Unsteady Flows in the Vortex Region Behind a Circular
Cylinder Started Impulsively *
(2nd Report, Velocity Fields and Circulations)

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The velocity field of the flow around a circular cylinder started impulsively from rest was studied by the aluminium dust method. The experiments were carried out at $R_e=1,200$ and when $\tau=U_0 t/d$ was from 0 to 5, where R_e is the Reynolds number based on the diameter d of the cylinder, U_0 the cylinder speed, and t the time after the start of the cylinder.

Circulations taken along several closed contours around a vortex-center, and the positions of vortex-center were measured during the motion of the cylinder. On the basis of these measured values, the velocity fields in the vortex region were calculated by using a simple model which approximates the vortex flow by a single viscous point-vortex. The analytical results were in a fairly good agreement with the experimental results.

Key Words : Unsteady Flow, Separated Flow, Vortex, Circulation, Vortex Filament, Vorticity, Flow Visualization, Flow Measurement

1. Introduction

In the previous paper⁽¹⁾, when a circular cylinder was started impulsively with a constant speed from rest in still water, the mechanism of the vorticity transfer from the surface of the cylinder to the vortex region was made clear assuming that the process of vortex formation can be interpreted as a process of vorticity transfer, in which the vorticity produced on the surface of the cylinder is conveyed through the separation points and located behind the cylinder. In the present study, the measurement of the circulations taken around the vortex region was performed with the aim of studying the overall structure of the vortex flow.

Since it is extremely difficult to measure the velocity field in an unsteady vortex flow, very few experimental values of the circulation taken around an unsteady vortex region have been reported so far. The velocity field of a starting flow around the circular cylinder was measured by Schwabe⁽²⁾, and Contanceau et al.⁽³⁾. Schwabe determined the velocity field around the cylinder by tracing the motions of Al-dust floating on the surface of the water to estimate the pressure distribution around the cylinder but he did not measure the values of circulation around the vortex region. Contanceau et al. made a careful study of the geometrical shape and velocity field of a vortex formed behind the

cylinder, especially the velocity on the center-line of the wake, but did not refer to the circulation, either. Later, Imaichi et al.⁽⁴⁾ developed a numerical image-processing method for the pattern visualized by Al-dust method and by this method they examined the flow field around the cylinder started impulsively in the case of Reynolds number, $R_e \approx 200$, in which Navier-Stokes equations can be solved numerically with sufficient accuracy. Since the main subject of their study was the development of an image-processing method with the aid of a micro-computer, the relation between the process of vortex formation and its circulation has not been discussed. Sarpkaya⁽⁵⁾ estimated the magnitude of circulation around the cylinder started with a constant acceleration by measuring the traces of Al-dust, but the resolution of the lens of a cine-camera he used to take the photographs of Al-dust seems to be poor to study the detailed structure of the vortex region.

In the present study, the velocity field of the vortex region and the magnitude of circulation for various integral circuits around the vortex region were obtained, and variation of the circulation with time was discussed in relation to the mechanism of vorticity transfer. Furthermore, in order to understand the overall structure of the vortex region, the vortex flow was represented approximately by a simple model in which the measured value of the circulation was used, and the calculated results were compared with the flow field obtained by the experiment. Recently the flow around a bluff body with separation vortices is often studied by a discrete vortex method⁽⁶⁾⁻⁽⁸⁾. This method is useful for determining the flow at high Reynolds numbers, for which the analytical solution

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of the Navier-Stokes equations cannot be obtained and their numerical solution needs a long calculation time. The discrete vortex method seems to be effective for throwing light on the mechanism of vorticity transfer. However, the relation between the circulation and the vorticity transfer remains unknown in many respects. The experimental results in this study may be expected to provide reliable information to make clear the relationship.

2. Experimental Apparatus and Method

2.1. Apparatus

The experiment was carried out by using the same apparatus as mentioned in the previous paper⁽¹⁾. The flow around a circular cylinder started impulsively with constant speed U_0 in still water was visualized by Al-dust method. The experimental conditions were $Re = U_0 d / \nu = 1200$ (mainly), $\tau = U_0 t / d = 0 \sim 5$, where d is the diameter of the cylinder, ν the kinematic viscosity, and t the elapsed time after the start of

the cylinder.

