Pressure Distribution Around an Airfoil Placed in
a Periodically Fluctuating Air Flow*

(2nd report, A thin airfoil approximation)

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This paper theoretically and experimentally investigates pressure distributions around an airfoil placed in a periodically fluctuating airflow. The calculated results agree fairly well with experimental results, because of the airfoil thickness being taken into account under the assumption of a thin airfoil. The following facts are, furthermore, found out: (1) The time averaged pressure distributions consist of quasi-steady components, independent of the reduced frequency, and nearly equal to those for steady flow conditions in the range of small fluctuating amplitudes. (2) The amplitude of the first harmonic component of the unsteady pressures is in proportion to that of the velocity fluctuation and comes under the influence of the virtual mass with the growth of the reduced frequency. (3) The phase of pressure fluctuation leads to that of velocity wave and is independent of fluctuating amplitudes. The phase difference becomes larger in the accelerating region and smaller in the decelerating region in the chordwise direction.

Key Words : Unsteady Flow, Pressure Distribution, Thin Airfoil,
Theoretical Analysis

1. Introduction

There are a few reports on the characteristics of airfoils placed in a periodically velocity-fluctuating flow. For instance, the fluctuating lift was worked out in references (1)-(5) and the fluctuating pressure distributions on airfoils in references (4), (6)-(8). There are, however, few reports on the uniformly fluctuating velocity field, especially fewer reports in which experimental results of fluctuating pressure distributions were compared with analytical results.

In our previous report⁽⁹⁾, pressure distributions on an airfoil placed in a periodically fluctuating flow were described, and experimental results were compared with analytical results obtained assuming flat plate airfoil. It was made

clear that the analytical results gave a good prediction of the pressure amplitudes, but did not give quantitatively so good prediction of the phase difference of pressure and fluid velocity.

In the present paper, so-called thin airfoil approximation is used, the boundary conditions of the airfoil surfaces are applied on the chord, the terms of the perturbation velocity higher than the first order are neglected and a theoretical analysis with consideration of the thickness distribution is worked out. Theoretical results of the time averaged pressure distributions and the phase difference of pressures and velocity agree fairly well with experimental results. And the influence of fluctuating velocity amplitude and frequency on the pressure distribution is made clear.

Nomenclature

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- b : half chord length = $c/2$
- c : chord length
- C_p : pressure coefficient
- k : reduced frequency = $\omega b/U_\infty$
- p : pressure
- t : time
- U : velocity of main flow = $U_\infty(1 + \delta \cos \omega t)$
- u, v : velocities in x, y directions, respectively
- x, y : Cartesian coordinates
- Y_c : thickness distribution of airfoil
- Y_c : camber distribution of airfoil
- α : attack angle

- δ : velocity amplitude ratio of fluctuating air flow
 θ : phase lag of pressure fluctuation to velocity
 ρ : density of air
 ω : angular frequency
 Π : pressure difference (defined by Eq.(7))
 K_c : pressure summation (defined by Eq.(8))
 Subscripts
 + : upper surface of airfoil ($y=+0$)
 - : lower surface of airfoil ($y=-0$)

2. Theoretical Analysis Assuming a Thin Airfoil Approximation

Two dimensional irrotational inviscid, incompressible fluid flow is assumed in order to calculate the flow field around an airfoil placed in a spatially uniform flow at sinusoidally varying velocity. As in the former report the mid-chord is taken as the origin of coordinates, the chord of the blade as x -axis and the perpendicular direction to the chord as y -axis. In Fig.1, the point C_0 is placed on the wind tunnel wall at distance l from the origin ($x=l \sin \alpha$, $y=-l \cos \alpha$), the point C_1 denotes the infinite upstream point ($x=-\infty$, $y=0$) and C_2 the infinite upstream point corresponding to the point C_0 .

