Liquid Films Flowing Concurrently with Air in Horizontal Duct*

(5th Report, Interfacial shear stress)

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Interfacial shear stress was measured for a thin liquid film flowing concurrently with air flow under wide ranges of gas and liquid flow rates in which various kinds of wave structures or flow patterns are realized. The results are summarized as follows.

(1) Interfacial shear stress depends on flow pattern besides the condition of air flow. (2) The waves on the liquid film are not equivalent to the roughness on a solid wall especially for a higher air flow even when the crests of waves on the film do not jut above the viscous sublayer. (3) A correlation for friction factor which is equivalent to shear stress was proposed for the cases of turbulent air flow.

Key Words: Multiphase flow, Film Flow, Interfacial Shear Stress, Horizontal Duct

1. Introduction

A thin liquid film flowing concurrently with a gas flow in a horizontal rectangular duct has been investigated to make clear the mechanism causing the liquid film breakdown on a heating surface. In the first part of this investigation(1), the flow patterns or the classification of waves, which are observed in the region surrounding the non-wetting region, are made clear. In the second and the third parts, the important roles of the disturbance wave and the viscous wave in the breakdown of the liquid film are discussed (2)~(5).

The parameters relevant to the film breakdown may be the surface tension, the roughness of the wall surface, the geometry of the duct etc. Especially, the interfacial shear stress is considered one of the most important parameters to control the liquid film flow. Therefore, further investigations on the interfacial shear stress should be done to explain the process leading to the film breakdown.

Assuming that liquid surface is equivalent to a rough solid surface, Leonard et al.(6) have defined and discussed an equivalent sand roughness. Sekoguchi et al.(7) proposed a correlation of a friction factor for the gas-liquid surface taking the characteristics of the ripple wave into consideration. It is, however, not practi-

cally useful because the information of the wave height and the passing frequency of the waves should be known.

It is probable that the interfacial shear stress depends on the characteristics of the surface, or the flow pattern. In this experiment the shear stress is measured over a wide range of flow conditions. And the measured shear stress is compared with the shear stress on a rough solid surface and examined paying attention on a relation between itself and the flow pattern. A correlation of the friction factor, which is equivalent to the interfacial shear stress, is proposed.

2. Nomenclature

B	: Width of duct	mm
c	: Wave velocity	m/s
D_e	: Hydraulic diameter	mm
h_c	: Wave height	mm
Δh_c	: Standard deviation of wave height	mm
Δh_c^+	: Nondimensional wave height	mm
	$[\Delta h_c u_i^* / \nu_g]$	
H	: Duct height	mm
H_{gw} , H_{gi}	: Distances from the top wall of the duct and from the gas-liquid interface to the maximum velocity point of a gas flow in a cross-section, respectively	mm
j_g	: Volumetric flux of gas	m/s
k_s	: Equivalent sand roughness	mm
k_s^+	: Dimensionless equivalent sand roughness	$[k_s u_i^* / \nu_g]$
N_{max}	: Maximum passing frequency of wave	1/s
Re_g	: Gas Reynolds number defined by superficial velocity	$[2H j_g c / \nu_g]$
Re_l	: Film Reynolds number	$[2\Gamma c / \nu_l]$
t_{fm}	: Time averaged thickness of liquid film	mm
u_{gmax}	: Maximum gas velocity in a cross-section	m/s
u_i^*	: Friction velocity at the interface	m/s $[\tau_i / \rho_g]$
u^+	: Dimensionless velocity	$[u / u^*]$
X	: Martinelli parameter	

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- $[\equiv (\lambda_l \rho_l j_{lc}^2 / \lambda_g \rho_g j_{gc}^2)^{1/2}]$
 x, y : Co-ordinate axes in the directions of flow and perpendicular to it, respectively mm
 y^+ : Dimensionless value of y [$\equiv y u^* / \nu_g$]
 Γ : Liquid volumetric flow rate in a duct width m^2/s
 K : Kármán constant
 λ_i : Friction factor at the interface
 λ_w : Friction factor at the solid wall
 μ : Viscosity $\text{Pa}\cdot\text{s}$
 ρ : Density kg/m^3
 τ_i, τ_w : Shear stresses at the interface and at wall, respectively Pa

