Investigation of the Aerodynamic Performance of a Transonic Axial-flow Turbine with Variable Nozzle*

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A single-stage transonic turbine was tested over a wide range of pressure ratios. The exit Mach numbers of nozzle and blade are supersonic at the design pressure ratio of 4.16. The measured performance was compared with the results calculated by a mean-line prediction method, using two kinds of loss systems. One is Ainley & Mathieson's system improved by Dunham & Came (AMDC), and the other is Craig & Cox's system (C&C). It was found that airflow characteristics and loss systems play an important role in estimating the performance of a transonic turbine, especially when nozzle and rotor row are choked alternately.

Key Words : Fluid Machine, Transonic Axial-flow Turbine, Variable Nozzle, Performance, Mean-line Prediction Method, Loss System, Choked Row

Introduction

In a simple cycle gas turbine, the elevated turbine inlet temperature improves the cycle efficiency. Recently, in industrial gas turbines, the increasing of turbine inlet temperature, about 1,000 to 1,100 deg C, tends to raise the cycle pressure ratio. Turbine efficiencies and stage loadings are determined by the usage and the operating conditions of the gas turbines. In medium size gas turbines as well as jet engines, a high cycle efficiency has to be attained by increasing the inlet temperature without changing the size. Thus, the stage loadings tend to increase.

An increase of heat drop in a stage is generally useful to reduce the amount of cooling air due to lower relative gas temperatures, to increase turbine inlet temperatures, and to demonstrate size, weight, and cost advantages readily. Large stage works can be accomplished by making the rotational speeds higher, making the deflection angle of blades larger, and/or using transonic cascades. However, the maximum rotational speeds are limited by material strength of blades in the operating conditions.

As a result, transonic turbines have come to be developed into advanced gas turbines with high cycle pressure ratios.⁽¹⁾⁽²⁾ Furthermore, a variable nozzle has been adopted to widen the operating range in mechanical drive gas turbines⁽³⁾.

However, it will take much cost and long time to develop transonic turbines and to perform their tests. Few experi-

mental results and references to be utilized for design are not available. Therefore, it is desirable to establish a method to evaluate the turbine performance accurately and simply.

To examine to a large extent the overall performance of turbines operating in a high-loading condition, a single stage transonic axial-flow turbine with variable nozzle was designed and tested, which had larger stage loadings than ordinary gas turbines.

The measured performance of the transonic axial-flow turbine with variable nozzle was compared with that calculated by a mean-line prediction method using two kinds of loss systems.

Nomenclature

- A_k : effective flow area in blade tip clearance space
- A_t : blade passage throat area
- C : blade chord
- C_0 : isentropic velocity, $\sqrt{2(H_{0T} - H_{1Ta})}$
- H : specific enthalpy
- ΔH : turbine stage work
- i : incidence
- k_c : tip clearance
- L : turbine power
- M : Mach number
- N : rotational speed
- O : blade opening
- P : pressure
- P_r : stage pressure ratio, P_{0T}/P_{2T}
- R_p : equivalent degree of reaction, $(P_1 - P_2)/(P_{0T} - P_2)$
- S : blade pitch
- T : temperature
- U : blade wheel speed at midspan
- W : airflow rate
- W : non-dimensional airflow rate
- α : gas flow angle
- δ : ratio of actual pressure to standard pressure, P_T/P_{st}
- θ : ratio of actual temperature to standard temperature, T_T/T_{st}
- $\Delta \xi$: offset angle of nozzle, deg.

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η_{TS} : turbine efficiency (total to static),
 $L/[W(H_{0T}-H_{2a})]$
 η_{TT} : stage efficiency (total to total),
 $L/[W(H_{0T}-H_{2Ta})]$

Subscripts

a : adiabatic
 R : relative
 S : static
 T : total (stagnation)
 0 : nozzle inlet
 1 : nozzle exit
 2 : rotor exit
 cr : critical ($M = 1.0$)
 st : standard condition, stall

Research Turbine and Test Procedure

Design velocity triangles and blade profiles

A cross section of the research turbine is shown in Fig.1, a detailed flow path is in Fig.2, the design parameters are in Table 1, and the velocity triangles at the design point are in Fig.3. This is a highly loaded transonic turbine, which is designed to operate at the stage pressure ratio of 4.16 and at the stage loading coefficient of 2.11. Furthermore, exit velocities of discharge from the nozzle and rotor are supersonic. The turning angle of the rotor blade is relatively large in 110 degrees. The stator vanes have smaller chords to reduce the varia-

Fig. 1 Research Turbine

Fig. 2 Flow path of research turbine

tion of nozzle tip clearance space with the nozzle rotation. On the other hand, the rotor blades have larger chords to increase the stage load, to decrease the exit swirl angle, to reduce the degree of reaction, and to suppress the increasing of flare angles in rotor.