2.2. Measurement of vorticity field and circulation

The flow behind the cylinder was visualized by Al-dust method and photographed with a short exposure time (below 1/4s.) during which the flow was considered to be approximately steady. The traces of Al-dust photographed on the film were enlarged and printed on RC paper about six times as large as the real ones. Figure 1 shows consecutive pictures of the flow. Many white, short traces in these photographs indicate the pathlines of Al-dust. From these pathlines the shape of streamline and velocity on the streamlines were determined as in the previous paper⁽¹⁾. The streamlines are drawn as smooth curves which are estimated by fitting these pathlines. The velocity was determined from the relation between the length of tracers of Al-dust and exposure time. The exposure time is determined from the time marks on the traces of Al-dust which are illuminated by a mercury vapor lamp (0.8kW) at the rate of 120 cps.

Fig.1 Development of a flow behind the cylinder at $Re=1200$ (Al-dust method)
 $\tau = 1.1(a), 1.3(b), 1.9(c), 2.4(d), 2.9(e), 3.1(f)$

3. Experimental Results and Discussion

3.1. Motion of center of vortex

Figure 1 shows the development of a flow behind a circular cylinder visualized by Al-dust method. As time passes, the vortex region grows larger and a downstream translation of the center of vortex can be observed. The position of the vortex center and the stream-wise length of the vortex region were measured to show the growth of a vortex. In Fig. 2, the position $(x_c/d, y_c/d)$ and the length of the vortex region, L/d (cf. Fig. 3 or 11), are plotted against the time τ , where the position of the center (x_c, y_c) is defined as the position of zero relative velocity to the cylinder, which may be called an apparent vortex center. The true vortex center should be a point around which a mass of fluid is circulating in the co-ordinates moving with the same velocity as the mass of fluid. Since the change in the values of y_c/d is very small as seen in the figure, the time variation of the values of x_c/d may be supposed to show approximately the velocity of the motion of vortex. The values of y_c/d were corrected to the values $(y_c/d)_c$ which indicate the vortex center observed in the co-ordinates

Fig.2 Location of the vortex-center and the length of vortex region

Fig.3 Movement of the vortex-center

moving with this velocity. The values of $(y_c/d)_c$ are represented by a broken line in the figure. As seen in the figure, the change of the values of $(y_c/d)_c$ is as small as that of y_c/d . Figure 3 shows the locations of the vortex center at consecutive instants shown by figures. In the figure, black circles, $\bullet$, indicate the locations (x_c, y_c) of the center of the vortex at $Re=1200$. The location (x_c, y_c) approaches the Föepfle Line⁽⁹⁾ corresponding to the non-dimensional time $\tau=2.2$, and thereafter the locations of the vortex-center are situated near this line. At $Re=800$, the location of the center of vortex is far from the Föepfle Line. Moreover, the center of vortex at $Re=2000$ comes nearer to this line than at $Re=1200$. Thus, it is suggested that the flow in the vortex region at Reynolds numbers higher than 1000 can be approximated by an inviscid flow model of a pair of point vortices.

3.2. Measurement of circulation

a) The variation of the circulation with integral circuits

The magnitude of circulation $\Gamma = \oint q ds$ (q : velocity on the line element ds on the circuit) was measured along several closed circuits of integration around the vortex center which are shown in Fig. 4. The integral circuit is composed of streamlines and their normals so as to have an accurate value of the circulation. The

Fig.4 Schematic diagram of integral circuits to calculate the magnitude of circulations

Fig.5 Variation of magnitude of circulation with the integral circuits

curve 0-n running from the vortex center toward the outer region of the vortex in the figure is drawn normal to the streamlines. Integration is started at a point on the curve, 0-n. The distance, n , measured along the curve 0-n from the vortex center to the starting point is used to indicate individual integration circuits. The values of circulations along several special integral circuits denoted by the marks (a), (b), (b_r), (c), and (d), in Fig. 4 are represented by the symbols, Γ , Γ_D , Γ_{Dm} , Γ_m , and Γ_m^* . The symbols, S and St, represent the separation points and the stagnation points respectively.