Fig.1 Coordinate system

It is assumed that the vortices in the wake are carried along the x -axis at the main flow velocity. The continuity equation, momentum equations and the

irrotational condition are the same as those of the former report. By assuming a thin airfoil and applying the boundary conditions of airfoil surfaces on the x -axis, the perturbation velocities on both upper and lower surfaces of the airfoil, $u'_{\pm} = u'(x, y=\pm 0, t)$, $v'_{\pm} = v'(x, y=\pm 0, t)$, become

$$u'_{\pm} = \pm \frac{1}{2} \gamma_a(x, t) - \frac{U \cos \alpha}{\pi} \oint_{-1}^1 \frac{Y'_i(\xi)}{\xi - x} d\xi \quad (1)$$

$$v'_{\pm} = -U \sin \alpha + U \cos \alpha \{Y'_c(x) \pm Y'_i(x)\} \quad (2)$$

(see Appendix A). By using the time averaged velocity U_{∞} , the fluctuating velocity ratio δ and the angular frequency ω , the velocity of uniform main flow U is expressed as

$$U = U_{\infty}(1 + \delta \cos \omega t) \quad (3)$$

and the vorticity γ_a on the blade surface is given below as the sum of the quasi-steady vorticity γ_0 and the vorticity γ_{sw} induced by the wake vortices,

$$\gamma_a(x, t) = \gamma_0(x, t) + \gamma_{sw}(x, t) \quad (4)$$

Y'_i and Y'_c respectively, denote the derivatives of the airfoil thickness distribution $Y_i(x)$ and the camber distribution $Y_c(x)$ with respect to x , and the integral symbol of Eq.(1) indicates the principal value integral.

By using the thin airfoil approximation, the induced velocity in the upstream of the blade is expressed as

$$u'(x, y=0, t) = \frac{U \cos \alpha}{\pi} \int_{-1}^1 \frac{Y'_i(\xi)}{x - \xi} d\xi \quad (5)$$

Pressure distributions on both upper and lower surfaces are derived from the momentum equations and the irrotational condition. In the paper, because of the terms being taken up to the first order of the induced velocity in the boundary conditions, the integral of the pressure is summed up to the first order. l is taken to be sufficiently large and the perturbation velocity is assumed to be zero at C_0 . Thus the pressure distributions are derived as

$$p_+ - p_{c_0} = -\rho \frac{\partial}{\partial t} \int_{-\infty}^{-1} u'(x, y=0, t) dx - \rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx - \rho \frac{\partial}{\partial t} \int_{-1}^x u'_- dx - \rho U \cos \alpha u'_+ \quad (6)$$

where p_{c_0} is the pressure at C_0 , $p(x=l \sin \alpha, y=-l \cos \alpha, t)$ (see Appendix B).

Using Eq.(6), the difference and the sum of the pressures on the upper and lower surfaces are given as

$$\Pi = p_+ - p_- = \rho \frac{\partial}{\partial t} \int_{-1}^x (u'_- - u'_+) dx + \rho U \cos \alpha (u'_- - u'_+) \quad (7)$$

$$K_c = (p_- - p_{c_0}) + (p_+ - p_{c_0}) = -2\rho \frac{\partial}{\partial t} \int_{-\infty}^{-1} u'(x, y=0, t) dx - 2\rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx - \rho \frac{\partial}{\partial t} \int_{-1}^x (u'_+ + u'_-) dx - \rho U \cos \alpha (u'_+ + u'_-) \quad (8)$$

Substituting Eqs.(1) and (5) into Eqs.(7) and (8), the following relations are derived.

$$\Pi = \rho \frac{\partial}{\partial t} \int_{-1}^x \gamma_a(x, t) dx + \rho U \cos \alpha \gamma_a(x, t) \quad (9)$$

$$K_{c_0} = -\frac{2\rho}{\pi} \frac{\partial}{\partial t} \int_{-1}^1 U \cos \alpha \int_{-1}^1 \frac{Y'_i(\xi)}{x-\xi} d\xi dx - 2\rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx + \frac{2\rho}{\pi} \frac{\partial}{\partial t} \int_{-1}^x U \cos \alpha \int_{-1}^1 \frac{Y'_i(\xi)}{\xi-x} d\xi dx + \frac{2\rho}{\pi} U^2 \cos^2 \alpha \int_{-1}^1 \frac{Y'_i(\xi)}{\xi-x} d\xi \quad (10)$$

