)$$

where,

σ_t and σ_c are the tensile and compressive

fillet stresses, respectively; (these stresses are converted from strains by multiplying E , if necessary) P is the jacking load; b is the face width; and m is the module.

Table 4 shows the nondimensionalized stresses computed by using Aida and Terauchi's Formulae and those obtained from the average of five measured strains across the face width, respectively. The stresses obtained from measured strains agree well with those computed, although the former was obtained based on the plane stress assumption.

Table 4 Nondimensionalized fillet stresses
(Experimental & computational)

Tooth	Pinion Tooth A1		Pinion Tooth B1	
	σ_{tn}	σ_{cn}	σ_{tn}	σ_{cn}
Method				
Computed by Aida & Terauchi's formula	2.84	-1.88	2.80	-1.91
Average of the values measured by the use of strain gages	2.94	-1.78	2.98	-1.87

It is obvious that the strain distribution for the tensile fillet is convex, while that for the compressive fillet is concave. The former phenomenon can be deduced from the previously obtained results. (12), (13) The latter phenomenon is considered to occur due to the fact that the effect of the local load distribution on the tensile side disappears on the compressive side in a rack with such a large module; therefore, the strain levels in the vicinity of the side which is almost under plane stress become higher than those at the center which is almost under plane strain. In fact, it was observed from the biaxial gages at the center of the face width that the value of k (the ratio of the strain in the face width direction to the tangential strain) ranged from 0 to -0.04 and from -0.1 to -0.14 for the tensile and compressive fillets, respectively. It can be seen that plane strain is especially dominant on the tensile side. This phenomenon is considered to be due to the fact that the constraint against the deformation in the face width direction is increased on the tensile side, since the contact area between the pinion and rack is adjacent to the fillet.

4.2 Strain distribution for rack teeth

Figures 15 and 16 show the tangential strain values for the tensile and compressive fillet surfaces obtained with strain gages and by using the FEM. The tangential angle " θ " denotes the angle between the line which is tangential to the fillet curve and the center line of the tooth as shown in Figure 15. " θ ", therefore, indicates the location of a point along the fillet curve. These strain values were nondimensionalized by using Equation (1) and then compared with those obtained by using the FEM. The strain values from the FEM shown in these figures are identical to those on the fillet surface, which are obtained by extrapolating six strain values at the centroids of elements in a row perpendicular to the surface through Lagrange's interpolation formula of the fifth order. It can be seen from Figure 15 that the measured strain values for the back-plate thickness $t_2=25\text{mm}$ agree fairly well with those computed by the FEM; however, it must be noted that the measured strains are ones obtained with the strain gages of 3mm in active gage length and conse-

Figure 15 Measured and computed values of tangential strains on the tensile fillet surface of the rack

Figure 16 Measured and computed values of tangential strains on the compressive fillet surface of the rack

quently are smaller than the actual strains.⁽¹⁴⁾ It is therefore concluded that the measured strains become greater than those computed for the strain gages of 1mm in gage length. Conversely, it can clearly be seen from Figure 16 that the computed strains are greater than the measured strains presumably due to the fact reported by Oda⁽⁷⁾ that the compressive fillet stresses (strains) are significantly affected by boundary conditions for the gears with small rim thickness. Consequently, it is concluded that the FEM values in this investigation which were obtained under plane stress assumption and by excluding the effect of the chord are markedly different from the measured values. Aida and Terauchi's formula, which is applicable to the gears with large stiff-rim thickness, was also used to obtain σ_{tn} and σ_{cn} at the respective locations of $\theta = 45^\circ$; subsequently, σ_{tn} of 3.50 and σ_{cn} of -4.36 were obtained, respectively. These values coincide with the corresponding measured values and are found to be slightly greater than, but in fairly good agreement with, the FEM values for $t_2 = 127\text{mm}$ which is equivalent to the rack with large rim thickness.

With regard to the FEM values only, it is clear that the most critically stressed points for $t_2 = 20\text{mm}/25\text{mm}$ are on the bottom land side in reference to the point ($\theta = 30^\circ$) determined by Hofer's method and consequently agree with the results obtained by Oda.⁽⁷⁾ It can also be seen that the most critically stressed points for the tensile and compressive fillets of the rack with $t_2 = 127\text{mm}$ are about $\theta = 40^\circ$ and 45° , respectively and thus are on the bottom land side in reference to the point determined by Hofer's method. This phenomenon is considered to be caused by the fact that the location of $\theta = 30^\circ$ is adjacent to the straight tooth surface because of the high pressure angle of 25° , and consequently the degree of stress concentration is higher at the locations on the bottom land side in reference to the location of $\theta = 30^\circ$.

Figure 17 shows the average of the strains ϵ_y in the y-direction for both side surfaces of each rack tooth and those obtained by using the FEM. Both strains were nondimensionalized by using Equation (1). It can be seen that these strains agree well with each other except in the vicinity of the surface of tooth fillets, while the difference in the strains between Measured Locations 1 and 13 in the direction of 45° to the y-direction is large especially on the compressive side. This is considered due to the previously stated reason.

Figure 18 shows the average of the strains ϵ_x in the x-direction obtained in the above mentioned manner. It can be seen that the measured strains are greater than the computed strains on the tensile side and vice versa on the compressive side. The strain values at Measured Locations 2 and 14 in the direction normal to the fillet surface naturally approach zero and the difference between measured and computed

strains is greater on the compressive side than on the tensile side; consequently, the strains (stresses) on the compressive side are deduced to be significantly affected by boundary conditions.

The k values at the center of the face width of the racks were found to range from -0.043 to -0.069 for the tensile fillet and from -0.074 to -0.096 for the compressive fillet.

5. Conclusions

A static loading test apparatus for the rack in service conditions was designed and manufactured, and a fracture test of the rack was conducted under excellent conditions. The FEM elastic stress analysis was also conducted under plane stress condition and the following conclusions were obtained:

- (1) It was found that the tangential strain distributions across the face width have convex and concave shapes for the tensile and compressive fillets respectively, for the investigated large scale pinions which are in mesh with rack teeth with a smaller face width.

Figure 17 Measured and computed strain values in the y-direction on rack tooth side surfaces

Figure 18 Measured and computed strain values in the x-direction on rack tooth side surfaces

- (2) The maximum tensile fillet strains were found to be greater than the maximum compressive strains for the investigated pinions with a large, positive offset factor. Even in this case, Aida and Terauchi's formulae were found to be effective.
- (3) The fillet strain values measured by using strain gages were found to be greater than those obtained by using the FEM for the tensile fillets of the rack teeth and vice versa for the compressive fillets. This is due to the fact that a chord was not included in the FEM analysis.
- (4) The FEM analysis led to the conclusion that the most critically stressed points in the fillets of the investigated rack of 25° in pressure angle are located on the bottom land side in reference to those determined by Hofer's method even for the rack with large rim thickness and these points approached the bottom land in the racks with decreased back-plate and rim thicknesses.
- (5) In the investigated pinions and rack with large face width, the plane strain characteristics were found to be marked at the center of the face width.

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