Flow pattern symbols

- D : Disturbance wave regime
 P : Peble wave regime
 S : Smooth surface regime
 V : Viscous wave regime
 NW : Nonwetting regime
 R : Ripple regime
 T : Two-dimensional wave regime

Subscripts

- c : Center of duct width
 g : Gas phase
 l : Liquid phase
 i : Gas-liquid interface
 w : Top wall

3. Experimental Apparatus and Procedure

The horizontal test section is made of transparent acrylic resin with rectangular cross-section of 10 mm in height, 80 mm in width and 7.1 m in total length. Air fed from a compressor and water from a constant head tank meet at an air-water joining section and they flow separately from each other inside the test section through the measuring stations for film thickness, pressure drop and the velocity profile of the gas phase, which are about 4 m (225 equivalent diameters) downstream of the joining section. At the exit of the test section the distribution of water flow rates in the duct width direction were measured by a collection method.

By a constant current method developed by the authors, the local film thickness at the center of the duct width and the mean film thickness averaged over the duct width were measured. The velocity distribution of gas phase in the vertical direction was measured at the center of the duct width by a total pressure tube of 0.73 mm outer diameter. The position of the tube was determined with a high accuracy by an optical microscope.

Fig.1 Flow model and nomenclature.

Experimental conditions are as follows: System pressure: (0.098 $\pm$ 0.105)MPa, superficial velocity of air j_g and air Reynolds number $Re_{g0}(\equiv j_g D_g / \nu_g)$: $j_g = (2\sqrt{50}) \text{ m/s}$, $Re_{g0} = (2200 \sim 57000)$, water volumetric flow rate Γ and its Reynolds number $Re_{l0}(\equiv \Gamma / \nu_l)$: $\Gamma = (1\sqrt{30}) \times 10^{-5} \text{ m}^2/\text{s}$, $Re_{l0} = 10^3 \sim 300$, liquid temperature : about 20 $^\circ\text{C}$.

Shear stress at the top wall τ_w and the gas-liquid interface τ_i were determined as follows. In the steady fully-developed two-dimensional separated flows of gas and liquid, the ratio of τ_i to τ_w is related to the ratio of H_{gw} to H_{gi} by a force balance (Fig.1) as,

$$\tau_i / \tau_w = H_{gi} / H_{gw} \quad (1)$$

For another equation for τ_i and τ_w , Hanratty and Engen(8) used a relation for pressure drop, Δp . It, however, gives a larger value of τ_i because it includes the effect of the side wall of the duct. So in this paper τ_w is first determined by the velocity distribution near the top wall. Then τ_i is obtained by Eq.(1) substituted by the measured values of H_{gw} and H_{gi} which is determined from the measured film thickness.

4. Results and Discussion

Figure 2 shows the range of the experiment and the area plot of flow patterns observed in this experiment. The symbol $\bigcirc$ denotes the data points. Notations S, T, P, V, D, R, A and NW stand respectively for the flow patterns. The hatched area shows the transition of air flow between laminar (left side of it) and turbulent (right side). It must be added that the gas Reynolds number defined by $(H_{gi} + H_{gw})$ and the averaged value of u_g measured at the center of the duct width is approximately equal to Re_{g0} , and it does not depend on Γ .

The air velocity profile is not completely the same as that of a single-phase air-flow even when it is laminar due to the influence of a roughened gas-liquid interface. In the region below the dotted line in Fig.2 entrainment could not be detected.

Fig.2 Experimental range and flow patterns.