Airfoil contours of nozzle vanes and rotor blades are illustrated in Fig.4, and the geometrical parameters are given in Table 2. Both of the blades have straight-back profiles. The maximum exit Mach number corresponding to the limit-load is 1.44 for the nozzle and 1.27 for the rotor, which were calculated according to reference (4).

The nozzle vanes are staggered constant and the pitch to chord ratio in the nozzle remains constant along a radial span. The rotor configuration was determined from a constant stage work distribution along a radial span. The nozzle vanes are designed to be rotatable around the

Table 1 Turbine design parameters

Mean Velocity Ratio, U/Co	0.446
Stage Loading Coefficient	2.11
Stage Pressure Ratio, P_{0T}/P_{2T}	4.16
Equivalent Stage Work, $\Delta H/\theta_{cr}$	82.0 kJ/kg
Degree of Reaction*	30.7 %
Corrected Airflow, $W/\sqrt{\theta_{cr}}/\sqrt{Scr}$	3.80 kg/s
Corrected Speed, $N/\sqrt{\theta_{cr}}$	156 s⁻¹

* Defined as $(H_1-H_2)/(H_{0T}-H_{2T})$

Table 2 Turbine physical dimensions

	Nozzle	Rotor
Root Diameter m	0.362	0.346
Blade Height m	0.0377	0.0567
Tip Clearance m	0.0006	0.0006
Trailing Edge Thickness m	0.0005	0.0005
Chord m	0.036	0.046
Number of Blades	50	42
Solidity, C/S	1.43	1.54
Blade Backbone Length to Chord Ratio	1.072	1.233
Maximum Blade Thickness to Chord Ratio	0.156	0.208
Stagger Angle rad (deg)	0.925 (53.0)	0.436 (25.0)
COS (O/S) rad (deg)	1.222 (70.0)	1.073 (61.5)

Fig. 3 Design point velocity triangles at midspan under the rig condition

static pressure tapings.

The offset angle, $\Delta\epsilon$, is a deviation from the design stagger angle, and it becomes positive as exit flow angles increase. The nozzle offset angle was detected by two synchros, axisymmetrically mounted on the turbine rig. These are connected directly with the vane shafts and amplified by a pair of gears (ratio 10 to 1). The measured offset angles were within ± 2 percent in an amplitude ratio of the gears, and within ± 0.1 degrees in the synchros.

Experimental condition

The test rig was connected directly to a ten-stage axial flow compressor (as an air source) and a steam turbine (as an auxiliary drive engine). The test was conducted to keep rotational speeds constant between 205 and 208 1/s. Therefore, corrected rotational speeds, $N/\sqrt{\theta_{cr}}$, varied with static pressure ratio, P_{01}/P , as shown in Fig. 6. The corrected speeds were 12 percent higher at low pressure ratios than at the design point. It is deduced from the experimental results indicated in reference (1) that these conditions bring about 1 to 1.5 percent increments in efficiency. On the other hand, there was no remarkable effect of the corrected speed variation on airflow characteristics, especially near a critical pressure ratio according to reference (8).

Blade Reynolds numbers based on blade chord and blade exit flow properties were 0.5×10^6 to 1.3×10^6 for the nozzle and 0.45×10^6 to 0.95×10^6 for the rotor under the test conditions.

Method for performance evaluation

The design and performance prediction of the transonic turbine were evaluated by a mean-line prediction method. In this method, velocity triangles and flow properties at midspan are calculated by incorporating a system of aerodynamic losses. This calculation is based on the assumption that the generation of various losses in a whole flow path can be represented by thermodynamic processes at midspan. A mean-line prediction method has some defects in accuracy, but it has been being widely utilized. It is a convenient method to carry out the preliminary design of

Fig. 6 Corrected speeds under the test conditions

turbines and to evaluate the off-design performance for a short time, because various loss systems proposed by many researchers can be applied to this method.

A program was developed to calculate airflow rates, velocity triangles, and other properties at midspan under the specified aerodynamic conditions - inlet total pressure, inlet total temperature, static exhaust pressure, and principal geometrical parameters.

