Dynamic Behavior of Thin-rimmed Helical Gears with Various Web Arrangements*

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This paper presents a study on dynamic behavior of thin-rimmed helical gears with various web arrangements.

Natural frequencies of flexural vibration of gear body were measured and compared with the calculated results by Mindlin's method. The relations between these natural frequencies and spectra of circumferential, radial and axial vibrations were investigated.

The circumferential, radial and axial vibration accelerations and the root stresses were measured under different running conditions by using a gear testing machine of a power absorbing type. On the basis of these results the effects of web arrangements on vibration and dynamic load were clarified to a considerable extent.

Key Words : Gear, Vibration, Helical Gear, Web, Rim, Rib, Vibration Acceleration, Frequency Analysis, Root Stress, Dynamic Load

1. Introduction

Recently with increased requirements for size-down and lighter weight, thin-rimmed gears have come increasingly into use and their dynamic behavior is now a matter of great concern. Although many studies on dynamic behavior of the solid gears and thin-rimmed spur ones have been published^{(1)~(5)}, those of thin-rimmed helical gears are only a few. In the case of helical gears, the flexural vibration, which plays an important role in the sound radiation, is considered different from the vibration of spur gears due to the thrust.

In the present paper, the dynamic behavior of thin-rimmed helical gears with various web arrangements was investigated. Natural frequencies of flexural vibration of these gears were measured and compared with the calculated results by Mindlin's method⁽¹⁾⁽⁶⁾⁽⁷⁾. The relations between these natural frequencies and the spectra of circumferential, radial and axial vibrations were examined. The circumferential, radial and axial vibrations and root stresses of thin-rimmed helical gears with various web arrangements were measured

under different running conditions by carrying out a running test and compared with those of thin-rimmed spur gears with web arrangements⁽¹⁾. On the basis of the results the effects of web arrangements on vibration and dynamic load were clarified to a considerable extent.

2. Experimental Method and Apparatus

2.1 Test gears

Dimensions of test gears used in this experiment are shown in Table 1 together with their materials and working and heat treatment conditions. Test gear G.F is a solid gear, G.G, G.H are symmetric types of single web gears which have a web at the middle of face width, and G.H has also ribs as shown in Fig.1(a). G.I, G.J are asymmetric types of single web gears which have a web at one end of face width, and G.J has also ribs as shown in Fig.1(b). The accuracy of these test gears is of 4th class in JIS. Gear P.B is a solid gear which meshes with each test gear. P.B is of SCM415 steel, and its tooth surfaces are ground after casehardening. The accuracy of P.B is of 1st class in JIS.

2.2 Testing method

The gear testing machine used in this experiment is of power absorbing type⁽¹⁾. In order to examine the natural frequencies of flexural vibration of G.F~G.H, the axial vibration accelerations of shock response under no load were measured. The effects of ribs and web arrangements on the dynamic behavior of thin-rimmed helical gears were investigated by measuring

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the circumferential, radial and axial vibration accelerations at the test gear speed $n = 400 \sim 1400$ rpm under the applied torque $T = 39.2, 68.6, 98$ Nm {4, 7, 10 kgfm}. These vibration accelerations of G.I, G.J were measured for two cases (case 1, case 2). In the cases 1, 2 the thrust is exerted on the web side and on the other side respectively. The accelerometers were attached to points separated by $r = 40, 50, 60$ mm from the gear center, and for ribbed gears G.H, G.J to points near (18 mm from rib) and between ribs,

(a) G.H

(b) G.J

Fig.1 Structures and dimensions of test gears

and the effects of the position of accelerometer on vibration acceleration were examined. The radial vibration accelerations of rim part (above and between ribs) of G.H were also measured. The vibration acceleration and root stress waveforms on compressive side were observed at the same time. Strain gages (gage length : 0.3 mm)

Fig.2 Block diagram for experiment

Fig.3 Flow chart for computer procedure

Table 1 Test gears

Gear sign	G.F	G.G	G.H	G.I	G.J	P.B
Normal module m_n			3			3
Normal pressure angle α_n			20°			20°
Helix angle β_0			20° (L)			20° (R)
Number of teeth z			48			24
Pitch circle diameter d_0			153.24 mm			76.62 mm
Face width b			20 mm			20 mm
Rim thickness l_w	∞ **		$2m_n^*$			∞ **
Web thickness b_w	20 mm		3 mm			20 mm
Gear structures	Symmetric	Symmetric	Symmetric (Ribbed)	Asymmetric	Asymmetric (Ribbed)	Symmetric
Materials			S45C			SCM415
Treatment conditions			Hobbed Normalized			Ground Casehardened
Remarks	* m_n : Normal module ** ∞ : refers to solid gear					

were pasted on the compressive fillet determined by Hofer's method in tooth height direction at the middle of the face width for G.F~G.H and one end of the face width (above web) for G.I, G.J. For ribbed gears G.H, G.J, strain gages were pasted on teeth just above and between ribs. A circumferential backlash $c_0 = 160 \mu\text{m}$, which is equal to the minimum value specified in JIS, was selected in this experiment.

