Experimental Study of Turbulence in a Pulsatile Pipe Flow*

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The effects of time-averaged Reynolds number, pulsation frequency and velocity amplitude ratio on the structure of turbulence in a pulsatile turbulent pipe flow were experimentally investigated. A turbulence model including a lag in response-time was presented and its applicability was examined from comparison with experimental results. The radial distribution of time-averaged component of the root mean square value of turbulence agrees well with that of steady pipe flow. The amplitude ratio and phase difference of oscillatory component were defined by choosing the cross-sectional mean velocity as a reference velocity. They are independent of velocity amplitude ratio but affected by pulsation frequency and time-averaged Reynolds number. The present turbulence model is applicable to the whole cross-section of a pipe except for the region very close to the wall.

Key Words: Unsteady Flow, Pulsatile Pipe Flow, Turbulence, Bursting period, Relaminarization

1. Introduction

The effect of unsteadiness on the structure of turbulence has experimentally been investigated mainly with pulsatile turbulent flows in a pipe⁽¹⁾⁻⁽⁴⁾. Mizushima, et al.⁽¹⁾⁽²⁾ used water as the working fluid and measured turbulence intensity, Reynolds shear stress, and mean bursting period using an electro-chemical method. Concerning the turbulence, they calculated the propagation time of a generated turbulence from the vicinity of the wall toward the pipe center line on the basis of the measurement of a cross-correlation coefficient. But the turbulence intensity distribution in the pipe cross-section was not considered. Also the Reynolds shear stress was measured indirectly by substituting the measured values of axial velocity and pressure into a momentum integral equation. This indirect method leads to an increasing error with the pulsation frequency. Ramaprian and Tu⁽³⁾ imposed puff type disturbances on pulsatile laminar oil flows and generated turbulent pulsatile pipe flows with time-averaged Reynolds number of about 2000. They measured the turbulence intensity and Reynolds shear stress, and found the Reynolds shear stress was frozen at its time-averaged value over one cycle when the pulsation frequency approached the mean burst frequency of the corresponding steady turbulent flow. Shemer and Wygnanski⁽⁴⁾ measured the

Reynolds shear stress in pulsatile pipe flows with various combinations of pulsation frequency and velocity amplitude ratio. But the values of time-averaged Reynolds number Re_{ta} were fixed at 4000. They proposed a model considering a lag in response time for Reynolds shear stress, and examined its applicability. They also observed the relaminarization of a flow under a certain experimental condition.

Though the results of Ramaprian and Tu⁽³⁾ and Shemer and Wygnanski⁽⁴⁾ are very attractive, it is not clear whether their results can be applied to a higher Reynolds number range or not. In both experiments the time-averaged Reynolds numbers are very low and fall into the transition region. Two of the present authors measured the Reynolds shear stress in pulsatile air flows with relatively high time-averaged Reynolds number using an X-type hot wire probe⁽⁵⁾. Since the range of pulsation frequencies was very limited, we could not reveal the effect of unsteadiness.

As mentioned above, the effects of three parameters of a pulsatile pipe flow such as time-averaged Reynolds number Re_{ta} , pulsation frequency f and velocity amplitude ratio A_1 have not been examined systematically. As a first step to clarifying the effect of unsteadiness on the structure of turbulence, we focus on the behaviour of turbulence intensity and investigate the effects of three parameters on the turbulence intensity distribution across a circular pipe.

2. Nomenclature

- A_1 : velocity amplitude ratio $\equiv |\bar{u}_{m.os.1}| / \bar{u}_{m.ta}$
 D : pipe diameter
 f : pulsation frequency
 k : coefficient of Eq.(5)
 L : length of test section

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n : exponent
 $\bar{p}$: pressure
 $\Delta \bar{p}/l$: pressure gradient
 R : pipe radius
 Re : instantaneous Reynolds number
 $= \bar{u}_m D / \nu$
 Re_{ta} : time-averaged Reynolds number
 $= \bar{u}_{m,ta} D / \nu$
 r : axial distance
 $r' = r/R$
 T_B : mean bursting period $= 2\pi/\omega_B$
 t : time
 u : axial velocity component $= \bar{u}_{ta} + \bar{u}_{os} + u'$
 $\bar{u}_m$: cross-sectional mean velocity
 u' : fluctuating component due to turbulence motion
 $\bar{u}^*$: friction velocity
 u'_{rms} : root mean square value of turbulence
 $y = R - r$
 $y^+ = y\bar{u}^*/\nu$
 θ : characteristic time
 ν : kinematic viscosity
 ρ : density
 $\bar{\tau}_w$: wall shear stress
 ω : angular frequency of pulsation
 $= 2\pi/T$
 ω' : dimensionless frequency $= R^2\omega/\nu$
 Subscripts and others
 cl : pipe center line
 os, ta : oscillatory and time-averaged components
 $-$: phase averaging
 1 : fundamental component of pulsation
 $|, \angle$: amplitude and phase angle

3. Experimental Apparatus and Procedure

The experimental apparatus is schematically shown in Fig.1. The test section consists of a smooth brass tube of 5 m in length and 49.9 mm in inner diameter. Air is used as a working fluid and a pulsatile flow is generated by a blower and a scotch yoke mechanism. Velocity measurements are made by using a hot wire anemometer at only one axial position 90 diameters downstream of the inlet. All pulsatile flows measured are ones fully developed. Output signals from the hot wire are recorded on a data recorder together with time marks and processed by a large scale computer after A-D conversion.