Two well-known loss systems have been introduced into this program. One is Ainley and Mathieson's system⁽¹¹⁾ improved by Dunham and Came (written in short, AMDC)⁽¹⁰⁾, and the other is Craig and Cox's system (C&C)⁽⁹⁾. Both systems have the capability to calculate off-design performance considering effects of exit Mach numbers. It is known that most of the calculated values of overall efficiency are substantially within ± 1.25 percent of the measured values in the C&C loss system and ± 2 percent in the AMDC loss system.

Assumptions employed in the procedure of performance estimation are as follows:

Fig. 7 Characteristics of corrected airflow

Fig. 8 Nozzle flow conditions in the tested range of nozzle offset angle

(a) if a blade row is choked, the flow coefficient is always constant for all the pressure ratios. (b) in a choked blade, pressure losses are generated between the blade throat and blade exit. (c) airflow rates in a subsonic exit flow condition are defined as follows:

$$W = W_N + (1-M)(W_L - W_N)$$

,where W_N is the flow rate through cascades without losses and W_L is the flow rate through cascades calculated from flow properties including the total losses. The former two assumptions are derived from the reaction test results of turbine nozzles examined by Keenan⁽¹²⁾ and Kraft⁽¹³⁾. However, to estimate the performance of turbines with extremely low aspect ratio rows, these assumptions are not always appropriate according to reference (14). The third assumption is introduced to make smooth the change in airflow characteristics at the Mach number of unity. These assumptions have a large effect on airflow rates as well as degree of reactions and efficiencies, especially, when nozzle row and rotor row are choked alternately.

The following correlation between flow coefficients and nozzle offset angles was deduced from the experimental results of nondimensional airflow characteristics:

$$W = 1 + 0.7234\xi \times 10^{-3}$$

However, this relation is applicable only to this test turbine.

Fig. 9 Correlation between nondimensional airflow rate and nozzle offset angle

Fig.10 Comparison of measured and predicted stage efficiency

Test Results and Discussions

Airflow characteristics

Corrected airflow characteristics are shown in Fig.7. The nozzle row is choked at the offset angle of zero, while a choking occurs in the rotor row at the offset angle of -12 degrees. This is verified from the correlation of the pressure ratio across the nozzle row, mean exit static pressures to inlet total pressures at midspan, with the offset angle as shown in Fig.8.

Above the choking, the measured airflow rates are larger by from 2.0 to 2.5 percent than the calculated values, which are coincident with each other regardless of the loss systems as assumed above. These differences are equivalent to about 0.5 degrees of stagger angle. In contrast, the differences between those airflow characteristics below the choking appreciably depend on the loss systems.

The relation between the nondimensional value, W , of the corrected airflow divided by $\cos(\alpha_{1cr})$ and the nozzle offset angle is presented in Fig.9. The critical exit flow angle, α_{1cr} , is the nozzle exit flow angle at the Mach number of unity, proposed by Ainley and Mathieson in reference (15). The angle, α_{1cr} , was nearly equal to $\cos^{-1}(0/S)$, because the ratios of spacing to back surface radius in the nozzle vane are of the order of 10^{-3} in the offset angle range of -12 to 6 degrees.

Corrected airflow rates are proportional to blade throat areas including tip clearance spaces and flow coefficients. The tip clearance space downstream of throat changes as the nozzle rotates around the axis of center of weight. The variation of area ratio, $(1+Ak/At)/(1+Ak/At)_{\alpha=0}$, is within ± 2 percent in the tested range of offset angles. It appears that W is subject to the variation of area ratio.

Flow coefficients are generally affected by not only incidences and Reynolds numbers but also flow inlet angles and flow outlet angles.⁽¹⁴⁾ Above the range of choking pressure ratios, these factors were hardly changed affecting the flow coefficient of nozzle. However, the experi-

Fig.11 Comparison of measured and predicted turbine efficiency

mental results show that the nondimensional airflow rate, $\dot{W}$, changes in proportion to the offset angle over the choking range. This can be interpreted as follows: even if a mean pressure ratio across the nozzle row is in excess of a critical value, the whole passage of nozzle row is not always choked due to three-dimensional distributions of exit pressures and pressure losses. Flow coefficients are a function of loss coefficients and not a function of pressure ratios until a choking occurs. Therefore, the flow coefficients of blade under these conditions are not equal to unity.