Figure 2 shows a block diagram for measurement of root stresses and vibration accelerations. The output signals from strain gages and accelerometer were amplified through the bridge circuit and dynamic strain meter and through the amplifier respectively and memorized in the transient recorder together with a synchronizing signal, and then the root stresses and vibration accelerations were measured on the picture tube of the syn-

chroscope and on the meter of the amplifier respectively. The frequency analyses were also carried out.

3. Theoretical Analysis of Vibration of Gear Body

The natural frequencies of flexural vibration of gear body were analysed by Mindlin's method^{(1) (6) (7)}. Figure 3 shows a flow chart for computer procedures and their details are given in ref.(1).

4. Experimental Results and Discussions

4.1 Results of frequency analyses

The measured results (shock spectra) of natural frequencies of flexural vibration of G.F G.H are shown in Fig.4. Each magnitude of the peaks of these vibration spectra changes with the position of applied shock but the frequencies of the peaks do not. The calculated values of natural frequencies of flexural vibration of G.F, G.G and the measured ones (Fig.4) are shown in Table 2. The boundary conditions for these calculations are selected as follows on the basis of the results of ref. (1). The outer radii of gear body are put as r_b for G.F (r_b : radius of tooth bottom circle) and as $r_p - m_n$ for G.G (r_p : radius of pitch circle, m_n : normal module), the mass of the teeth is considered at the outer circumference and has an effect only in axial direction, and the

Table 2 Natural frequencies of flexural vibration

			G.F		G.G	
			Calculated values	Measured values	Calculated values	Measured values
Outer radius	mm		72.87		73.62	
Teeth mass	g		0.520		0.420	
Elastic constants	k_w N/mm		2×10^5			
	k_r Nmm/rad		4×10^9		∞ *	
	k_g Nmm/rad		4×10^9			
Natural frequencies Hz	(m,s)	(1,0)	1585	1500	637	600
		(2,0)	4106	4360	1452	1520
		(3,0)	8706	8780	4241	4380
		(4,0)	13985	14020	8021	8160
		(5,0)	19545	19520	12398	12440
		(6,0)	—	—	17125	17060

* ∞ : Fixed support

Fig.4 Vibration spectra

inner circumference of G.F is elastically supported while the outer circumference of the boss of G.G is fixed. It is seen from Table 2 that these calculated values agree well with the measured ones. The peaks near 15 kHz and 20 kHz of G.H in Fig.4 are considered to be due to the effects of ribs.

The vibration spectra in three directions of G.F~G.H under $T = 98 \text{ Nm}$, $n = 1400 \text{ rpm}$ are also shown in Fig.4. The frequencies of the peaks of these vibration spectra correspond with the natural frequencies of flexural vibration, mesh frequency and its higher order frequencies. Since the frequencies of the peaks in axial direction coincide with those in other directions, the axial vibration is considered to be coupled with the circumferential and radial ones.

(a) G.F

4.2 Effect of gear speed on vibration

Figure 5(a)~(e) show the effect of gear speed on the vibration accelerations in three directions of test gears for $T = 98 \text{ Nm}$. The vibration acceleration levels (VAL) dB are given as $20 \log(a_i/a_0)$ (a_i : vibration acceleration (rms value), $a_0 = 10^2 \text{ mm/s}^2$). The VALs of ribbed gears G.H, G.J shown are those between ribs which are almost equal to those near rib. The VALs of asymmetric types of single web gears

(b) G.G

(c) G.H

(d) G.I

(e) G.J

1 Nm = 0.102 kgfm

Fig.5 Effect of gear speed on vibration acceleration ($T = 98 \text{ Nm}$)

G.I, G.J are those for case 2 which are almost equal to those for case 1. It is seen from Fig.5 that the VAL in each direction has a tendency to increase with an increasing gear speed for each gear and shows a higher peak at each resonance speed for G.G~G.J. The circumferential, radial and axial VALs of G.F are smaller in the order mentioned, but the axial VALs of G.G~G.J are larger than the circumferential and radial ones. This might be because the axial vibration displacements of thin-rimmed gears with web arrangement are larger compared with the case of solid gears, since the rigidity of the web of thin-rimmed gears is smaller.