The non-dimensional values of the circulation $\Gamma^* = \Gamma/U_0 d$ at the instant, $\tau = 2.9$, are plotted against the non-dimensional distance n/d in Fig. 5. In the figure are shown the distributions of the dye generated from the surface of the cylinder and hydrogen bubbles generated from the whisker-like wire (the wire extends to the outer edge of the boundary layer near the separation point as shown in Fig. 7 in the previous paper⁽¹⁾). The distributions of dye and hydrogen bubbles closely correspond to the distribution of vorticity on the normal 0-n to the streamlines as described in the previous paper⁽¹⁾. The magnitude of circulation, however, does not change drastically for different integral circuits and is not affected directly by the local concentration of vorticities corresponding to the dye or the hydrogen bubbles, because its magnitude is the result of integration of vorticity. As shown by a broken line in the figure, the values of Γ^* are approximately expressed by the following empirical equation which has the same form as the circulation around a viscous vortex-filament

$$\Gamma^* = \Gamma_m^* (1 - e^{-k(n/d)^2}) \quad (1)$$

where Γ_m^* and k are empirical constants. The constant Γ_m^* is determined by the value of circulation for the integral circuit (c) which lies outside of the dividing streamline. The procedure to determine the value of k will be described later. The value of Γ_m^* for the integral circuit (d) which encloses the main and the secondary vortex regions is larger than that of Γ_m^* . When

Fig. 6 Variation of the magnitudes of circulations with time

the measured values of circulation, Γ^* , are examined in detail, the value of Γ_D^* for the circuit (b) is the minimum, because the integral circuit (b) consists of the dividing streamline and the surface of the cylinder, and inside the circuit a vorticity opposite in sense to one in the main vortex region is produced on the surface of the cylinder. The circulation for the integral circuit (b_r), which is slightly inside the circuit (b), has the maximum value denoted by Γ_{Dm}^* , because the circuit does not enclose the boundary layer behind the cylinder.

b) The variation of circulation with time

In Fig. 6 the non-dimensional circulations Γ/Γ_m for various integral circuits are plotted against the non-dimensional distance $n/n_{1/2}$ for the non-dimensional time τ , where Γ_m is the circulation on the above integral circuit (c), and $n_{1/2}$ is the distance from the vortex center to the position where the value of Γ is half the value of Γ_m , that is, the half distance of Γ_m . Variation in circulation for various integration circuits is almost the same at any non-dimensional time τ as that at the time $\tau = 2.9$ in Fig. 5. The circulation for the integration circuit (b) has the minimum values Γ_D shown by arrows in the figure. As a whole, the value of circulation can be approximated by a modified expression of the empirical equation (1) as follows

$$\Gamma/\Gamma_m = 1 - e^{-0.69(n/n_{1/2})^2} \quad (2)$$

On closer examination, the difference between the experimental values of circulation and the approximate values represented by a broken line for eq. (2) are found to become a little larger at the time passes. This fact suggests that the approximation of the vorticity distribution

Fig. 7 Variation with time of the magnitudes of circulations and the amount of vorticity fed to the vortex region

by the distribution with a single peak becomes poorer as time goes on, because the distance between two peaks in the vorticity distribution becomes larger with time as shown in the previous paper⁽¹⁾ and eq.(2) of circulation is obtained from the vorticity distribution with a single peak.

c) The relation between circulation and vorticity flux

In Fig. 7, the non-dimensional magnitudes of circulations Γ_m^* , Γ_m^* , Γ_D^* , and $\Gamma_{S_2}^*$ for various circuits are plotted against the non-dimensional time τ , where Γ_m^* , Γ_m^* , and Γ_D^* denote the magnitudes of the circulations on the integral circuits indicated by (d), (c), and (b) in Fig. 4, respectively, and $\Gamma_{S_2}^*$ is the presumed magnitude of the circulation around the secondary vortex obtained approximately by

$$\Gamma_{S_2} \approx 2 \int_{S_2}^{S_{24}} q ds, \Gamma_{S_2}^* = \Gamma_{S_2} / U_0 d \dots \dots \dots (3)$$

