On the Characteristics of Bolted Joints with Gaskets*

(Stress Analysis of Full-Face Metallic Gaskets Compressed by a Cover of Pressure Vessel)

By Hiroyuki KUMANO**, Toshiyuki SAWA***,
Kazuya GONDA† and Fujiya KANAI††

This paper deals with a contact stress distribution in bolted joints when a full-face metallic gasket is used between a cover and a base of pressure vessel. Replacing the cover, the gasket and the base with a finite cylinder and two finite hollow cylinders respectively, the contact stress distribution is analyzed using three-dimensional theory of elasticity. Effects of the thickness and the stiffness of gaskets on the contact stress distribution are shown by numerical computations. Next, the force ratio of bolted joints with gaskets, which is a ratio of an increment of bolt axial force to an inner pressure, and the sealing performance are analyzed. A bending moment is produced in bolts by the inner pressure. The maximum stress produced in bolts is also analyzed taking the bending moment into consideration. For verification of the analytical method, experiments are performed. Analytical results are in a fairly good agreement with experimental ones.

Key Words: Contact Stress, Metallic Gasket, Pressure Vessel, Bolted Joint, Sealing Performance

1. Introduction

Bolted joints with gaskets are widely used in connections of the mechanical devices in order to prevent leakage of fluid. Since a bolted joint between a cover and a base of pressure vessel is of practical importance to sealing performance, they are joined with bolts and nuts inserting gaskets. Design of bolted joints with gaskets is standardized in the codes of JIS and so on⁽¹⁾⁽²⁾. But, in order to make a rational design of bolted joints with gaskets, it is necessary to examine the distribution of the contact stresses on the contact surfaces of gaskets due to an initial clamping force, which governs the sealing performance, and the force ratio (the ratio of an increment of bolt axial force to an external load) in detail, when the external load caused by the fluid pressure is applied to joints. It seems that many investigations^{(3)~(8)} have been carried out mainly with respect to gaskets themselves, but the problems mentioned above have not been solved sufficiently.

Therefore, in a series of our investigations^{(9)~(12)} on the characteristics of bolted joints with gaskets, effects of the stiffness and the thickness of gaskets on the sealing performance and the force ratio are studied by analytical theories and experiments in the case where two hollow cylinders⁽⁹⁾ with a gasket are fastened with a set of bolt and nut, in the cases of T-flanged joints⁽¹⁰⁾, circular flanged joints⁽¹¹⁾ and pipe flanged connections⁽¹²⁾.

In the present paper, discussion is made on the characteristics of a connection between a cover and a

base of pressure vessel with a full-face metallic flat gasket, which is important in practice but few investigations have been done up to now. The purpose of the present paper is to make a contribution to the rationalization of the design of bolted joints with gaskets. In an analysis, the distribution of contact stresses on gasket surfaces is made clear using three-dimensional theory of elasticity when a connection is joined with an initial clamping force. Furthermore, the force ratio is analyzed when an internal pressure is applied to the connection. Effects of the stiffness, the thickness of gaskets and the pitch circle diameter on the characteristics of joints are also examined by numerical analyses and experiments.

2. Theoretical analysis

2.1 Analysis of contact stress distribution

Figure 1 shows a connection in pressure vessels in which a full-face metallic flat gasket is interposed

Fig. 1 Model of a joint between cover and base of pressure vessel with a full-face metallic flat gasket fastened with initial clamping force

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** Associate
The Metropolitan College of Technology,
Asahigaoka 6-6, Hino, Tokyo, Japan 191.

*** Associate Prof.
Yamanashi University,
Takeda 4-3-11, Kofu, Yamanashi, Japan 400.

† Engineer
Toyota Body Co., Ltd.,
Kanayama 100, Ichiriyama-cho, Kariya, Aichi,
Japan 448.

†† Prof.
The Metropolitan College of Technology,
Asahigaoka 6-6, Hino, Tokyo, Japan 191.

between a cover and a base and which is fastened by N sets of bolts and nuts with the initial clamping force F_f . In order to analyze the contact stress on gasket surfaces, the cover, the gasket and the base of the pressure vessel are modeled as shown in Fig. 2. Namely, replacing the cover, the gasket and the base with a finite cylinder and two finite hollow cylinders as shown in Fig. 2, respectively, the contact stress distribution on gasket surfaces is analyzed. In the analysis it is assumed that the initial clamping force $N \times F_f$ acts annularly on the bearing surface as a uniform compressive stress P_1 . The outer diameter of the cylinder and that of the hollow cylinder for the cover [I], the gasket [II] and the base [III] are designated as $2a$ and $2b$, the inner diameter of the hollow cylinders for the gasket [II] and the base [III] as $2a$, the thickness as $2h_1$, $2h_2$, $2h_3$, Young's modulus as E_1 , E_2 , E_3 and Poisson's ratio as ν_1 , ν_2 , ν_3 , respectively. The diameter of the bearing surface is designated as $2c$, and the pitch circle diameter as $2d$. Since bolt holes are neglected in the model for the analysis shown in Fig. 2, the distribution of contact stresses in the neighbourhood of bolt holes is analyzed using the model of a hollow cylinder demonstrated in the previous paper⁽⁹⁾ as shown in Fig. 3. In Fig. 3, the inner diameter $2a_0$ of the hollow cylinder is equated to the diameter of bolt holes, the thicknesses $2h_1$, $2h_2$ and $2h_3$ of the hollow cylinders to those of the cover, the gasket and the base, respectively. In order to obtain the compressive spring constant for clamped parts which is treated in Section 2.2, the diameter of this hollow cylinder is determined as shown in Fig. 3(b). Namely, denote by $2b'_0$ the diameter of the circle when one circle with its center at the center of a bolt hole comes into contact with an adjacent circle with its center at the center of an adjacent bolt hole. And denote by $2b''_0$ and $2b'''_0$ the diameters of the circles which come into contact with the inner and the outer circumferences of the flange of base. Then, comparing the values of $2b'_0$, $2b''_0$ and $2b'''_0$, the smallest one is taken as the diameter $2b_0$ of the hollow cylinder.