4.1 Shear stress ratio, τ_i/τ_w

Figure 3 shows the ratio of the shear stress at the interface, τ_i , to that at the top wall, τ_w . Data are linked by dotted lines for a laminar air flow and by solid lines for a turbulent flow. Data on the left side of the two dotted line and the one dotted line are for the S-region and the T- and P-regions, respectively. Those on the right side of the one dotted line are for the R- and the D-regions. This figure clearly shows that the interfacial shear stress is larger than the wall shear stress without an exception. τ_i/τ_w becomes larger with an increasing Γ when they are compared with j_g kept constant. The behavior of τ_i/τ_w with respect to j_{gc} depends on the condition of air flow, laminar or turbulent, and also on the flow pattern. It should be noticed that τ_i/τ_w takes the minimum value at $j_{gc} \sim 12$ m/s where the film breakdown takes place most easily.

4.2 Shear stress in the case of laminar air flow

In Fig.4 the experimental data on the velocity profile of air for $j_g = 4$ m/s are compared with a theoretical result obtained by assuming that both the air and water flows are two-dimensional and laminar. The theory agrees quite well with the data on the single phase air flow shown by mark $\circ$. Existence of water film, however, modifies the laminar solutions. As already shown in Fig.3, τ_i/τ_w takes a larger value in the T-region, where waves exist, than in the S-region, where both Γ and j_{gc} are small. This means that the roughened interface gives rise to a turbulence in the air flow. So the data points are fitted by the following equation,

$$u_g = \frac{u_{gmax}y}{H_{gw}} \left(2 - \frac{y}{H_{gw}} \right) + ay(H_{gw} - y)^2 \quad (2)$$

where the constant, a , was decided such as to fit the data points on the upper side of the maximum air velocity point. τ_w was decided by differentiating Eq.(2), and then τ_i by Eq.(1).

As shown in Fig.5, τ_i thus obtained is larger than the laminar flow solution,

Fig.3 Ratio of interfacial shear stress to wall shear stress.

τ_{ith} , by about 20%. Because of this increase in shear stress the interfacial liquid velocity is larger than that of the laminar flow solution(5).

4.3 Shear stresses for turbulent air flow

Shear stresses in the case of turbulent air flow were decided as follows. First, by assuming that the velocity profile of air flow on the wall side is expressed by the logarithmic velocity profile with Karman constant equal to 0.4, the measured values of the gas velocity were plotted on Clauser's chart. Then a skin friction coefficient, which is equivalent to τ_w was read from the chart.

4.3.1 Velocity profile of an air flow

A plot of the dimensionless air velocity, u^+ , versus the dimensionless distance from the top wall or from the interface, y^+ as a function of Γ is shown in Fig.6. The thick solid line expresses the logarithmic wall law, with which the measured velocities on the top side agree quite well. On the interfacial side, however, the velocity profile shifts below as Γ increases because τ_i increases with Γ . The Karman constant,

Fig.4 Velocity profile of air layer (laminar flow).

Fig.5 Comparison of τ_i (air flow $\pm$ laminar).

K , evaluated by the slope of each line in the figure is plotted in Fig.7. If entrained droplets are not included in an air flow the term K takes values between 0.30-0.45 on both the upper and the interfacial surfaces although that on the upper side has a tendency of decreasing with j_g .

Fig.8 shows a plot of K versus j_g for a vertical upward annular flow in a circular pipe with an inner diameter of one inch (9). A.R and A.D in the figure stand for annular, ripple and disturbance flows, respectively. The boundary between them is expressed by a dotted line. K in the A.R-regime is 0.3-0.4, which agrees well with the present experimental value. On the other hand in A.D-regime it takes considerably small values. This may be due to the effect of entrainment. Taking these facts into consideration K is assumed to be 0.4 in the following sections.

4.3.2 Equivalent sand roughness, k_s

As will be discussed later (refer to Fig.11) the roughness Reynolds number, k_s^+ , obtained in the present experiment takes a value which corresponds to that for the

Fig.6 Velocity profile of air layer (turbulent flow).