Characteristics of efficiency and work

Fig.10 shows the manner in which stage efficiency varies with the pressure ratios at two constant nozzle offset angles, zero and -12 degrees. At the offset angle of zero, the measured efficiency decreases with an increase of pressure ratios to a value 3.5 percent lower at the design point than the maximum. The efficiency calculated by use of the C&C loss system varies slightly over the range of pressure ratios from 2.0 to 3.8 and is larger by from 1 to 4 percent than the measured data. On the other hand, the efficiencies calculated by the AMDC loss system are lower from -2 to -5 percent than the measured values.

At the offset angle of -12 degrees, a sudden change in the measured efficiency appeared over the range of pressure ratios from 2.2 to 2.7. This can be deduced from the calculated results that the rotor blades operate at a large negative incidence of more than -40 degrees and under a transonic flow condition, as shown in Fig. 15. However, there is no definite information about the flow condition in rotor to explain this change adequately.

At the offset angles of zero and -12 degrees, the efficiency was estimated much lower by the AMDC loss system, especially over 6 percent lower at the offset angle of -12 degrees, than the measured values.

In the calculation of stage efficiency, the C&C loss system was more suitable than the AMDC loss system. In the same figure, the calculated curves of stage efficiency at a constant corrected speed of 156 1/s are presented. (dotted line) The curve with the offset angle of zero has two peaks at both pressure ratios of 1.9 and 3.8, but the peak at the stage pressure ratio of 3.8 is not so clear as in the experimental results reported in reference (1). This is reflected in an increase of degree of reaction as explained in the following.

Fig.11 shows the variation of turbine efficiency (total to static) with pressure ratio, which includes exhaust losses. The variation of the measured efficiency is similar to the calculated values at the offset angles of zero and -12 degrees, but in the C&C loss system the maximum difference in efficiency between them becomes about 4 percent with an increasing pressure ratio at the offset angle of zero, and 6 percent at the offset angle of -12

degrees. This is because the predicted degree of reaction is lower than the measured value in the high pressure ratio range. In the AMDC loss system, the discrepancy between them is extremely great in the lower range of pressure ratios, being over 8 percent at the offset angle of -12 degrees.

The turbine with variable nozzle is characterized in that it can maintain the efficiency high over a wide range of pressure ratios. Fig.12 shows the correlations of stage efficiency with the nozzle offset angle at five pressure ratios. The optimum offset angle which gives the maximum stage efficiency changes from -6 to zero degree with an increasing pressure ratio. It is notable that the stage efficiency for the turbine with variable nozzle can remain higher over a wide range of pressure ratios than that for the turbine with fixed nozzle.

Fig.13 shows the variation of corrected work with pressure ratio and the good agreement of the calculated results with the measured data as the result of a trade-off between airflow rate and efficiency.

Equivalent degree of reaction

The equivalent degree of reaction based on pressures rises with an increase of pressure ratio as shown in Fig.14. These characteristics can be inferred from the calculated variation of exit Mach number at the nozzle and rotor as shown in Fig.16. At the offset angle of zero, the

Fig.12 Variation of stage efficiency with nozzle offset angle at five pressure ratios

nozzle exit Mach number grows more rapidly than the rotor exit Mach number below the nozzle choking. Therefore, degree of reaction decreases. Above the nozzle choking, the rotor exit Mach number continues to rise because the nozzle exit Mach number is limited by a rotor blade choking. This leads to an increase of degree of reaction. An increase in flow acceleration in rotor blades apparently contributes to efficiency improvement. At the offset angle of -12 degrees, the same phenomenon occurs but the nozzle is not choked. The rotor exit Mach number increases over a critical Mach number because the nozzle exit Mach number is restricted by the low blade choking Mach number of about 0.8.

The degree of reaction is mainly decided from the throat area ratio of nozzle and rotor, and it is influenced by the magnitude and the ratio of losses in nozzle and rotor. Furthermore, a choking when it occurs at either row depends largely on airflow characteristics too. The test results at the offset angle of zero indicate that the assumption about airflow characteristics is appropriate for performance estimation of transonic turbines. However, according to the test results at the offset angle of -12 degrees, the flow coefficient of blades operating under the condition of large negative incidences should be studied more in detail.

Other characteristics and properties

The blade choking Mach number has an important effect on the variation in degree of reaction, at which a rotor row is fully choked. At the offset angle of zero, the design blade incidence is -2.6 degrees, at which the ratio of throat area to inlet area for the rotor blade is 1.034. Therefore, the choking Mach number is about 0.78 according to reference (17). At the offset angle of -12 degrees, the blade incidence is estimated to be about -40 degrees for the pressure ratio of 2.8 as shown in Fig.15. Then the choking Mach number is about 0.45. Fig.16(a) shows that the nozzle exit Mach number increases gradually with pressure ratio, which was derived from isentropic relations to the measured pressure ratio, P_{07}/P_1 . The choking Mach numbers calculated from the measured pressure ratios, P_{07}/P_1 , are 0.67

Fig.13 Comparison of measured and predicted corrected-work

and 0.44 at the offset angles of zero and -12 degrees respectively, neglecting the losses in nozzle.