Fig. 2 Model for analysis and dimensions of a pressure vessel

In the analyses of the cover, the gasket and the base of pressure vessel shown in Fig. 2, Michell's stress function Φ is used⁽⁹⁾⁽¹¹⁾. Then, expanding the stress distribution on the bearing surface into Bessel functions, the boundary condition for the case of Fig. 2 is expressed as follows:

For the finite cylinder [I] (cover of pressure vessel),

$$r=b: \sigma_r=\tau_{rz}=0$$

$$z=h_1: \sigma_z=f_0+\sum_{s=1}^{\infty} f_s J_0(\gamma_s r), \tau_{rz}=0$$

For the finite hollow cylinder [II] (gasket),

$$r=a, r=b: \sigma_r=\tau_{rz}=0$$

On the contact surface between $z=-h_1$ and $z=h_2$,

$$\tau_{rz}=0, \left(\frac{\partial w}{\partial r}\right)_{z=-h_1}=\left(\frac{\partial w}{\partial r}\right)_{z=h_2}$$

$$(\sigma_z)_{z=-h_1}=(\sigma_z)_{z=h_2}, a \leq r \leq b$$

For the finite hollow cylinder [III] (base),

$$r=a, r=b: \sigma_r=\tau_{rz}=0$$

$$z=-h_3: \sigma_z=g_0+\sum_{s=1}^{\infty} g_s C_0(\bar{\gamma}_s r), \tau_{rz}=0$$

On the contact surface between $z=-h_2$ and $z=h_3$,

$$\tau_{rz}=0, \left(\frac{\partial w}{\partial r}\right)_{z=-h_2}=\left(\frac{\partial w}{\partial r}\right)_{z=h_3}$$

$$(\sigma_z)_{z=-h_2}=(\sigma_z)_{z=h_3}, a \leq r \leq b$$

..... (1)

where,

$$f_0=-\frac{d_1^2-d_2^2}{b^2}P_1, g_0=-\frac{d_1^2-d_2^2}{b^2-a^2}P_1$$

$$f_s=-\frac{2\{d_2 J_1(\gamma_s d_2)-d_1 J_1(\gamma_s d_1)\}}{\gamma_s b^2 J_0^2(\gamma_s b)}P_1$$

$$g_s=-\frac{2\{d_2 C_1(\bar{\gamma}_s d_2)-d_1 C_1(\bar{\gamma}_s d_1)\}}{\gamma_s \{b^2 C_0^2(\bar{\gamma}_s b)-a^2 C_0^2(\bar{\gamma}_s a)\}}P_1$$

$$d_2=d+c, d_1=d-c$$

$J_n(\gamma_s r)$: Bessel function of 1st kind in order n

$Y_n(\gamma_s r)$: Bessel function of 2nd kind in order n

$\gamma_s, \bar{\gamma}_s$: the S -th positive roots satisfying $J_1(\gamma_s b)=0$ and $C_1(\bar{\gamma}_s a)=C_1(\bar{\gamma}_s b)=0$, respectively

$I_n(r)$: Modified Bessel function of 1st kind in order n

$K_n(r)$: Modified Bessel function of 2nd kind in order n

sh: sinh, ch: cosh

In order to analyze the finite cylinder and the finite hollow cylinders under the boundary condition of Eq. (1), Michell's stress function Φ is put as follows: For the cylinder [I], $\Phi_I = \Phi_{I1} + \Phi_{I2}$, for the hollow cylinder [II], $\Phi_{II} = \Phi_{II1} + \Phi_{II2}$ and for the hollow cylinder [III], $\Phi_{III} = \Phi_{III1} + \Phi_{III2}$, where we omit expressions for Φ_{II1} and Φ_{II2} , because they correspond to Eqs. (4) and (5) in the reference (9).

(a) model of hollow cylinder (b) determination of outer diameter cylinders of hollow