Figure 6 shows the effect of gear speed on the VALs of the thin-rimmed spur gear G.B⁽¹⁾ whose dimensions are almost equal to those of G.G. The VALs of G.B are largest in circumferential direction

Fig.6 Effect of gear speed on vibration acceleration (spur gear : G.B)

(a) Circumferential direc.

(b) Radial direc.

(c) Axial direc.

1 Nm = 0.102 kgfm

Fig.8 Relation between gear speed and vibration acceleration (T = 98 Nm)

but those of G.G are in axial direction. The circumferential and radial VALs of G.B are larger than those of G.G, but the axial VAL of G.B is smaller than that of G.G. These phenomena are known from comparison of the cases of thin-rimmed spur and helical gears with other web arrangements. This might be because the axial vibration displacement of helical gear is larger compared with that of spur gear due to the thrust and because the circumferential VAL of helical gear is smaller compared with that of spur gear since the variation of the stiffness of helical gear teeth is smaller.

Figure 7 shows a torsional vibration system corresponding to the gear testing machine used in this experiment considering the tooth stiffness, and the natural frequency of this system is calculated by Holzer's method. The tooth stiffness at pitch point of the virtual spur gear of helical gear is converted into the

Fig.7 Torsional vibration system

torsional rigidity. In this case, the torsional rigidity K_t of torsion bar is neglected, since it is much smaller compared with other torsional rigidities. The natural frequencies f_{ni} (i : modal number) of the system shown in Fig. 7 are calculated to be $f_{n1} = 463$ Hz, $f_{n2} = 979$ Hz, $f_{n3} = 3628$ Hz for G.F, $f_{n1} = 464$ Hz, $f_{n2} = 1224$ Hz, $f_{n3} = 3818$ Hz for G.G and $f_{n1} = 464$ Hz, $f_{n2} = 1211$ Hz, $f_{n3} = 3805$ Hz for G.H. The peaks of VAL of G.F~G.H appear at the gear speed where the natural frequencies of flexural and torsional vibrations coincide with the mesh frequency f_z or its higher order frequencies. The VAL of G.F has a peak at $n = 650$ rpm because of the coincidence of $3f_z$ and $f_{1,0}$.

Figure 8(a)~(c) show the relations between the gear speed and the VALs of test gears for the applied torque $T = 98$ Nm. It is seen from this figure that the VALs in three directions of G.F, on the whole, are smaller compared with those of other gears and this tendency is more remarkable in axial direction than in other directions. Therefore the effects of web arrangements on the VAL are considered largest in axial direction. The order of VAL in each direction of thin-rimmed gears G.G~G.J changes depending on the gear speed.

Figure 9 shows the radial VALs of web (gear body) and rim parts (above and between ribs) of G.H for $T = 98$ Nm. The VAL of each part has a tendency to increase with an increasing gear speed, and the VAL of web part is almost equal to that of rim part above rib but smaller than that of rim part between ribs. This might be because the radial vibration displacement of web part is almost equal to that of rim part above rib but smaller than that of rim part between ribs.

Fig. 9 Comparison of radial vibration accelerations of web and rim parts

4.3 Effect of applied torque on vibration

Figure 10(a)~(c) show the relation between the applied torque T and the VAL in each direction of G.F, G.G, G.I at $n = 500, 1400$ rpm. It is seen from this figure that the VAL in each direction shows a tendency to increase with an increasing applied torque for each test gear but its increasing rate is smaller compared with that of spur gear⁽¹⁾. This might be attributed to the fact that the variations

(a) G.F

(b) G.G.

(c) G.I

1 Nm = 0.102 kgfm

Fig. 10 Effect of applied torque on vibration acceleration

of the stiffness of gear teeth and load have great influence on the vibration in the case of spur gears but the vibration due to the variations of the stiffness and load is smaller in the case of helical gears.