The airfoil used in the experiments is symmetrical so that $Y_c(x)=0$, and then the quasi-steady vorticity γ_0 is given as

$$\gamma_0(x, t) = 2U \sin \alpha \sqrt{\frac{1-x}{1+x}} \quad (11)$$

γ_{0w} induced by wake vortices has the same relation as that of the former report,

$$\gamma_{0w}(x, t) = \frac{1}{\pi} \sqrt{\frac{1-x}{1+x}} \int_1^\infty \frac{\gamma_w(\xi, t)}{\xi-x} \sqrt{\frac{\xi+1}{\xi-1}} d\xi \quad (12)$$

where γ_w denotes the vorticity in the wake ($1 \leq x$) and it is given, taking account of the terms up to δ^2 , as

$$\gamma_w(x, t) = \frac{1}{2} \delta^2 \Gamma_s \operatorname{Re} \left\{ \frac{e^{-ikx}}{K_0(ik) + K_1(ik)} \right\} - \delta \Gamma_s \operatorname{Re} \left\{ \frac{e^{-ikx} e^{i\omega t}}{K_0(ik) + K_1(ik)} \right\} - \frac{1}{2} \delta^2 \Gamma_s \operatorname{Re} \left\{ \left[\frac{e^{-ikx}}{K_0(ik) + K_1(ik)} - \frac{e^{-i2kx}}{K_0(i2k) + K_1(i2k)} \right] e^{i2\omega t} \right\} \quad (13)$$

In Eq. (13), $\Gamma_s = 2\pi U_\infty \sin \alpha$, $k = \omega b / U_\infty$, K_0 and K_1 are modified Bessel functions of the second kind of the 0-th and 1st orders respectively, $\operatorname{Re}\{\}$ indicates the real part of a complex number, and i denotes an imaginary unit.

The relations $\cos \alpha \cong 1$, $\sin \alpha \cong \alpha$ are hereafter assumed because of a small attack angle α . Substituting Eqs. (11)–(13) into Eqs. (9) and (10), the following relations are obtained by neglecting the terms higher than δ^2 ,

$$\Pi = G \sqrt{1-x^2} \delta \operatorname{Re} \{ i k e^{i\omega t} \} + G \sqrt{\frac{1-x}{1+x}} \left[\delta \operatorname{Re} \{ [c(k)-1] e^{i\omega t} \} + \frac{\delta^2}{2} \operatorname{Re} \{ [2c(k) - c(2k)-1] e^{i2\omega t} \} \right] + G \sqrt{\frac{1-x}{1+x}} \left[1 + \frac{\delta^2}{2} + 2\delta \operatorname{Re} \{ e^{i\omega t} \} + \frac{\delta^2}{2} \operatorname{Re} \{ e^{i2\omega t} \} \right] \quad (14)$$

$$K_{c_0} = -\frac{G}{\alpha} x \delta \operatorname{Re} \{ i k e^{i\omega t} \} - \frac{G}{\alpha} \delta \operatorname{Re} \{ i k e^{i\omega t} \} \left[\frac{Q}{\pi} - \frac{P(x)}{\pi} \right] + \frac{G}{\alpha} \left[1 + \frac{\delta^2}{2} + 2\delta \operatorname{Re} \{ e^{i\omega t} \} + \frac{\delta^2}{2} \operatorname{Re} \{ e^{i2\omega t} \} \right] \frac{F(x)}{\pi} \quad (15)$$

where,

$$C(k) = K_1(ik) / [K_0(ik) + K_1(ik)] \\ G = \rho U_\infty \Gamma_s / \pi$$

$$C_{p_1} = (p_1 - p_a) / \left(\frac{1}{2} \rho U_\infty^2 \right) \quad (19)$$

and the terms associated with the airfoil thickness are

$$\left. \begin{aligned} Q &= \int_{-1}^1 \int_{-1}^1 \frac{Y'_i(\xi)}{x-\xi} d\xi dx \\ F(x) &= \int_{-1}^1 \frac{Y'_i(\xi)}{\xi-x} d\xi \\ P(x) &= \int_{-1}^x \int_{-1}^1 \frac{Y'_i(\xi)}{\xi-x} d\xi dx \end{aligned} \right\} \quad (16)$$

The integrals of Eq. (16) are calculated by expanding the airfoil thickness into a Fourier series (see Appendix C).