Fig. 3 Model for analysis about bolt hole

$$\begin{aligned}
& \Phi_{11} \{ \bar{A}_0^I, \bar{C}_0^I, \bar{A}_n^I, \bar{C}_n^I, \beta_n(h_1), \Delta_n(h_1), \Omega_s(h_1) \} \\
& = \bar{A}_0^I z^3 / 6 + \bar{C}_0^I z r^2 / 2 \\
& + \sum_{n=1}^{\infty} \frac{\bar{A}_n^I}{\Delta_n \beta_n^3} [\{ 2(1-\nu_1) I_1(\beta_n a) + \beta_n a I_0(\beta_n a) \} I_0(\beta_n r) - I_1(\beta_n a) \beta_n r I_1(\beta_n r)] \sin(\beta_n z) \\
& + \sum_{n=1}^{\infty} \frac{\bar{C}_n^I}{\Omega_n \gamma_n^3} [- \{ 2\nu_1 \operatorname{sh}(\gamma_n h_1) + \gamma_n h_1 \operatorname{ch}(\gamma_n h_1) \} \operatorname{sh}(\gamma_n z) + \operatorname{sh}(\gamma_n h_1) \gamma_n z \operatorname{ch}(\gamma_n z)] J_0(\gamma_n r) \dots \dots \dots (2) \\
& \Phi_{12} \{ \bar{A}_n^I, \bar{C}_n^I, \beta_n'(h_1), \Delta_n'(h_1), \Omega_s'(h_1) \} \\
& = - \sum_{n=1}^{\infty} \frac{\bar{A}_n^I}{\Delta_n \beta_n^3} [\{ 2(1-\nu_1) I_1(\beta_n' a) + \beta_n' a I_0(\beta_n' a) \} I_1(\beta_n' r) - I_1(\beta_n' a) \beta_n' r I_1(\beta_n' r)] \cos(\beta_n' z) \\
& + \sum_{n=1}^{\infty} \frac{\bar{C}_n^I}{\Omega_n \gamma_n^3} [- \{ 2\nu_1 \operatorname{ch}(\gamma_n h_1) + \gamma_n h_1 \operatorname{sh}(\gamma_n h_1) \} \operatorname{ch}(\gamma_n z) + \operatorname{ch}(\gamma_n h_1) \gamma_n z \operatorname{sh}(\gamma_n z)] J_0(\gamma_n r) \dots \dots \dots (3) \\
& \Phi_{111} = \Phi_{11} \{ \bar{A}_0^{II}, \bar{B}_0^{II}, \bar{C}_0^{II}, \bar{A}_n^{II}, \bar{B}_n^{II}, \bar{C}_n^{II}, \beta_n(h_2), \Delta_n(h_2), \bar{A}_1^I, \bar{A}_2^I, \dots, \bar{A}_s^I, \Omega_s(h_2) \} \dots \dots \dots (4) \\
& \Phi_{112} = \Phi_{12} \{ \bar{A}_n^{II}, \bar{B}_n^{II}, \bar{C}_n^{II}, \beta_n'(h_2), \Delta_n'(h_2), \bar{A}_1^I, \bar{A}_2^I, \dots, \bar{A}_s^I, \Omega_s'(h_2) \} \dots \dots \dots (5)
\end{aligned}$$

where $\bar{A}_0^I, \bar{C}_0^I, \bar{A}_n^I, \bar{C}_n^I, \dots, \bar{A}_n^{II}, \bar{B}_n^{II}$ and $\bar{C}_n^{II}$ are undetermined coefficients to be determined from the boundary conditions.

Substituting Φ_I, Φ_{II} and Φ_{III} into equations of the stresses and the displacements, and equating the obtained results to the boundary condition of Eq. (1), the relationships among undetermined coefficients are obtained as follows:

$$\left. \begin{aligned}
& \bar{A}_0^I \nu_1 + \bar{C}_0^I (2\nu_1 - 1) = 0, \bar{A}_0^I (1 - \nu_1) + \bar{C}_0^I (2 - \nu_1) = f_0, \bar{A}_0^{II} \nu_2 + \bar{B}_0^{II} / a^2 + \bar{C}_0^{II} (2\nu_2 - 1) = 0 \\
& \bar{A}_0^{II} \nu_2 + \bar{B}_0^{II} / b^2 + \bar{C}_0^{II} (2\nu_2 - 1) = 0, \bar{A}_0^{II} (1 - \nu_2) + 2\bar{C}_0^{II} (2 - \nu_2) = g_0 \\
& \bar{A}_0^{III} \nu_3 + \bar{B}_0^{III} / a^2 + \bar{C}_0^{III} (2\nu_3 - 1) = 0 \\
& \bar{A}_0^{III} \nu_3 + \bar{B}_0^{III} / b^2 + \bar{C}_0^{III} (2\nu_3 - 1) = 0, \bar{A}_0^{III} (1 - \nu_3) + 2\bar{C}_0^{III} (2 - \nu_3) = g_0
\end{aligned} \right\} \dots \dots \dots (6)$$