In other words, the values of Γ_{S_2} are two times as large as the value of the line integral of the velocity q on the streamline from the separation point S_2 to the stagnation point S_{24} , as shown in Fig. 4. In Fig. 7, I_1 and I_2 represent the amounts of vorticity fed from the angular positions $\theta_1=90^\circ$ and $\theta_2=150^\circ$ measured from the front stagnation point, which are nearly the positions of the two separation points S_1 and S_2 . The values of I_1 and I_2 were obtained by time integral of the vorticity fluxes K_1 and K_2 , that is,

$$I_n = \int_0^\tau K_n d\tau \quad (n=1, 2),$$

where K_1 and K_2 are defined by

$$K = (U_\theta / U_0)^2 / 2, \quad K_1 = (K)_{\theta=90^\circ}, \quad K_2 = (K)_{\theta=150^\circ},$$

All of Γ_m^* , Γ_m^* , and Γ_D^* increase as the time elapses. Then, we refer to the relation between the mechanism of vorticity transfer and the circulation Γ_m^* for the integral circuit (d) which encloses the whole vortex region. Since the circulation integrated along a closed curve is equal to the surface integral of vorticity enclosed by the curve in general, the magnitude of Γ_m^* is considered to be surface integral of the vorticity transferred from the outer into the inner region of the integral circuits and produced on the surface of the cylinder.

At the time $\tau=0$ when the cylinder starts, since the flow around the cylinder is irrotational except on the surface of the cylinder where no-slip condition of the flow should be satisfied, the magnitude of Γ_m^* is equal to 1.0 theoretically. The measured values of Γ_m^* are plotted on the solid line connecting with 1.0 on the ordinate at $\tau=0$ as shown in Fig. 7. This means that measured values are precise and reliable. After the start of the cylinder, firstly a reversed flow occurs at the rear stagnation point of the cylinder, then the flow develops subsequently as follows; the point of the onset of reversed flow moves upstream and the boundary layer separates from the cylinder, and the separation vortices are formed behind the cylinder. Immediately after the start of the

cylinder, the fluid particles with vorticity are concentrated only in the region near the surface of the cylinder and the interaction of the flow with the surface wall is so complex that the details of the vorticity production have not been made clear. After this stage, that is, $\tau > 1.3$ when the main symmetrical vortices and the secondary vortices are formed clearly, the wall interaction is considered to become smaller and the amount of the vorticity fed from the rear surface of the cylinder can be approximated by the value of vorticity flux K_2 which is the vorticity flux past the angular position $\theta_2=150^\circ$ (near the separation point S_2 in Fig. 4). And also, the rate of feeding of vorticity from the outer into the inner region of the vortex can be approximated by the value of vorticity flux K_1 (opposite in sense to K_2) which represents the vorticity flux through the angular position $\theta_1=90^\circ$. From this point of view, the time $\tau=1.3$ is taken as the lower limit of the integral and the values of I_1' and I_2' are calculated as follows

$$I_1' = \int_{1.3}^\tau K_1 d\tau = I_1 - \alpha \dots \dots \dots (4)$$

$$I_2' = \int_{1.3}^\tau K_2 d\tau = I_2 - \beta \dots \dots \dots (5)$$

As shown in Fig. 7, the values of $I_1' - I_2'$ are a little larger than those of $I_1 - I_2$ and slightly smaller than Γ_m^* . When the time $\tau=2.0$ is taken as the lower limit of the integral, the values of $I_1' - I_2'$ are represented by a dash-and-double dotted line and these values are in good agreement with the values of Γ_m^* . The agreement suggests that in the case of $\tau > 2$, the rate of feeding the vorticity to the vortex region behind the cylinder can be approximated by the values of K_1 and K_2 and

Fig. 8 Time variation of the ratios of the magnitudes of circulations Γ_m^* , Γ_m^* , Γ_D^* , and Γ_{Dm} to the amount of vorticity fed to the vortex region, I_1 , and to the magnitude Γ_m^* of circulation

the wall interaction is negligibly small.