The pressure is considered as a difference from the atmospheric pressure p_a , in the same manner as in the previous report, hence the relation is obtained as

$$K_a = (p_- - p_a) + (p_+ - p_a) \\ = K_{c_0} + 2\rho l' \frac{\partial U}{\partial t} \quad (17)$$

where l' is the distance from C_0 to the tunnel exit. Also the pressure on the blade surface is calculated from Eqs. (14)–(17), that is,

$$p_- - p_a = (K_a \mp \Pi) / 2 \quad (18)$$

Then the pressure coefficient C_p is given by the following relation,

3. Comparison of Experimental with Calculated Results

As described in the former report, the experimental pressure waves are approximated by the time averaged values and the fundamental waves, and we follow the same procedure as that used in the report. In calculation the airfoil is set at $-1 \leq x \leq 1$, but at $0 \leq x/c \leq 1$ for comparison of experiment and prediction. Considering the end effect, l' is taken equal to 258 mm in Eq. (17).

Table 1 Test conditions

Symbol	⊖	+	△	×	◇	⬆
k	0.5	0.4	0.3	0.2	0.2	0.2
δ	0.1	0.1	0.2	0.2	0.1	0.1

Table 1 shows round numbers of experimentally obtained reduced frequency k and velocity amplitude ratio δ of fluctuating velocity. Symbols in the table are used in Figs. 3–5.

3.1 Pressure waves

An example of pressure waves on the suction side is shown in Fig.2 under the condition of $\alpha=7^\circ$, $k=0.32$ and $\delta=0.20$. The

Fig.2 Pressure waves

origin of the time is set at the instant when the velocity takes the maximum value. In the figure, the symbols represent the experimental pressure coefficients, where

the higher harmonics more than 20 Hz were cut off by Fourier transform, and solid lines indicate the calculated results. The figure shows the fact that the waves are approximated by the time averaged values and the fundamental waves.

The effects of α , k and δ on the time averaged values and the fundamental component of the pressure distribution are considered as follows.

3.2 Time averaged pressure distribution

Fig.3 shows the time averaged pressure coefficients $\bar{C}_p$. The figure (a) is for $\alpha=0^\circ$, (b) for $\alpha=5^\circ$ and (c) for $\alpha=7^\circ$. The symbols represent the experimental results corresponding to the cases in Table 1. The solid lines in the figure indicate the calculations under the conditions of $\delta=0.1$ and $k=0.1-0.6$. It is clear that calculation results depend on quasi-steady components and not on k in Eqs.(14) and (15), and that the time averaged pressure distributions are equal to those under steady flow conditions by neglecting the term δ^2 . In the case (a) the attack angle is 0.1° in calculation, and then two solid lines appear in the figure. Furthermore the calculation is carried out only in the case of $\delta=0.1$, because there is about 2 % difference in $\bar{C}_p$ within the experimental range of $\delta=0.1-0.2$. It is reasonable to compare the calculation for $\delta=0.1$ with the test results.

In the former report the calculation of pressure distribution at $\alpha=0^\circ$ and on the pressure side at $\alpha=0^\circ$ differ largely from the experiments, but the calculations taking into account the airfoil thickness distribution agree well with the experiments on the whole. Especially the calculations well represent the transition from the accelerating region to the decelerating region.

From the above it is clear that the

Fig.3 Time averaged pressure distribution

Fig.4 Distribution of pressure amplitudes

Fig.5 Distribution of phase differences

time averaged pressure distributions are nearly equal to the distributions under the steady flow conditions and they are not affected by the reduced frequency k in the region of relatively small δ ($\delta < 0.3$).

3.3 Amplitude distribution of pressure fluctuation

Figure 4 shows the amplitude distributions of fundamental pressure coefficient $|\Delta C_p|$ divided by δ . The figure (a) is for $\alpha=0^\circ$, (b) for the pressure side at $\alpha=7^\circ$ and (c) for the suction side at $\alpha=7^\circ$. The solid lines indicate the calculated results under the same conditions as those of Fig.3 and the symbols represent the experimental results. The calculation of figure (a) is for the suction side at $\alpha=0.1^\circ$.