$$\left. \begin{aligned}
& \bar{A}_n^I + \sum_{s=1}^{\infty} \bar{C}_s^I \bar{Q}_{sn}^{(1)} = 0, \bar{A}_n^I + \sum_{s=1}^{\infty} \bar{C}_s^I \bar{Q}_{sn}^{(1)} = 0, \bar{A}_n^{II} + \sum_{s=1}^{\infty} \bar{C}_s^{II} \bar{P}_{sn}^{(2)} = 0, \bar{A}_n^{II} + \sum_{s=1}^{\infty} \bar{C}_s^{II} \bar{P}_{sn}^{(2)} = 0 \\
& \bar{B}_n^{II} + \sum_{s=1}^{\infty} \bar{C}_s^{II} \bar{Q}_{sn}^{(2)} = 0, \bar{B}_n^{II} + \sum_{s=1}^{\infty} \bar{C}_s^{II} \bar{Q}_{sn}^{(2)} = 0, \bar{A}_n^{III} + \sum_{s=1}^{\infty} \bar{C}_s^{III} \bar{P}_{sn}^{(3)} = 0, \bar{A}_n^{III} + \sum_{s=1}^{\infty} \bar{C}_s^{III} \bar{P}_{sn}^{(3)} = 0 \\
& \bar{B}_n^{III} + \sum_{s=1}^{\infty} \bar{C}_s^{III} \bar{Q}_{sn}^{(3)} = 0, \bar{B}_n^{III} + \sum_{s=1}^{\infty} \bar{C}_s^{III} \bar{Q}_{sn}^{(3)} = 0, \bar{C}_s^I + \bar{C}_s^I + \sum_{n=1}^{\infty} (\bar{A}_n^I \bar{R}_{ns}^{(1)} + \bar{A}_n^I \bar{R}_{ns}^{(1)}) = f_s \\
& \bar{C}_s^{II} - \bar{C}_s^{II} + \sum_{n=1}^{\infty} (\bar{A}_n^{II} \bar{R}_{ns}^{(2)} + \bar{B}_n^{II} \bar{S}_{ns}^{(2)} - \bar{A}_n^{II} \bar{R}_{ns}^{(2)} - \bar{B}_n^{II} \bar{S}_{ns}^{(2)}) = g_s \\
& \bar{C}_s^I - \bar{C}_s^I + \sum_{n=1}^{\infty} (\bar{A}_n^I \bar{R}_{ns}^{(1)} - \bar{A}_n^I \bar{R}_{ns}^{(1)}) - \sum_{n=1}^{\infty} \bar{b}_m (\bar{C}_s^I + \bar{C}_s^I + \sum_{n=1}^{\infty} (\bar{A}_n^I \bar{R}_{ns}^{(2)} + \bar{B}_n^{II} \bar{S}_{ns}^{(2)} + \bar{A}_n^I \bar{R}_{ns}^{(2)} + \bar{B}_n^{II} \bar{S}_{ns}^{(2)})) = 0 \\
& \bar{C}_s^{II} + \bar{C}_s^{II} + \sum_{n=1}^{\infty} (\bar{A}_n^{II} \bar{R}_{ns}^{(2)} + \bar{B}_n^{II} \bar{S}_{ns}^{(2)} - \bar{A}_n^{II} \bar{R}_{ns}^{(2)} - \bar{B}_n^{II} \bar{S}_{ns}^{(2)}) - \bar{C}_s^{III} - \bar{C}_s^{III} \\
& - \sum_{n=1}^{\infty} (\bar{A}_n^{III} \bar{R}_{ns}^{(3)} + \bar{B}_n^{III} \bar{S}_{ns}^{(3)} + \bar{A}_n^{III} \bar{R}_{ns}^{(3)} + \bar{B}_n^{III} \bar{S}_{ns}^{(3)}) = 0 \\
& \bar{b}_m (\bar{C}_s^I \bar{T}_s^{(1)} + \bar{C}_s^I \bar{T}_s^{(1)}) + \bar{C}_s^I \bar{T}_s^{(2)} + \bar{C}_s^I \bar{T}_s^{(2)} = 0, \bar{C}_s^{II} \bar{T}_s^{(2)} + \bar{C}_s^{II} \bar{T}_s^{(2)} + \bar{C}_s^{III} \bar{T}_s^{(3)} + \bar{C}_s^{III} \bar{T}_s^{(3)} = 0
\end{aligned} \right\} \dots \dots \dots (7)$$