Fig.7 Kármán constant.

transition of flow from a hydraulically smooth to a fully rough wall flow. Then k_s was determined by the roughness function plotted with respect to k_s^+ (10).

In Fig.9 k_s thus obtained is plotted with respect to wave height, $h_c [=(t_{fmax})_c - (t_{fmin})_c]$. The term k_s seems to be correlated by the wave height h_c in T- and P-regions, which are on the right side of a

Fig.8 Kármán constant for a vertical upward annular flow (Gill et al. (9)).

Fig.9 Relation between equivalent sand roughness and wave height.

Fig.10 Wave height.

one dotted line in the figure. However, the data on R- and D-regimes, which are distinguished by short oblique lines in the figure, show that k_s depends on j_g too besides h_c . As shown in Fig.10, h_c decreases with j_g when $j_g > 15$ m/s. This means that with an increasing j_g , k_s becomes larger although h_c decreases.

Fig.11 shows the relation between the dimensionless equivalent sand roughness $k_s^+ (\equiv k_s u_i^* / \nu_g)$ and the dimensionless wave height $\Delta h_c^+ (\equiv \Delta h_c u_i^* / \nu_g)$, Δh_c being the standard deviation of wave height and u_i^* the friction velocity at the gas-liquid interface. The experimental data obtained by Leonard et al.(6) in T- and P-regimes are also plotted by mark $\circ$. These data show that the term k_s^+ in T- and P-regimes can be correlated by Δh_c^+ .

On the other hand, the values of Δh_c^+ for almost all the data in R- and D-regimes are small, $\Delta h_c^+ < 3$. Therefore, interfacial waves should be within a sublayer of the air flow, if the air-water interface is considered to be equivalent to a solid boundary. The term k_s^+ , however, takes a rather large value comparable to that in the transition zone to a fully turbulent flow.

Fig.12 shows the comparison between the equivalent sand roughness, k_s/k for a solid wall with regular roughness pattern obtained by Schlichting (11) and k_s/h_c for gas-liquid interface. Where k and D are

Fig.11 Relation between equivalent sand roughness and wave height.

Fig.12 Comparison of equivalent sand roughness at solid wall with that at the interface.

the height of projections and the distance between the projections, respectively, and c/N_{max} is the wave length λ . In Fig.12 h_c and c/N_{max} are considered to correspond respectively to k and $2D$. In the case of the solid boundary, the term k_s/k decreases with a decreasing $k/2D$, while in the case of the air-water interface the trend of k_s/h_c with h_c/λ is opposite especially in R- and D-regimes.

These experimental results signify that (1) the factor controlling shear stress at a gas-liquid interface changes depending closely on flow pattern, and (2) it is unreasonable to consider that the gas-liquid interface is equivalent to a solid boundary.

4.3.3 Relation between the characteristics of wave and the equivalent sand roughness

Figure 13 shows the wave velocity, c , determined by the cross-correlation of film thickness variations with time measured at two axially different stations. Fig.14 shows the passing frequency of waves, N_{max} , determined by the spectrum density distribution of film thickness variation. In D- and R-regimes, c and N_{max} increase sharply with j_{gc} .

As shown in Fig.15, in T- and P-regimes the wave velocity c is approximately equal to the liquid velocity at the surface, $2\Gamma c/(t_{fm})_c$, obtained by assuming a linear velocity profile in the liquid film.

Fig.13 Wave velocity.

Fig.14 Maximum wave passing frequency.

Meanwhile, c is smaller than $2\Gamma_c/(t_{fm})_c$ in R- and D-regimes. This is closely related to the occurrence of disturbance waves(4).