Fig.17 shows exit swirl angles at midspan. In all cases of the offset angles of zero and -12 degrees, the calculated values vary moderately with pressure ratio unlike the measured data. This tendency is similar to that of degree of reaction.

Fig.18 shows the variation of pressure ratios which is related to the absolute Mach number at rotor exit. In a high pressure ratio range, the measured values become smaller than the calculated ones. In both cases of the offset angles of zero and -12 degrees, these trends correspond to the variation of the degree of reaction already mentioned.

Comparison of the C&C loss system with the AMDC loss system

The accuracy of performance estimation is largely restricted by the propriety of flow characteristics and loss systems. Fig.19 shows the differences between the two loss systems in terms of enthalpy loss coefficients composed of profile losses, secondary losses and tip clearance losses. The secondary loss in rotor evaluated from the AMDC loss system was only 1 to 2 percent greater than that from the C&C loss system, and the loss in nozzle calculated with both loss systems was almost the same.

For the C&C loss system, the tip clearance loss correlation proposed by Ainley and Mathieson is recommended under

Fig.14 Comparison of measured and predicted equivalent degree of reaction

Fig.15 Calculated results of incidence of rotor with pressure ratio

the conditions described in reference (9). Therefore, the tip clearance losses in the C&C loss system are equal to those in the AMDC loss system, multiplied by $1.064 (kc/C)^{0.22}$ in the form of total pressure loss coefficient.

A large difference between the two loss systems appeared in profile losses. This resulted from the difference in estimation of Mach number losses and in blade stalling incidences. At the offset angle of zero, the profile losses in the C&C loss system increase by only one percent with an increasing pressure ratio. In the AMDC loss system, the losses at the design

pressure ratio become 2.2 times as large as those at the pressure ratio of 2.1. This implies that the exit Mach numbers have a strong effect on the profile losses.

In the rotor, especially, at the offset angle of -12 degrees, the profile losses in the AMDC loss system differ extremely from those by the C&C loss system with a factor of 2 to 5. This can be seen from the calculated results showing that the rotor blades operate under the condition of large negative incidences as shown in Fig.15 and that the stalling incidences in both systems are greatly different from each other as follows; in the C&C loss

(a) at the offset angle of zero

(b) at the offset angle of -12 degrees

Fig.16 Variation of exit Mach numbers at nozzle and rotor with pressure ratio

Fig.17 Comparison of measured and predicted stage exit flow angle

Fig.18 Comparison of measured and predicted absolute exit Mach numbers at rotor

(a) at the offset angle of zero

(b) at the offset angle of -12 degrees

Fig.19 Variation of enthalpy loss coefficients with pressure ratio

system, the negative incidence ($-i_{st}$) is -45 degrees, and the positive incidence ($+i_{st}$) is 16 degrees, while in the AMDC loss system, $-i_{st}$ is -32 degrees, and $+i_{st}$ is 15 degrees, where the stalling incidence is defined as one when a profile loss is twice the minimum value.

In the prediction of stage efficiency at the offset angle of zero, the efficiency calculated by the AMDC loss system is lower than that by the C&C loss system. This contributes to the differences in tip clearance loss system. The discrepancies between them become remarkable in the low pressure ratio range because the negative stalling incidence in the AMDC loss system is smaller than that in the C&C loss system. The same relationship held at the offset angle of -12 degrees. It was demonstrated that the performance predicted by the C&C loss system was almost similar to the measured one. However, it can be expected that the improvements in the negative stalling incidence will make the AMDC loss system better.

Conclusions

The following conclusions were obtained from the foregoing considerations.

1. The mean-line prediction method presented in this paper is generally suitable for the performance estimation of the transonic turbine. The assumptions for airflow characteristics are appropriate for performance calculation, considering the results of the variation in degree of reaction and exit Mach number.

2. In the highly-loaded transonic turbine with variable nozzle, the stage efficiency of 82 percent can be attained at the design point.

3. It is shown that the calculated results using the C&C loss system have better consistency in off-design performance with the experimental results than those using the AMDC loss system.

4. The variable nozzle makes the stage efficiency keep higher over a wide range of pressure ratios.

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