where,

$$\begin{aligned}
& \bar{Q}_{sn}^{(1)} = \frac{4(-1)^n \gamma_s h_1 (n\pi)^2 \operatorname{sh}^2(\gamma_s h_1) J_0(\gamma_s b)}{\bar{Q}_s \{ (\gamma_s h_1)^2 + (n\pi)^2 \}^2}, \bar{Q}_{sn}^{(1)} = \frac{(-1)^{n-1} \gamma_s h_1 (2n-1)^2 \pi^2 \operatorname{ch}^2(\gamma_s h_1) J_0(\gamma_s a)}{\bar{Q}_s \{ (\gamma_s h_1)^2 + \{ (2n-1)\pi/2 \}^2 \}^2} \\
& \bar{R}_{ns}^{(1)} = \frac{4(-1)^{n+1} \beta_n b \lambda_s^2 I_1^2(\beta_n b)}{\Delta_n \{ (\beta_n b)^2 + \lambda_s^2 \}^2 J_0(\gamma_s b)}, \bar{R}_{ns}^{(1)} = \frac{4(-1)^n \beta_n b \lambda_s^2 I_1(\beta_n b)}{\Delta_n \{ (\beta_n b)^2 + \lambda_s^2 \}^2 J_0(\gamma_s b)} \\
& \bar{P}_{sn}^{(2)} = \bar{Q}_{sn}^{(1)} \frac{C_0(\bar{\gamma}_s a)}{J_0(\bar{\gamma}_s b)}, \bar{P}_{sn}^{(2)} = \bar{Q}_{sn}^{(1)} \frac{C_0(\bar{\gamma}_s a)}{J_0(\bar{\gamma}_s b)}, \bar{Q}_{sn}^{(2)} = \bar{P}_{sn}^{(2)} \frac{C_0(\bar{\gamma}_s b)}{C_0(\bar{\gamma}_s a)}, \bar{Q}_{sn}^{(2)} = \bar{P}_{sn}^{(2)} \frac{C_0(\bar{\gamma}_s b)}{C_0(\bar{\gamma}_s a)} \\
& \bar{R}_{ns}^{(2)} = \frac{2(-1)^n}{\Delta_n \{ b^2 C_0^2(\bar{\gamma}_s b) - a^2 C_0^2(\bar{\gamma}_s a) \}} \frac{\bar{S}_s}{\beta_n^2 + \bar{\gamma}_s^2}, \bar{R}_{ns}^{(2)} = \frac{2(-1)^{n+1}}{\Delta_n \{ b^2 C_0^2(\bar{\gamma}_s b) - a^2 C_0^2(\bar{\gamma}_s a) \}} \frac{\bar{S}_s}{\beta_n^2 + \bar{\gamma}_s^2} \\
& \bar{S}_{ns}^{(2)} = \bar{R}_{ns}^{(2)} \frac{\bar{S}_s}{\bar{S}_s}, \bar{S}_{ns}^{(2)} = \bar{R}_{ns}^{(2)} \frac{\bar{S}_s}{\bar{S}_s}, \bar{P}_{sn}^{(3)} = \bar{P}_{sn}^{(2)}(h_2), \bar{P}_{sn}^{(3)} = \bar{P}_{sn}^{(2)}(h_2), \bar{Q}_{sn}^{(3)} = \bar{P}_{sn}^{(3)} \frac{C_0(\gamma_s b)}{C_0(\gamma_s a)} \\
& \bar{Q}_{sn}^{(3)} = \bar{P}_{sn}^{(3)} \frac{C_0(\bar{\gamma}_s b)}{C_0(\bar{\gamma}_s a)}, \bar{T}_s^{(1)} = \frac{2(1-\nu_1^2)}{E_1} \frac{\operatorname{sh}^2(\gamma_s h_1)}{\gamma_s \bar{Q}_s}, \bar{T}_s^{(1)} = -\bar{T}_s^{(1)} \frac{\operatorname{ch}^2(\gamma_s h_1)}{\operatorname{sh}^2(\gamma_s h_1)} \\
& \bar{T}_s^{(2)} = \bar{T}_s^{(1)}(h_2), \bar{T}_s^{(2)} = \bar{T}_s^{(1)}(h_2), \bar{T}_s^{(3)} = \bar{T}_s^{(1)}(h_2), \bar{T}_s^{(3)} = \bar{T}_s^{(1)}(h_2) \\
& \bar{b}_m = -\frac{2\gamma_s a_0 C_0(\bar{\gamma}_s a) J_1(\gamma_m a)}{b^2 J^2(\gamma_m b) (\gamma_m^2 - \bar{\gamma}_s^2)}, \bar{b}_m = \frac{2\gamma_s \{ a C_1(\bar{\gamma}_m a) J_0(\gamma_s a) - a C_0(\bar{\gamma}_m a) J_1(\gamma_s a) \}}{(\gamma_s^2 - \bar{\gamma}_m^2) \{ b^2 C_0^2(\bar{\gamma}_m b) - a^2 C_0^2(\bar{\gamma}_m a) \}}
\end{aligned}$$

Accordingly, solving Eqs. (6) and (7), the undetermined coefficients are determined and by using those coefficients the stresses and the displacements are obtained. Where symbols used in Φ_I are the same as those denoted in the reference (11), and symbols in $\Phi_{II}, \Phi_{III}, S_A, S_A, S_B$ and S_B in the reference (9). In the above mentioned equations, upper suffixes I, II and III correspond to unknown coefficients of the cylinder [I], the hollow cylinders [II] and [III], and upper suffixes (1), (2) and (3) correspond to known coefficients of the same cylinders.

2.2 Analysis of force ratio

As shown in Fig. 4(a) when an internal pressure P is applied to a pressure vessel of which a bolted joint is fastened with the initial clamping force F_f as shown in Fig. 1, an increment F_t of the bolt axial force is produced in bolts and a compressive force F_c is eliminated from the contact surfaces. The relationships among F_t, F_c and the internal pressure P for a bolt, that is the force ratio, is obtained. In such range that the contact surfaces are completely in contact, defining the force ratio Φ_g per bolt by the equation

$F_t/(W/N)$ and the axial load W caused by the pressure by $P\pi a^2$, Φ_g is given as follows⁽⁹⁾ (11):

$$\Phi_g = \frac{F_t}{(W/N)} = \frac{K_t}{K_t + K_{cg}} \left(\frac{K_{cg}}{K_{tg}} \right) \quad \dots\dots\dots (8)$$

where K_t is the spring constant⁽¹⁵⁾ of a bolt-nut system. K_{cg} is the compressive constant⁽⁹⁾ of clamped parts which is obtained from the equation $K_{cg} = F_f/(e_0 + e'_0)$ analyzing the case shown in Fig. 3. K_{cg} is the compressive one which is obtained from the equation $K_{cg} = F_f/(e_1 + e'_1)$ in the model shown in Fig. 2 using the mean displacement $e_1 + e'_1$ on the bearing surface. K_{tg} is the tensile spring one which is obtained from the equation $K_{tg} = (W/N)/(e_2 + e'_2)$ using the mean displacement $e_2 + e'_2$ on the bearing surface as shown in Fig. 4(b).