From Eqs.(14) and (15) it is clear

that the amplitude ratio δ has no effect on $|\Delta C_p|/\delta$ and that the effect of α is fairly small except in the region near the leading edge on the suction side.

Figure 4 shows good agreement between the calculated and experimental results except in the region near the leading edge on the suction side. In the leading edge region, the velocity becomes high and the degree of approximation is poor. However, the calculation shows the tendency that the pressure amplitude on the suction side rapidly grows toward the leading edge. Also Fig.4 shows that the amplitude increases over the whole region with the growth of k . This is due to the fact that the first term in Eq.(14) and the first and the second terms in Eq.(15) become larger than the other terms in those

equations.

From the above it is clear that the amplitude of the fundamental component of the fluctuating pressure is proportional to the velocity fluctuating ratio δ and that the change of the amplitude on the suction side from the leading edge to the trailing edge is larger than that on the pressure side.

3.4 Phase difference of pressure wave and velocity fluctuation

Figure 5 shows the phase difference ($-180^\circ \leq \theta \leq 180^\circ$) defined in the same manner as in the former report. The test and calculation conditions are the same as those of Figs. 1 and 4. All the values of θ are negative, which implies that the pressure fluctuation leads to a velocity wave. In the former report the flat plate airfoil approximation cannot predict the results about the phase difference at $\alpha=0^\circ$ and on the pressure side, but the present calculation agrees well with the experimental results.

On the suction side, as k becomes larger, the lead of phase becomes smaller except in the region near the leading edge. And it seems that the phase differences on the suction side show a tendency to produce uniform distribution of $\theta=-90^\circ$ with an increasing k . As α becomes larger, in the calculation, the lead of phase on the suction side becomes larger, and this agrees well with the experimental results.

On the pressure side, in the region near the leading edge, the lead of phase becomes larger with an increasing k , but in the mid region, a contrary tendency is observed. However, as k becomes larger, the phase difference shows a tendency to produce uniform distribution of $\theta=-90^\circ$. As α becomes larger, the lead of phase becomes smaller in the whole region, and it agrees well with the experimental results too.

The tendency to produce uniform distribution of $\theta=-90^\circ$ with an increasing k seems to come under the dominant influence of the virtual mass. In calculation δ has little effect on θ , and then it is reasonable to compare the calculation for $\delta=0.1$ with the test results.

4. Conclusions

The pressure distributions around an airfoil placed in a uniform air flow with a sinusoidally fluctuating velocity were calculated by the thin airfoil approximation method and compared with experimental results. According to the results, it is clear that the time averaged pressure distribution, the amplitude of the fluctuating pressure and the phase difference are predicted with a good accuracy by the present method.

The conclusions are as follows:

(1) The fluctuating pressures are approximated by the time averaged values and the fundamental waves in the range of small reduced frequencies k and small

fluctuating velocity ratios δ .

(2) The time averaged pressure distributions consist of quasi-steady components and they are nearly equal to those for steady flow conditions in the range of small δ .

(3) The amplitude of the first harmonic component of fluctuating pressure is proportional to δ , is little affected by the attack angle α , and it comes under the dominant influence of the virtual mass.

(4) The phase of pressure fluctuation leads to that of the velocity wave and is independent of δ . The phase difference becomes larger in the accelerating region and smaller in the decelerating region along the chord. As α becomes larger, the lead becomes larger on the suction side, and smaller on the pressure side.

Appendix A

When the boundary of the airfoil surface is given by

$$F(x, y, t) = y - Y_2(x) = 0$$

the boundary condition is expressed as

$$\frac{\partial F}{\partial t} + u \frac{\partial F}{\partial x} + v \frac{\partial F}{\partial y} = 0$$

that, is, the boundary condition is described as

$$v(x, y=y_2, t) = u(x, y=y_2, t) \cdot Y_2'(x)$$

where Y_2 and Y_2' indicate the upper and lower surfaces, respectively and Y_2' implies the derivative of Y_2 with respect to x .