In R- and D-regimes, the interfacial shear stress depends on the gas flow besides the characteristics of waves such as velocity, length and passing frequency. It is suggested by Figs.13 and 14 that the number of waves in a unit axial length is not much affected by j_{gc} . Therefore, $\Delta h_c c/v_l$ and $(2Hu_w^*/v_g)$ were introduced as the dimensionless parameters representing the wave characteristics and the gas flow condition, respectively, to correlate the equivalent sand roughness for the interface. As shown in Fig.16, the data of k_s^+ in R-regime, which are expressed by solid marks, are passably well correlated by the product of the above two parameters. If the disturbance waves, which have a large wave height, exist, the data deviate from the solid line as shown by dashes in the figure.

5. Correlation of Friction Factor

5.1 Laminar air flow case

As a basis of the correlation of the friction factor for the air-water interface, a wall friction factor attained under a single air flow with the same flow rate as that in two-phase flow is preferable because τ_w , which was discussed in Fig.3, may include the effect of the gas-liquid

Fig.15 Relation between wave velocity and interfacial velocity.

Fig.16 Relation between sand roughness and $(\Delta h_c c/v_l)(2Hu_w^*/v_g)$.

interface. The terms λ_i and λ_w are defined in the next equations, respectively.

$$\lambda_i = 8(u_i^*/\bar{u}_{gc})^2 \quad (3)$$

$$\lambda_w = 8(u_w^*/j_{gc})^2 \quad (4)$$

where u_i^* and u_w^* are respectively the friction velocities at the interface and at the wall, $\bar{u}_{gc}$ is the velocity averaged over the distance from the interface to the point of the maximum air velocity. It should be noticed that the ratio λ_i/λ_w is equal to $(\tau_i/\tau_w)/(j_{gc}/\bar{u}_{gc})$ and not equal to τ_i/τ_w .

As shown in Fig.17 the data on a single air flow, which are expressed by mark $\bigcirc$, agree well with the relation for a single phase laminar flow, $\lambda_w = 96/Re$ when $Re_g (=2Hj_{gc}/v_l) \leq 6000$. It was tried to correlate the increase in λ_i over λ_w in this Reynolds number range by the parameters introduced in §4.3. Fig.18 shows a satisfactory correlation between these parameters. This signifies that the increase of λ_i is due to the increase of turbulence level in the gas flow caused by the increased roughness and deformability of the gas-liquid interface.

Wallis : $\lambda_i = \lambda_g [1 + 360((t_{fm})_c/D)]$
 Robert : $\lambda_i = \lambda_g + 1.5[(t_{fm})_c/D] - (5/Re_g)\sqrt{8/\lambda_g}$
 Sekoguchi : $\lambda_i = \lambda_g + 5500h_c N_{max}/j_{gc} Re_g^{0.5}$

Fig.17 Friction factor.

Fig.18 λ_i/λ_w vs. $(\Delta h_c c/v_l)(2Hu_w^*/v_g)$.

5.2 Turbulent air flow case

Both λ_i and λ_w in the case of a turbulent air flow are already shown in Fig.17. One dotted line, two dotted line and broken line in the figure show λ_i for $\Gamma = 5 \times 10^{-5}$ m²/s calculated from the correlations proposed respectively by Wallis, (12) Robert et al. (13) and Sekoguchi et al. (7). The present experimental values were used for $(t_{fm})_c$, h_c and N_{max} in these correlations. According to the present experimental result the dependence of λ_i on Re_g in T- and P-regimes is different from that in R- and D-regimes. Only the film thickness is and the effect of flow pattern is not taken into consideration in the correlations by Wallis and Robert et al. The correlation by Sekoguchi et al. for a vertical upward ripple flow gives considerably smaller values than those in the present experiment when $Re_g > 40,000$.