In the analysis of K_{tg} , assuming that the load W ($W = P\pi a^2$) acts on the hub of the base as a uniform tensile stress P_2 in the axial direction, the boundary conditions are expressed as follows:

For the cylinder [I] (cover),

$$r=b: \sigma_r = \tau_{rz} = 0$$

$$z=h_1: \sigma_z = 0$$

$$z=-h_1: \sigma_z = f'_0 + \sum_{s=1}^{\infty} f'_s J_0(\gamma_s r), \tau_{rz} = 0$$

$$0 \leq r \leq a$$

For the hollow cylinder [II] (gasket),

$$r=a: \sigma_r = P, \tau_{rz} = 0$$

$$r=b: \sigma_r = \tau_{rz} = 0$$

For the hollow cylinder [III] (base),

$$r=a: \sigma_r = P, \tau_{rz} = 0$$

$$r=b: \sigma_r = \tau_{rz} = 0$$

$$z=-h_3: \sigma_z = g'_0 + \sum_{s=1}^{\infty} g'_s C_0(\bar{\gamma}_s r), \tau_{rz} = 0$$

..... (9)

where,

$$f'_0 = \frac{a^2}{b^2} P_2, f'_s = \frac{2aJ_1(\gamma_s a)}{\gamma_s b^2 J_0^2(\gamma_s b)} P_2$$

$$g'_0 = \frac{e^2 - a^2}{b^2 - a^2} P_2$$

$$g'_s = \frac{2eC_1(\bar{\gamma}_s e)}{\bar{\gamma}_s \{b^2 C_0^2(\bar{\gamma}_s b) - a^2 C_0^2(\bar{\gamma}_s a)\}} P_2$$

The boundary conditions on the contact surfaces are omitted because they are the same as those shown in Eq. (1). The analysis of the case of Fig. 4(b) is the same as mentioned in Section 2.1. When the value of Φ_g is obtained from Eq. (8), the compressive force F_c , which is eliminated from the contact surfaces, is obtained by the equation $F_c = (1 - \Phi_g) W$.

2.2.1 Estimation of internal pressure when separation of the contact surfaces begins

With an increase of the internal pressure P , the separation of the contact surfaces begins from the inner circumference. By analyzing both cases of Figs. 2 and 4(b) and by superposing the results at the contact surfaces, the internal pressure P is determined such that the stress σ_z at $r=a$ becomes zero when the separation of the contact surfaces begins.

2.2.2 Analysis in the case where contact surfaces are separated

With further increase of the internal pressure P , the separation of the contact surface proceeds. It is supposed that the contact surfaces of the cover and

(a) model of a joint between cover and base of pressure vessel

(b) model for analysis of the tensile spring constant K_{tg}

Fig. 4 A joint between cover and base when pressure P is applied

(a) model of a joint between cover and base of pressure vessel

(b) model for analysis of the tensile spring constant K_{tg}

Fig. 5 A joint between cover and base of pressure vessel in the case where contact surfaces are separated

the base are separated from the inner circumference to $r = R_1$ and $r = R_2$, respectively as shown in Fig. 5(a). For simplicity, we put $R_1 = R_2 = R$ assuming that R_1 nearly equals to R_2 . Thus, the case of Fig. 5(b), in which the inner diameter of the hollow cylinder is R , is analyzed in the similar way to the cases of Fig. 2 and Fig. 4(b). At the same time, it is supposed that the value of K_{cg}/K_{tg} in Eq. (8) changes with the proceeding of the separation. In the analysis, a model, in which the pressure is applied to the separate portion of the interfaces, is approximately assumed as shown in Fig. 5(b). Namely, K_{tg} is obtained from the equation $K_{tg} = (W/N)/(\epsilon_3 + \epsilon'_3)$ ($W = P\pi R^2$) in Fig. 5(b). K_{cg} is obtained in the similar way to the case of K_{tg} . Using the spring constants mentioned above, the force ratio is obtained when the contact surfaces are separated. When the contact surfaces are separated under the position $r = (R_c)_{\max}$ in Fig. 5(a), namely under the bearing surface, it is assumed that the leakage of fluid is caused. The leakage is influenced by the other factors such as the characteristics of internal fluid and the surface roughness on gasket and so on, but in this investigation the effect of stress at the contact surfaces on the leakage is discussed.

2.3 Analysis of a bending moment produced in bolts

With the application of the internal pressure P a bending moment M_b is produced in bolts besides an increment F_t of bolt axial force, because the bearing surface is inclined. In order to evaluate the bending moment M_b , the bending compliances γ_p , γ_c of clamped parts and γ_b of bolts are defined by the equations $\gamma_p = (W/N)/\theta_p$, $\gamma_c = M_b/\theta_c$ and $\gamma_b = M_b/\theta_b$ respectively as shown in Fig. 6 as demonstrated in the previous paper⁽¹²⁾. Using the bending compliances mentioned above, the following equation is obtained from the condition that the inclination of the bearing surface of the bolts equals to that of the clamped parts when the internal pressure P is applied.^{(11) (12)}

$$\psi_g = \frac{M_b}{W/N} = \frac{\gamma_b}{\gamma_b + \gamma_c} \left(\frac{\gamma'_c}{\gamma_p} \right) \quad (10)$$

Accordingly, using Eqs. (8) and (10) the maximum increment $\sigma_{t\max}$ of stress produced in the bolts is expressed as follows:

$$\sigma_{t\max} = \left(\frac{\Phi_g}{A} + \frac{\psi_g}{Z} \right) \left(\frac{W}{N} \right) \quad (11)$$

where A is a cross sectional area of bolt axis and Z is section modulus. In the case where contact surfaces are separated the analysis is done in the same way.