By assuming a thin airfoil, the velocity on the surfaces is expressed by

$$\begin{aligned} u(x, y=y_2, t) &= u(x, y=\pm 0, t) \\ &+ \frac{\partial u(x, y=\pm 0, t)}{\partial y} y_2 + \dots \\ &= u(x, y=\pm 0, t) \\ &+ \frac{\partial v(x, y=\pm 0, t)}{\partial x} y_2 + \dots \\ v(x, y=y_2, t) &= v(x, y=\pm 0, t) \\ &+ \frac{\partial v(x, y=\pm 0, t)}{\partial y} y_2 + \dots \\ &= v(x, y=\pm 0, t) \\ &- \frac{\partial u(x, y=\pm 0, t)}{\partial x} y_2 + \dots \end{aligned}$$

Introducing a small number ϵ , except in the region near the leading edge where Y_2' is great, we have the following relation.

$$O(u'/U_\infty) = O(v'/U_\infty) = O(y_2/c) = O(\epsilon)$$

Thus the velocities u, v are approximated as

$$\begin{aligned} u(x, y=y_2, t) &= u(x, y=\pm 0, t) + O(\epsilon^2) \\ v(x, y=y_2, t) &= v(x, y=\pm 0, t) + O(\epsilon^2) \end{aligned}$$

Furthermore, by neglecting the terms higher than $O(\epsilon)$, the velocity components on both upper and lower surfaces ($-1 \leq x \leq 1$),

u_+ and v_+ are approximated as those on the upper and lower sides of the x -axis, that is,

$$\begin{aligned} u(x, y=\pm 0, t) &\cong u(x, y=\pm 0, t) \\ &= U \cos \alpha + u'(x, y=\pm 0, t) \\ v(x, y=\pm 0, t) &\cong v(x, y=\pm 0, t) \\ &= U \sin \alpha + v'(x, y=\pm 0, t) \end{aligned}$$

then the boundary condition becomes

$$\begin{aligned} v(x, y=\pm 0, t) \\ = u(x, y=\pm 0, t) \cdot Y'_c(x) \end{aligned}$$

Therefore, by neglecting the terms higher than $O(\varepsilon)$, the perturbation velocity v' becomes

$$\begin{aligned} v'(x, y=\pm 0, t) \\ = U \cos \alpha Y'_c(x) - U \sin \alpha \end{aligned}$$

By introducing the thickness distribution Y_c and the camber distribution Y_c which have the following relations,

$$\begin{aligned} Y_c(x) &= \{Y_+(x) - Y_-(x)\} / 2 \\ Y_c(x) &= \{Y_+(x) + Y_-(x)\} / 2 \end{aligned}$$

we have

$$\begin{aligned} v'_+ + v'_- &= 2U \cos \alpha Y'_c(x) - 2U \sin \alpha \\ v'_+ - v'_- &= 2U \cos \alpha Y'_c(x) \end{aligned} \quad (20)$$

Finally, we have

$$v'_\pm = -U \sin \alpha + U \cos \alpha \{Y'_c(x) \pm Y'_c(x)\} \quad (21)$$

Representing a thin airfoil by a vortex distribution $\gamma_a(x, t)$ and a source distribution $m(x, t)$ and expressing the wake vortices ($1 \leq x$) by $\gamma_w(x, t)$, the perturbation velocities on the surfaces ($-1 \leq x \leq 1$), u' and v' , are obtained as

$$\begin{aligned} u'(x, y=\pm 0, t) &= \pm(1/2)\gamma_a(x, t) \\ &+ \frac{1}{2\pi} \oint_{-1}^1 \frac{m(\xi, t)}{x-\xi} d\xi \quad (22) \\ v'(x, y=\pm 0, t) &= \pm(1/2)m(x, t) \\ &- \frac{1}{2\pi} \oint_{-1}^1 \frac{\gamma_a(\xi, t)}{x-\xi} d\xi - \frac{1}{2\pi} \int_1^\infty \frac{\gamma_w(\xi, t)}{x-\xi} d\xi \quad (23) \end{aligned}$$