In the previous paper⁽¹⁾, it was shown that the rate of feeding the vorticity to the vortex region is variable with time corresponding to the behavior of the secondary vortex and a shedding of the secondary vortex causes a local concentration of the vorticities in the separated shear layer for $Re > 10^3$. As seen in Fig. 7, however, the magnitudes of Γ_m^* , Γ_m^* , and Γ_b^* do not change drastically with time, that is, the formation and the shedding of the secondary vortex do not directly affect the values of Γ_m^* , Γ_m^* , and Γ_b^* . This means that the vorticity distribution is, so to speak, averaged by the integral procedure to obtain the circulation, and as the result, the local change of vorticity does not appear explicitly in the values of circulation. The presumed values of Γ_{s2}^* (the circulation around a secondary vortex) are extremely smaller than the magnitude Γ_m^* (the circulation around the whole region of the main vortex). Thus, the behavior of the secondary vortex does not affect significantly the rotating velocity of the fluid in the whole vortex region, though it relates closely to the mechanism of vorticity transfer as well as the mechanism of the occurrence of turbulence.

Figure 8 shows the ratios of the various circulations Γ_m^* , Γ_m^* , and Γ_b^* to the integral value I_1 of the vorticity flux K , and the ratios of Γ_m^* , Γ_b^* , and Γ_{s2}^* to Γ_m^* against elapsed time. In order to apply the discrete vortex method, which is useful for studying the flow around a bluff body, it is necessary to know details of vorticity transfer. However, many problems to be solved about the vorticity transfer remain unknown. One of the

problems is the ratio of the vorticity produced on the surface of the body to the vorticity which is kept in the wake region. The discrete vortex method predicts that more than 80% of the original vorticity fed from the surface of the body remains in the wake region, while the experimental results give a percentage of about 50 to 60. The results in Fig. 8 are considered to give a fundamental information for these problems about the vorticity transfer.

3.3 Approximate theory to express the flow

The simplest model to approximate the vortex flow behind the cylinder may be the potential flow model with a pair of vortex points behind the cylinder. The potential flow model, however, does not represent the flow near the center of the vortex, because it has a singularity at the center of vortex where the velocity is infinite, though in the region far away from the vortex center it represents satisfactorily a real flow. Therefore, in this study, we intend to represent approximately the flow around the cylinder by a viscous vortex-filament which has the zero velocity at the center of vortex and induces the same flow as the potential flow in the region far from the vortex center. The velocity induced by a viscous vortex-filament with the circulation Γ_0 is expressed by

$$u_\theta = \Gamma_0 / 2\pi n (1 - e^{-k'n^2}) \quad (6)$$

where n denotes the distance from the center of vortex and u_θ the peripheral velocity on the circle concentric with the vortex-center⁽¹⁰⁾. The circulation Γ around the vortex-filament is expressed by

$$\Gamma / \Gamma_0 = 1 - e^{-k'n^2/d^2}, \quad k = k'd^2$$

This equation coincides with the experimental equation (1). When the half distance of circulation is denoted by $n_{1/2}$, we have

$$k = \ln 2 / (n_{1/2}/d)^2 \quad (7)$$

The relation between the constant k and the vorticity ω_0 at the vortex center ($n=0$) is obtained from eq. (6) with substitution of $k = k'd^2$, and we have

Fig. 9 Schematic diagram of the viscous vortex-filament model

Fig. 10 Time variation of the characteristic values, Γ_m^* , k , $n_{1/2}$ of the vortex region

Fig. 11 Variation of the length and width of the vortex region with time

$$k = \omega_0^* \pi / \Gamma_0^* \quad \dots\dots\dots (8)$$

where ω_0^* denotes $\omega_0 d / U_0$ and Γ_0^* denotes $\Gamma_0 / U_0 d$.