3. Experiments

Figure 7 shows dimensions of covers and bases of the pressure vessel used in experiments in order to obtain the force ratio. The material of pressure vessel is steel (S45C, JIS) and three types of flanges of which the pitch circle diameters $2d$ are 135, 150,

Fig. 6 Analysis of bending moment caused in bolts

165 mm are prepared as shown in Fig. 7. The size of bolts⁽¹⁰⁾ is M16 and the material is SCM3 (JIS). Two strain gauges (Kyowa Electronic Instrument Co., Ltd., KFC-2-C1-11) are attached to the shank of the bolts in 180° phase to each other. The thickness $2h_2$ of gaskets is taken as 5, 10, 20 mm and the material is mild steel (SS34, JIS) and aluminum (A2-H, JIS). The maximum surface roughness $R_{\max}$ of gaskets is below $5\mu\text{m}$ for mild steel and below $7\mu\text{m}$ for aluminum. After the cover with a gasket is fastened by eight sets of bolts and nuts with the initial clamping force $F_f = 20\text{ kN}$ to the base of pressure vessel, the internal pressure is applied to the assembly by using an oil pressure pump (Riken Precision Machine Co., Ltd., MP-5C) as shown in Fig. 8. Then, the magnitude of the internal pressure is measured with a pressure transducer (Kyowa Electronic Instrument Co., Ltd., PG-500KU) which is attached to the cover, and the increment of bolt axial force and the maximum stress are picked up by the strain gauges. The output signals are recorded by an X-Y recorder (Yokogawa Electric Works Co., Ltd., 3036) through dynamic amplifiers (Kyowa Electronic Instruments Co., Ltd., DPM-110B). The fluid used in experiments is spindle oil (#150) which is used at the room temperature.

4. Results of analyses and comparison with experimental ones

4.1 Results of analyses

The numerical computations are done using the

Fig. 7 Cover and base of pressure vessel used in experiments

Fig. 8 Sketch of experimental apparatus

same dimensions of pressure vessel and gaskets as used in the experiments shown in Fig. 7 in order to compare numerical results with experimental ones. The contact stress on $z = h_2$ between the cover and the gasket is obtained when the value of $2h_1$ equals to that of $2h_3$, and the contact stress ($z = h_2$) between the cover and the gasket fairly coincide with those ($z = -h_2$) between the base and the gasket. In the computations, the number of terms of series is 20. It is assumed that Poisson's ratio of covers and bases of pressure vessels and gaskets is 0.3. Young's modulus E_2 of gaskets is 206 GPa for mild steel, 68.7 GPa for aluminium, and that of covers and bases of pressure vessels is $E_1 = E_3 = 206$ GPa. The number N of bolts is 8.

Figure 9 shows the effects of the ratio E_1/E_2 of Young's modulus of covers to that of gaskets on the contact stress caused by the initial clamping force. It is seen that the contact stress becomes large on the

(case of $d/a = 2.0$, $h_1/a = 0.44$, $h_2/a = 0.29$, $c/a = 0.35$, $h_1 = h_3$, $E_1 = E_3$)

Fig. 9 Effects of ratio of Young's modulus of cover of pressure vessel to that of gaskets on distribution of contact stresses

(case of $d/a = 2.0$, $h_1/a = 0.44$, $E_1/E_2 = 3$, $c/a = 0.35$, $h_1 = h_3$, $E_1 = E_3$)

Fig. 10 Effects of thickness of gaskets on distribution of contact stresses

inner circumference of contact surface with an increase of the ratio E_1/E_2 . Figure 10 shows the effects of the thickness h_2/a of gaskets on the contact stress when the ratio of E_1/E_2 is held constant. With an increase of the thickness h_2/a of gaskets, it is seen that the distribution of contact stresses becomes uniform. In the case of $h_1/a = 0$, that is in the case where a gasket is not used, the stress is low on the inner circumference ($r/a = 1.0$), and the contact surfaces are separated on the outer circumference ($r/a = 3.0$) when the joint is fastened with the initial clamping force. Figure 11 shows effects of the pitch circle diameter d/a . The smaller the pitch circle diameter d/a is, the larger the contact stress becomes on the inner circumference ($r/a = 1.0$). Figure 12

(case of $h_1/a = 0.44$, $h_2/a = 0.29$, $E_1/E_3 = 3$, $c/a = 0.35$, $h_1 = h_3$, $E_1 = E_3$)

Fig. 11 Effects of pitch circle diameter on distribution of contact stresses

(case of $h_1/a = 0.44$, $E_1/E_2 = 3$, $e/a = 1.44$, $h_1 = h_3$, $E_1 = E_3$)

Fig. 12 Analytical results for the case of Fig. 4 (distribution of contact stresses)

shows the analytical results of the contact stress distribution for the case of Fig. 4. The contact stress distribution and the internal pressure when a separation of the contact surfaces is caused are estimated by superposing the results of Fig. 10 upon those of Fig. 12 when the internal pressure is applied to the connection.