Figure 19 shows the relation between the ratio, $\Delta h_c / (t_{fm})_c$ and Martinelli parameter which is defined by the next equation,

$$X^2 = \left(\frac{\lambda_l \rho_l j_l c^2}{D_e} \right) / \left(\frac{\lambda_g \rho_g j_g c^2}{D_e} \right) \quad (5)$$

where λ_l and λ_g are calculated from the existing correlation for a parallel plate flow assuming the gas and liquid flows are turbulent and laminar, respectively, because the liquid Reynolds number is always less than 300. As shown by open marks in Fig.19 the term $\Delta h_c / (t_{fm})_c$ for T- and P-regimes takes approximately a constant value with respect to X .

Figure 20 shows that $(t_{fm})_c$ can be correlated by X , although the relation changes depending on whether the flow pattern is P- and T-regimes or R- and D-regimes. The results in Figs.19 and 20 besides the fact that the parameter controlling the equivalent sand roughness is the wave height in T- and P-regimes (Fig. 11) signify that the interfacial friction factor λ_i in those flow regimes can be correlated by X only.

On the other hand it is shown in Fig. 16 that the equivalent sand roughness in R-regime can be correlated by the parameters $(\Delta h_c / v_l)$ and $(2Hu_w^* / v_g)$. From the linear dependences of the velocity, c , on the liquid velocity at the interface $\Gamma_c / (t_{fm})_c$, and of the term $\Delta h_c / (t_{fm})_c$ on X , the parameter $(\Delta h_c / v_l)$ is expressed as,

Fig.19 $\Delta h_c / (t_{fm})_c$ vs. X .

$$\frac{\Delta h_c c}{v_l} \propto \frac{\Delta h_c \Gamma_c}{(t_{fm})_c v_l} \propto X Re_l \quad (6)$$

Meanwhile, u_w^* , which is included in the other parameter is expressed by Re_g . Then, λ_i in R-regime is a function of Re_l and Re_g besides X .

The ratios λ_i / λ_w for respective flow regimes are correlated by the following correlations,

$$\lambda_i / \lambda_w = 1 + 4X^{1.2} \quad (\text{T- and P-regimes}) \quad (7)$$

$$\lambda_i / \lambda_w = 1 + 6.1 \times 10^{-5} X^{0.52} Re_l^{0.65} Re_g^{0.7} \quad (\text{R-regime}) \quad (8)$$

Figure 21 shows that from the above correlations, the interfacial friction factor can be estimated within an accuracy of $\pm 10\%$. Although the use of Eq.(8) is limited to R-regime in the above, it is applicable to the flow in D-regime also unless disturbance waves are extremely large in that flow.

Fig.20 $(t_{fm})_c$ vs. X .

Fig.21 Comparison of estimated friction factor with the measured.

6. Conclusions

Interfacial shear stress of the thin water film flow accompanied by an air flow in a rectangular horizontal duct is examined. The results are summarized as follows:

- (1) Shear stress at the air-water interface, τ_i , is always larger than that on a smooth solid wall, τ_w , when compared under the same air flow rate regardless of the flow pattern and the condition of air flow, laminar or turbulent. The ratio τ_i/τ_w increases with the liquid flow rate.
- (2) The velocity profile in the air layer deviates from that of a single-phase air flow due to the existence of a liquid film. The increase in the interfacial shear stress over that of a single phase flow is about 20% when the air flow is laminar.
- (3) In the case of a turbulent air flow, the dependence of τ_i/τ_w on j_g changes depending on the flow pattern. In T- and P-regimes that ratio decreases linearly with an increase of j_g , while in R- and D-regimes the relation is reversed. The boundary of the two groups is expressed by $j_g \approx 12$ m/s, in which τ_i/τ_w takes the minimum value compared with Γ kept constant.
- (4) In T- and P-regimes, the equivalent sand roughness, k_s , for a gas-liquid interface is controlled mainly by the wave height, and the interface is equivalent to a solid rough wall. In R- and D-regimes k_s depends on the wave velocity and the friction

velocity of a solid wall besides the wave height.

(5) Correlations (7) and (8) are introduced to estimate the interfacial friction factor in the case of a turbulent gas flow, taking the wave characteristics and flow patterns into consideration.

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