As shown in Fig. 9, when a pair of viscous vortex-filaments are put behind the cylinder and their mirror images are taken into account so as to have no normal velocity on the surface of the cylinder, the velocity field (u, v) around the cylinder can be determined by the following expressions

$$\begin{aligned} u = & 1 - \frac{1}{r_0^2} \cos 2\theta_0 \\ & + K \sum_{i=1}^2 \frac{(-1)^{i+1}}{r_i} (1 - e^{-k r_i^2}) \sin \theta_i \\ & + K \sum_{i=3}^4 \frac{(-1)^i}{r_i} (1 - e^{-k r_{ic}^2}) \sin \theta_i \quad \dots\dots\dots (9) \end{aligned}$$

$$\begin{aligned} v = & \frac{1}{r_0^2} \sin 2\theta_0 + K \sum_{i=1}^2 \frac{(-1)^i}{r_i} (1 - e^{-k r_i^2}) \cos \theta_i \\ & + K \sum_{i=3}^4 \frac{(-1)^i}{r_i} (1 - e^{-k r_{ic}^2}) \cos \theta_i \quad \dots\dots\dots (10) \end{aligned}$$

where K is defined by $K = \Gamma_0 / 2\pi$. The suffixes 1 and 2 indicate the vortices behind the cylinder and the suffixes 3 and 4 the mirror images of the above vortices (cf. Fig. 9). The nomenclature in the above equations is as follows. r_0 and θ_0 : the radial distance and argument measured from the origin; r_i and θ_i ($i=1,2,3,4$): the radial distance and argument measured from the complex location z_i ($i=1,2,3,4$) of each vortex-filament; and r_{ic} : the radial distance measured from origin to the location z_i of each vortex-filament. This velocity field is identical with that of the potential flow with an inviscid vortex-filament when the value of k tends to infinity. When the locations (z_1 and z_2) and the strength Γ_0 of the vortex-filaments, and the constant k are given, we can calculate the velocity field from the above equations. Then, we calculated the velocity field around the cylinder by substituting the measured values $[x_c, (y_c)_c]$ of the main vortex center (cf. Fig. 2) into the locations of vortex center, and substituting the measured values of Γ_m^* (cf. Fig. 4) into the

strength of a vortex-filament. The values of k can be determined by eq.(7) or (8). The values of ω_0 in eq.(8) are determined as double the value of velocity gradient at the center of vortex. Figure 10 shows the values of k obtained by the above procedure. In this figure, the values of Γ_m and $n_{1/2}$ are also plotted as non-dimensional values Γ_m^* and $n_{1/2}^*$. The constants k obtained from eq.(7) are plotted as black circles "●" and those obtained from eq.(8) are marked with "Δ". These values are plotted immediately near a single dotted line as seen in Fig. 10.

In the following section the velocity field calculated by the above procedure will be discussed.

3.4. Geometrical configuration of vortex

The length of the vortex was calculated using the viscous vortex-filament model mentioned above. In Fig. 11, the calculated results are plotted against time together with the experimental values of the length L and width b of the vortex. The values of L are independent of the value of k and variable with time in the same way as the experimental values but they are a little smaller than the experimental values. This means that the magnitude of circulation is overestimated by the vortex-filament model in which the measured length of vortex is used as an empirical constant.

In the figure, the lengths of vortices obtained in the numerical solution at $Re=1000$ by Ta Phuoc Loc⁽¹¹⁾ are also plotted. These lengths are slightly shorter than those obtained in this experiment. This discrepancy may be due to the effect of the side walls, by which the growth of the vortex is increased in the longitudinal direction. In this experiment the ratio d/h (h : distance between the side walls) is not zero but has a finite value of 0.14, though the numerical solution was obtained for $d/h=0$.

Fig.12 Velocity distribution on the symmetric axis of the wake

Fig.13 Variation of the maximum velocity of the reversed flow with time

3.5. Velocity distribution on the symmetrical axis

Figure 12 shows the velocity distribution on the symmetrical axis for $\tau = 2.9$. The non-dimensional velocity u/U_0 is taken as the ordinate and as the abscissa taken is the non-dimensional distance, $x/d - 0.5$, measured from the end point of the diameter on the x-axis (symmetrical axis). In Fig. 12, the values calculated by the above model are indicated by the broken line for $k=22.1$ and by the dash and dotted line for $k \rightarrow \infty$. The calculated values for $k=22.1$ are closer to the experimental values than those for $k \rightarrow \infty$, but the discrepancy between the calculated values and experimental values increases with the downstream distance from the vortex-center in both cases.