4.4 Relation between vibration acceleration and position of accelerometer

Figure 11(a), (b) show the relation between the position of accelerometer and the VAL in each direction of G.F and G.I respectively for $T = 98 \text{ Nm}$. The abscissa denotes the gear speed and the ordinate $\delta = 20 \log (a_r/r_i/a_r=40)/(r_i/40)$ ($r_i = 50, 60 \text{ mm}$). $\delta = 0 \text{ dB}$ when the increasing rate ($a_r/r_i/a_r=40$) of VAL is equal to that ($r_i/40$) of r , and $\delta = -1.9 \text{ dB}$ for $r = 50 \text{ mm}$ and $\delta = -3.5 \text{ dB}$ for $r = 60 \text{ mm}$ when the VAL is constant irrespective of r . The VALs of G.F, G.I increase proportionally to r in circumferential direction and are almost constant irrespective of r in radial direction, and the increasing rates of axial VALs of G.F, G.I are larger compared with those of r and the increasing rate of axial VAL of G.F with r is smaller

(a) G.F

(b) G.I

1 Nm = 0.102 kgfm

Fig.11 Relation between vibration acceleration and position of accelerometer

than that of G.I. The relation between the position of accelerometer and the VAL in each direction of G.G, G.H, G.J is almost the same as in the case of G.I.

4.5 Effects of gear speed and applied torque on root stress

Figure 12 shows the relation between the gear speed and the maximum root stresses of test gears for $T = 98 \text{ Nm}$. For ribbed gears G.H, G.J the maximum root stresses of the tooth between ribs are shown, which are larger than those of the tooth just above rib, and for asymmetric types of single web gears G.I, G.J the maximum root stresses for case 2, which are almost equal to those for case 1. The maximum root stress of each test gear, on the whole, is almost constant irrespective of gear speed, but that of G.I shows peak at $n = 750 \text{ rpm}$ and that of G.J at $n = 1400 \text{ rpm}$. Since the gear speeds for these peaks coincide with those of the VALs (Fig.5), the dynamic root stress is considered to be related to gear vibration.

Figure 13 shows the relation between the applied torque and the maximum root stresses of test gears at $n = 700 \text{ rpm}$. The maximum root stress of each test gear shows a tendency to increase with an increasing applied torque. This tendency is also observed at other gear speeds.

1 Nm = 0.102 kgfm, 1 MPa = 0.102 kgf/mm²

Fig.12 Relation between root stress and gear speed

1 Nm = 0.102 kgfm, 1 MPa = 0.102 kgf/mm²

Fig.13 Relation between root stress and applied torque

5. Conclusions

The main results obtained from this investigation are summarized as follows.

- (1) The calculated natural frequencies of flexural vibration of solid and thin-rimmed helical gears by Mindlin's method agree well with the measured ones.
- (2) In the circumferential, radial and axial vibration spectra of helical gears, peaks are observed at the mesh frequency, its higher order frequencies and the natural frequencies of flexural vibration. The vibrations in three directions are considered to be coupled with each other as in the case of spur gears.
- (3) The vibration accelerations in three directions of solid and thin-rimmed helical gears increase with an increasing gear speed. The vibration acceleration in each direction of thin-rimmed helical gears with web arrangement becomes considerably larger at the gear speed where the natural frequencies of flexural vibration coincide with the mesh frequency or its higher order frequencies and this tendency is more remarkable compared with the case of thin-rimmed spur gears.
- (4) The vibration accelerations of solid spur and helical gears and thin-rimmed spur gears are largest in circumferential direction, but those of thin-rimmed helical gears are in axial direction.
- (5) The circumferential and radial vibration accelerations of helical gears are smaller than those of spur gears, but the axial vibration acceleration of helical gears is larger than that of spur gears.
- (6) The vibration acceleration levels dB in three directions of thin-rimmed helical gears with web arrangement, on the whole, are higher than those of solid helical gears as in the case of spur gears. The order of vibration acceleration level dB in each direction of thin-rimmed helical gears with web arrangement changes depending on the gear speed.
- (7) The vibration accelerations in three directions of helical gears increase with an increasing applied torque and their increasing rates are smaller compared with those of spur gears.
- (8) The circumferential vibration accelerations of solid and thin-rimmed helical gears increase proportionally to r (r : distance between the position of accelerometer and gear center) and the radial ones are almost constant regardless of r . The increasing rate ($a_r = r_i / a_r = 40$) of axial vibration acceleration is larger compared

with that ($r_i/40$) of r and the increasing rate of axial vibration acceleration of thin-rimmed helical gears with r is larger than that of solid helical gears.

(9) The radial vibration acceleration of rim part (between ribs for ribbed gear) of thin-rimmed gears with web arrangement is larger than that of web part.

(10) Both the vibration accelerations in three directions and the dynamic root stress just above web of an asymmetric type of single web helical gear are almost equal regardless of the direction of the thrust.

(11) The dynamic root stresses of helical gears are almost constant regardless of the gear speed, and in some cases of thin-rimmed helical gears the dynamic root stress shows peaks at the gear speed where the vibration accelerations in three directions have peaks. The dynamic root stresses of helical gears show a tendency to increase with an increasing applied torque.

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