From Eqs. (20) and (23), we have

$$\begin{aligned} v'_+ - v'_- &= m(x, t) \\ &= 2U \cos \alpha Y'_c(x) \end{aligned}$$

Finally by substituting this into Eq. (22), we have

$$\begin{aligned} u'_\pm &= \pm \frac{1}{2} \gamma_a(x, t) \\ &- \frac{U \cos \alpha}{\pi} \oint_{-1}^1 \frac{Y'_c(\xi)}{\xi-x} d\xi \quad (24) \end{aligned}$$

Appendix B

The momentum equations of the two-dimensional flow of incompressible inviscid fluid and the irrotational condition are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (25)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (26)$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad (27)$$

Integrating Eq. (25) with respect to x by using Eq. (27), we have

$$p = -\rho \int_{-\infty}^x \frac{\partial u}{\partial t} dx - \frac{1}{2} \rho [u^2 + v^2]_{-\infty}^x + C(y, t) \quad (28)$$

Also, integrating Eq. (26) with respect to y , we have

$$p = -\rho \int_0^y \frac{\partial v}{\partial t} dy - \frac{1}{2} \rho [u^2 + v^2]_0^y + C'(x, t) \quad (29)$$

From Eq. (28), since $p(x=-\infty, y, t) = C(y, t)$, the pressure p_{c_1} at the point C_1 , and the pressure p_{c_2} at the point C_2 become

$$\begin{aligned} p_{c_1} &= p(x=-\infty, y=0, t) = C(y=0, t) \\ p_{c_2} &= p(x=-\infty, y=-l \cos \alpha, t) \\ &= C(y=-l \cos \alpha, t) \end{aligned}$$

And from Eq. (29) they become

$$\begin{aligned} p_{c_1} &= C'(x=-\infty, t) \\ p_{c_2} &= -\rho \frac{\partial}{\partial t} \int_0^{-l \cos \alpha} v(x=-\infty, y, t) dy \\ &- \frac{1}{2} \rho [u(x=-\infty, y, t)^2 \\ &+ v(x=-\infty, y, t)^2]_0^{-l \cos \alpha} \\ &+ C'(x=-\infty, t) \end{aligned}$$

From Eq. (28) the pressure difference p_+ on the surfaces ($-1 \leq x \leq 1$) becomes

$$\begin{aligned} p_+ &= p(x, y=\pm 0, t) \\ &= -\rho \frac{\partial}{\partial t} \int_{-\infty}^x u(x, y=\pm 0, t) dx \\ &- \frac{1}{2} \rho [u(x, y=\pm 0, t)^2 \\ &+ v(x, y=\pm 0, t)^2]_{-\infty}^x \\ &+ C(y=\pm 0, t) \end{aligned}$$

The pressure p_{c_0} at the point C_0 on the wind tunnel wall becomes

$$\begin{aligned} p_{c_0} &= p(x=l \sin \alpha, y=-l \cos \alpha, t) \\ &= -\rho \frac{\partial}{\partial t} \int_{-\infty}^{l \sin \alpha} u(x, y=-l \cos \alpha, t) dx \\ &- \frac{1}{2} \rho [u(x, y=-l \cos \alpha, t)^2 \\ &+ v(x, y=-l \cos \alpha, t)^2]_{-\infty}^{l \sin \alpha} \\ &+ C(y=-l \cos \alpha, t) \end{aligned}$$

Therefore, rewriting the velocity components on the surface by $u_\pm = u(x, y=\pm 0, t)$ and $v_\pm = v(x, y=\pm 0, t)$, we have

$$\begin{aligned} p_+ - p_{c_0} &= -\rho \frac{\partial}{\partial t} \int_{-\infty}^{l \sin \alpha} u(x, y=0, t) dx \\ &- \rho \frac{\partial}{\partial t} \int_{-1}^1 u_\pm dx \\ &+ \rho \frac{\partial}{\partial t} \int_0^{-l \cos \alpha} v(x=-\infty, y, t) dy \end{aligned}$$

$$\begin{aligned}
& + \rho \frac{\partial}{\partial t} \int_{-\infty}^{l \sin \alpha} u(x, y = -l \cos \alpha, t) dx \\
& - \frac{1}{2} \rho [u_z^2 + v_z^2] \\
& + \frac{1}{2} \rho [u(x = l \sin \alpha, y = -l \cos \alpha, t)^2 \\
& + v(x = l \sin \alpha, y = -l \cos \alpha, t)^2]
\end{aligned}$$