4.2 Comparisons with respect to the force ratio and the internal pressure when a leakage of fluid is caused

Figure 13 shows comparisons of numerical results with experimental ones with respect to the force ratio. The abscissa is the internal pressure P and the ordinate is the bolt axial force $F_f + F_t$. The initial clamping force F_f is 20kN. With an application of the internal pressure P the increment F_t of bolt axial force increases linearly, and yields a curve when the separation of contact surfaces begins from the inner circumference. Furthermore, the separation of the contact surfaces proceeds with an increase of the internal pressure P and then, a leakage of internal fluid is caused from bolt holes. The leakage of fluid is confirmed with our eyes and in fact, an increment of the internal pressure is not produced after the leakage of fluid is caused. The experimental value of the pressure when the leakage is caused is indicated with a turning point between the loading and the unloading of the pressure in Fig. 13. The numerical values of the pressure when the leakage is caused are a little smaller than the experimental ones. It is said that the numerical results are in a fairly good agreement with experimental ones. The force ratio is obtained using straight lines which indicate the relationship [Eq. (8)] between the internal pressure P and the increment F_t of bolt axial force. The numerical values of the force ratio are 0.058 in Fig. 13(a)

(a) case of $2h_2 = 10\text{mm}$, $2d = 150\text{mm}$ and gasket is aluminium ($E_1/E_2 = 3$, $E_1 = E_3$)

(b) case of $2h_2 = 10\text{mm}$, $2d = 150\text{mm}$ and gasket is mild steel ($E_1/E_2 = 1$, $E_1 = E_3$)

Fig. 13 Comparisons of numerical results with experimental ones with respect to force ratio

and 0.036 in (b). Numerical results are in a fairly good agreement with experimental ones and it seems that the value of the pressure when the leakage is caused is fairly well estimated by numerical analyses. The internal pressure when the leakage is caused is 10.5MPa for the case of Fig. 13(a) and 9.5MPa for the case of (b). It is noticed that the internal pressure when the leakage is caused becomes large with an increase of the ratio E_1/E_2 . It is also seen that the increment F_t of bolt axial force makes a rapid increase when the separation of contact surfaces begins. This fact is undesirable for the fatigue strength of bolts. The numerical value of the force ratio in the case where a gasket is not used and the pitch circle diameter $2d$ of a joint is 150mm is 0.032, while the experimental value is 0.02.

Table 1 shows comparisons of the limited internal pressure obtained by the experiments when the leakage is caused. The comparisons among the cases No. 1 to 3 show the effects of the pitch circle diameter $2d$. It is seen that the internal pressure when the leakage is caused becomes large and the sealing performance becomes better with a decrease of the value $2d$. Also the comparison among the cases No. 2, 4 and 5 show the effect of the thickness $2h_2$ of the gaskets, and the comparison between the cases No. 2 and 6 shows the effect of the material of the gaskets. It is shown that the sealing performance becomes better with an increase of the value $2h_2$ and the value E_1/E_2 . The case of No. 7 shows the result in the case where a gasket is not used. Comparing the case No. 7 with the case No. 1 the internal pressure when the leakage is caused becomes small and the sealing performance becomes worse. From the comparisons mentioned above, it is considered desirable to make the stiffness of gaskets small, that is, make an increase of the value E_1/E_2 , the thickness of gaskets large and the pitch circle diameter $2d$ small in

Table 1 The limited internal pressure by experimental results in the case where the leakage of fluid is caused

No.	$2d(\text{mm})$	$2h_2(\text{mm})$	Material	Pressure of leakage (MPa)
1	135	10	Al	12.7
2	150	10	Al	11.8
3	165	10	Al	7.2
4	150	5	Al	9.7
5	150	20	Al	12.0
6	150	10	Fe	11.2
7	135	—	—	11.6

[case of $2h_2 = 10\text{mm}$, $2d = 135\text{mm}$ and gasket is aluminium ($E_1/E_2 = 3$, $E_1 = E_3$)]

Fig. 14 Comparison in the case where bending moment is taken into consideration

order to improve the sealing performance. These facts match the results (Figs. 9 ~ 11) described in Section 4.1. Namely, it is seen that the stress on the inner circumference ($r/a = 1.0$) becomes large in the case where the ratio E_1/E_2 is large (Fig. 9), the thickness of the gasket is large (Fig. 10) and the pitch circle diameter $2d$ is small (Fig. 11). It is noticed that these cases above mentioned are effective for prevention of the leakage.

Figure 14 shows a comparison of the maximum stress taking account of the bending moment produced in bolts. The numerical result is in a fairly good agreement with the experimental one.

5. Conclusions

In the present paper, discussion is made on the distribution of contact stresses, the force ratio and the sealing performance in the case of a connection between a cover and a base of pressure vessel in which a full-face metallic flat gasket is interposed in order to obtain the characteristics of bolted joints with gaskets. The following results are obtained.

(1) Replacing the cover, the gasket and the base with a finite cylinder and two finite hollow cylinders respectively, a method of analysis of the contact stress distribution is demonstrated by using three-dimensional theory of elasticity.

(2) A method of analysis is demonstrated with respect to the force ratio when an internal pressure is applied to an assembly and a method of analysis of spring constants is also demonstrated. Furthermore, a method to obtain an increment of bolt axial force is shown when the contact surfaces are separated.

(3) A method of analysis to obtain a bending moment caused in bolts is shown with an application of an internal pressure and a calculation method of the maximum stress taking account of the bending moment is also shown.

(4) Based on the method mentioned (1), the effects of the thickness, the stiffness of a gasket and the pitch circle diameter on the contact stress are made clear by numerical computations. It is shown that the sealing performance becomes better with an increase of the ratio of Young's modulus and the thickness of a gasket and with a decrease of the pitch circle diameter. But it is shown that conversely the force ratio becomes large.

(5) For verification of the results obtained from (2) and (3), experiments are performed. It is shown that numerical results are in a fairly good agreement with experimental ones.

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