Numerical Analysis of Pneumatic Capsule Pipeline System*

(Continuous Loading with Short Intervals)

By Yuji TOMITA**

A numerical analysis by the method of characteristics is presented which enables a successful estimation by capsule collision. The pattern of collision for high speed capsules differs from that for low speed ones. Relation between time histories of pressure at fixed points of pipeline and pathes of capsules is discussed.

Key Words : Compressible Flow, Unsteady Flow, Fluid Transportation System, Capsule Transport, Numerical Analysis, Collision

1. Introduction

In the previous paper⁽¹⁾, motions of capsules as well as air flow is analysed in such a case when the capsules are periodically injected into a pipeline. The time increment and the space mesh size in the analysis are taken sufficiently small in compared respectively with the capsule loading interval and distance between neighboring capsules. Thus we could avoid a collision of capsules. In this paper we analyze a case when the collision can occur.

2. Basic Equations

We use the same assumptions as those in the previous paper⁽¹⁾. Then, the equations of state, continuity and motion for air and the equation of motion for the capsules are expressed, respectively, as,

$$p/\rho = RT = \text{const.}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0, \quad (2)$$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial p}{\partial x} - \frac{\lambda}{2d}\rho u^2, \quad (3)$$

$$M_i \ddot{Z}_i = (p_{Ri} - p_{Fi})A - fM_i g, \quad (4)$$

where p : pressure, ρ : air density, R : gas constant, T : absolute temperature, t : time, x : axial coordinates, u : air velocity, λ : pipe friction factor of air, d : inside pipe diameter, M_i : mass of i -th capsule, Z_i : i -th capsule coordinates, p_{Ri} and p_{Fi} : pressures at the rear and front sides of i -th capsule, A : pipe cross sectional area, f : frictional coefficient of capsule against the pipe wall, and g : acceleration of gravity.

The boundary conditions for air stream at the capsule are

$$\rho_{Ri} u_{Ri} = \rho_{Ri} \dot{Z}_i + (1 - A_c/A) Q_{si} \quad (5)$$

$$\rho_{Fi} u_{Fi} = \rho_{Fi} \dot{Z}_i + (1 - A_c/A) Q_{si} \quad (6)$$

where A_c the cross sectional area of the capsule, Q_{si} the mass velocity of air through the clearance between the capsule and the pipe wall. We assume that air leaks from the

higher pressure side to the lower and that Q_{si} is given by an orifice flow model. The boundary conditions at the inlet and exit of pipeline are

$$x = 0 ; p = p_t, \quad x = l ; p = p_o \quad (7)$$

where p_t is the inlet pressure, p_o is the exit pressure, and L is the length of pipeline. The initial conditions are

$$\begin{aligned} t = 0, x > 0 ; p &= p_o, u = 0 \\ t = 0, x = 0 ; p &= p_t \end{aligned} \quad (8)$$

The initial conditions for i -th capsule, loaded at a time interval of τ , are

$$t = (i-1)\tau ; Z_i = 0, \dot{Z}_i = 0 \quad (9)$$

3. Procedure for Calculation

Equations (1) to (3) are solved by the method of characteristics. When the capsule trajectory crosses the characteristic, we set the pressures at the capsule front and the rear as⁽²⁾

$$p_{Ri} = p_{k-1}^{m-1}, \quad p_{Fi} = p_k^{m-1} \quad (10)$$

By this assumption the calculation is significantly simplified if the distance between neighboring capsules is smaller than the mesh size $\Delta x = x_k - x_{k-1}$. When there are two capsules in Δx as shown in Fig. 1, we put

$$p_{Fi} = p_{Ri-1} = (p_{k-1}^{m-1} + p_k^{m-1})/2 \quad (11)$$

If two capsules, the i -th and the $(i-1)$ th, collide with each other in a region DCGF, we take the two capsules as a single capsule.

Fig. 1 A case when there are two capsules in Δx .

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** Professor, Kyushu Institute of Technology, Sensuicho 1, Tobata, Kitakyushu.

In such case we suppose that the two capsules are coupled and that there still remains a finite space between the two. Air flow at F can be influenced by the coupled capsules, but we assume as if the i -th capsule crosses the left running characteristic FC separately; the mesh point behind the coupled capsules is modified by the back capsule and the point in front of the coupled capsules by the preceding capsule. After the coupling of two capsules the velocity at $t = \Delta t$ is given by using the conservation of momentum at the collision, and the coordinate Z^m is given by the mean value of each coordinates which would take without collision at $t = \Delta t$.

4. Results and Discussion

At first we calculate the case when the previous procedure could not apply. Figure 2 shows the result when three capsules of which diameter d is 0.18m and $M=20$ kg are loaded at $\tau = 0.5$ s for $d=0.2$ m, $L=500$ m, $p_t=2.5p_0$, $p_0=101.3$ kPa, $T=293$ K and $c=0.5$ or $c=0$, where c is the orifice discharge coefficient. When there is no air leakage, a collision occurs between the 2nd and the 3rd capsules. Since air mass between two capsules is constant, repulsion by compressibility of air works when the distance between two capsules decreases and the distance again increases. Figure 3 is the time histories of rear and front pressures at each capsule for $c=0$. It is seen that the variation of the front pressure is great and that it takes peak values when two capsules close with each other. When the number of capsules injected into the pipeline is 9 for $c=0$, it was found out that the collision occurs from the last capsule as in Fig. 2. In this case an intensive expansion wave returns from the pipe exit for the first capsule and then it gets great acceleration as compared with the 2nd. Thus the 2nd capsule cannot catch up to the 1st. It was confirmed that when there is air leakage the collision does not occur and that the variations of pressure and velocity become smooth.

Otaki⁽³⁾ already mentioned the collision when the capsule velocity is lower. We calculated under an approximately equal condition to his case. Figures 4 and 5 show the result where we put $M=2$ kg, $d_c=0.100$ m, $d=0.1035$ m, $c=0.8$, $L=200$ m, $\tau=1$ s, $p_t=p_0$, $p_0=101.3$ kPa and $T=293$ K. The collision occurs even if there is air leakage, and moreover it starts from the first capsule. Figure 5 is the time histories of pressure at $x=40$ m, 120m and 160m. The numeric on the graph means capsule number which passes x . For the graph of $x=40$ m the pressure rapidly decreases when capsules pass. However, for the graph of $x=120$ m the pressure does not always drop rapidly when capsules pass. In some cases the pressure increases. Thus it is said that relation between capsule passing and pressure change at the fixed point is not simple. When the collision approaches, pressure between the capsules increases, which is inferred from the graph of $x=40$ m near $t=7$ s when the collision soon occurs between the 3rd capsule and the 1st and 2nd capsules. The same situation can be seen for the graph of $x=120$ m near $t=14$ s.

References

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Fig.2 Capsule trajectories.

Fig.3 Time histories of pressure at the rear and front side of capsule.

Fig. 4 Capsule trajectories with air leakage.

Fig.5 Time histories of pressure at the fixed points of pipeline.

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