The maximum velocity of the reversed flow on the symmetrical axis was measured for the elapsed time. In Fig. 13, the non-dimensional, absolute value of the maximum velocity, $|(u/U_0)_{\min}|$, is plotted as ordinate against the time τ in comparison with the values calculated by the above model. The measured length of the vortex L/d is also plotted in the figure to show the growth of vortex. Maximum velocity of the reversed flow calculated by using the relation $k = \ln 2 / n_{1/2}^*$ is in good agreement with experimental value, but the calculated value for infinite value of k increases with time over the experimental value.

3.6. Velocity distribution at the section passing the vortex-center

Figure 14 shows the distribution of the velocities q on the line in y -direction through the vortex-center for several instants. The ordinate shows the non-dimensional distance n/d (n denotes $y - y_c$) from the vortex-center in y -direction and the abscissa shows non-dimensional velocity q/U_0 . In the figure, the broken lines indicate the values calculated by the above viscous vortex-filament model. The calculated values are in good agreement with the experimental values except in a small part inside the dividing streamline

indicated by arrows (D.S.L.) in the figure. It suggests that the velocity distribution at this section can be calculated with good accuracy by the viscous vortex-filament model.

3.7 Growth of the vortex region

a) Stream function

The rate of flow across a line connecting the vortex-center with a point on the dividing stream line was measured and variation with time of its value is shown as a value of the stream function in Fig. 15. The stream function is nondimensionalized by referring to U_0 and d and is denoted by ψ_0 . The black square "■" in the figure indicates the value of ψ_0 calculated by the above model. These values are in good agreement with the experimental values. As seen in the figure, the values of ψ_0 increase almost linearly with time. This means that the rate of rotating flow induced by the vortex increases at a constant rate.

b) Area of the vortex region and average vorticity

The area of the vortex region and the average vorticity over the vortex region were measured and plotted in Fig. 16. The average vorticity $\bar{\omega}$ is defined as the ratio of the magnitude Γ_0 of the circulation to the area A_1 enclosed by the integral circuit used in the calculation of the circulation, that is, $\bar{\omega} = \Gamma_0 / A_1$. In the figure, the area A_1 of the large vortex region (the area enclosed by the closed curve (b) in Fig. 4) and the area A_2 of the secondary vortex region (shown in Fig. 16) are plotted in non-dimensional form against time. The area A_1 increases linearly with time and the average vorticity $\bar{\omega}^*$ remains nearly constant, that is, $\bar{\omega}^* \approx 4$. These facts mean that the average rotating velocity in the vortex region does not change significantly with an increase in the area of the vortex region.

The average vorticity in the secondary vortex region ranges between about 0.7 and 1.3, when the value of average vorticity is

Fig.14 Velocity distribution at the section passing the vortex-center in y -direction

Fig.15 Variation of the stream function ψ_0 with time

estimated by taking the ratio of the magnitude Γ_2^* to the area $A_2^*/2$. In other words, the average rotating velocity in the secondary vortex region is about 1/4 of that in the main vortex region.

4. Conclusions

The magnitude of circulation of the vortices formed behind a circular cylinder started impulsively was measured. The main experimental results are summarized as follows.

(1) The magnitudes of circulation along various integral circuits around the vortices were determined in relation to the elapsed time.

(2) The magnitudes Γ_b , Γ_m , Γ_m^* of the circulation increase with time in the period of τ between about 1 and 4, and do not change drastically corresponding to the shedding of the secondary vortex when τ is about 2.4.

(3) The whole flow field around the cylinder containing the vortex region can be approximated by a simple viscous vortex-filament model using the measured values of the circulation and the location of vortex-center as the empirical constants.

(4) The relation between the amount of vorticity fed to the vortex region behind the cylinder and that of vorticity remaining in the vortex region was made clear by comparing the magnitude of the circulation with the integral values of vorticity fluxes through the sections near two separation points.

Fig.16 Time variation of the area of the vortex region and the average vorticity in the vortex region

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