Assuming that l is sufficiently large, we may assume $u(x, y = -l \cos \alpha, t) = U \cos \alpha$ and $v(x, y = -l \cos \alpha, t) = U \sin \alpha$. And we may put $v(x = -\infty, y, t) = U \sin \alpha$. By introducing the perturbation velocity components, u' and v' , we obtain

$$\begin{aligned}
u_z &= U \cos \alpha + u'(x, y = \pm 0, t) \\
&= U \cos \alpha + u'_z \\
v_z &= U \sin \alpha + v'(x, y = \pm 0, t) \\
&= U \sin \alpha + v'_z \\
u(x, y = 0, t) & \\
&= U \cos \alpha + u'(x, y = 0, t)
\end{aligned}$$

Considering the integral relation

$$\int_{-\infty}^{l \sin \alpha} = \int_{-\infty}^{-1} + \int_{-1}^0 + \int_0^{l \sin \alpha}$$

and neglecting the terms higher than $O(\varepsilon)$, we have

$$\begin{aligned}
p_z - p_{c0} &= -\rho \frac{\partial}{\partial t} \int_{-\infty}^{-1} u'(x, y = 0, t) dx \\
&- \rho \frac{\partial}{\partial t} \int_0^x U \cos \alpha dx - \rho \frac{\partial}{\partial t} \int_{-1}^x u'_z dx \\
&- \rho U \cos \alpha u'_z, \dots \dots \dots (30)
\end{aligned}$$

Appendix C

Assume that the thickness distribution $Y_t(\xi)$, where $\xi = \cos \varphi$, may be represented by Fourier cosine series as

$$Y_t(\varphi) = \sum_n A_n \sin n\varphi \dots \dots \dots (31)$$

the coefficients A_n are given by

$$A_n = \frac{2}{\pi} \int_0^\pi Y_t(\varphi) \sin n\varphi d\varphi \dots \dots \dots (32)$$

Table 2 shows the results of A_n up to the 19th term derived by integrating Eq.(32), where the thickness of NACA 65A010 airfoil used in the experiments is indicated by 27 coordinate data, and approximated by 13 quadratic equations.

By considering

$$Y_t(\varphi) = \sum_n A_n \cos n\varphi$$

and putting $x = \cos \theta$, finally we have the

following relations of Q , $F(x)$ and $P(x)$ in Eq.(16).

$$\begin{aligned}
Q &= \int_{-\infty}^{-1} \int_{-1}^1 \frac{Y_t(\xi)}{x - \xi} d\xi dx \\
&= \pi \sum_n (-1)^n A_n \\
F(\theta) &= - \int_0^\pi \frac{\sum_n A_n \cos n\varphi}{\cos \varphi - \cos \theta} d\varphi \\
&= - \frac{\pi}{\sin \theta} \sum_n A_n \sin n\theta \\
P(\theta) &= \int_{-\pi}^\pi F(\theta) (-\sin \theta) d\theta \\
&= -\pi \sum_n A_n (\cos n\theta - \cos n\pi)
\end{aligned}$$

Table 2 Fourier coefficients

n	A_n	n	A_n
1	8.61×10^{-2}	11	-5.37×10^{-5}
2	-1.92×10^{-2}	12	9.95×10^{-6}
3	-1.01×10^{-2}	13	-3.56×10^{-5}
4	-1.15×10^{-3}	14	6.61×10^{-5}
5	6.53×10^{-4}	15	-9.30×10^{-5}
6	-4.39×10^{-4}	16	7.39×10^{-5}
7	-1.37×10^{-4}	17	-6.95×10^{-5}
8	-3.15×10^{-4}	18	5.92×10^{-5}
9	1.47×10^{-4}	19	-4.79×10^{-5}
10	-1.64×10^{-4}		

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