The axial velocity component u is expressed by

$$u = \bar{u} + u' = \bar{u}_{ta} + \bar{u}_{os} + u' \quad \left. \begin{array}{l} \\ \bar{u}_{os} = \sum_{n=1}^{\infty} |\bar{u}_{os,n}| \cos(n\omega t + \angle \bar{u}_{os,n}) \end{array} \right\} \dots\dots\dots (1)$$

where $\bar{u}$ is the phase averaged value, u' is the fluctuating component due to turbulent motion and the subscripts ta and os denote the time-averaged and oscillatory components respectively.

The phase averaged value $\bar{u}$ is obtained from the data for 200 periods as follows:

$$\bar{u} = \sum_{i=0}^{199} u_i / 200, \quad u_i = u(r, t + iT) \dots\dots\dots (2)$$

Hence the root mean square value u'_{rms} is

$$u'_{rms} = \left[\sum_{i=0}^{199} (u_i - \bar{u})^2 / 200 \right]^{1/2} \dots\dots\dots (3)$$

Both $\bar{u}$ and u'_{rms} are obtained at 12 phases

Fig.1 A schematic of the experimental apparatus

Fig.2 Turbulence intensity distributions in steady flows

in a cycle. The measured values of $\bar{u}$ are approximated by Eq.(1) and those of u'_{rms} by the following finite Fourier series.

$$u'_{rms} = u'_{rms,ta} + u'_{rms,os} \quad \left. \begin{array}{l} \\ u'_{rms,os} = \sum_{n=1}^{\infty} |u'_{rms,os,n}| \cos(n\omega t + \angle u'_{rms,os,n}) \end{array} \right\} \dots\dots\dots (4)$$

In a preliminary measurement we compare the data for 200 periods with those for 1000 periods and notice there is little difference between them.

4. A Turbulence Model Including a Lag in Response Time

The following relation between u'_{rms} and $\bar{u}_m$ holds with a steady pipe flow.

$$u'_{rms} = k \bar{u}_m \dots\dots\dots (5)$$

Strictly speaking, the proportional constant k is a function of the dimensionless axial distance $r' = r/R$ and Reynolds number Re . Under the present Reynolds number range of $10^4 \leq Re \leq 10^5$, u'_{rms} varies according to Re as shown in Fig.2. This variation is relatively small, and we can neglect the effect of Reynolds number dependence on k when we consider a narrower Reynolds number range. From Fig.2, it is clear that the present measuring method is good because the present data agree well with the previous ones.

Since u'_{rms} pulsates behind $\bar{u}_m$ in a pulsatile flow, we assume the following relation on the basis of Eq.(5).

$$u'_{rms} = k\bar{u}_m - \theta \partial u'_{rms} / \partial t \quad (6)$$

where θ is the characteristic time. Though θ is also a function of r' and Re , the Reynolds number dependence is weak as can be seen from Fig.18 shown later. For the sake of simplicity, we perform the analysis assuming θ is a function of r' only. The effect of Reynolds number on θ will be considered separately in a later section.

Here, we express u'_{rms} and $\bar{u}_m$ as follows:

$$u'_{rms} = u'_{rms,ta} + |u'_{rms,os,i}| \times e^{i(\omega t + \angle u'_{rms,os,i})} \quad (7)$$

$$\bar{u}_m = \bar{u}_{m,ta} + |\bar{u}_{m,os,i}| e^{i(\omega t + \angle \bar{u}_{m,os,i})} \quad (8)$$

Substituting Eqs.(7) and (8) into Eq.(6) and after some arrangement we can get

$$u'_{rms,ta} = k\bar{u}_{m,ta} \quad (9)$$

for the time-averaged component and

$$\frac{|u'_{rms,os,i}|}{|\bar{u}_{m,os,i}|} = \frac{k}{\sqrt{1 + \omega^2 \theta^2}} \quad (10)$$

$$\omega \theta = \tan(\angle \bar{u}_{m,os,i} - \angle u'_{rms,os,i}) \quad (11)$$

for the oscillatory component. Equations (9) through (11) are called the turbulence model including a lag in response time.

5. Experimental Results and Discussion

Experiments are made in eleven runs shown in Table 1. The flows except Run 3 are fully turbulent and fully developed pulsatile pipe flows. Run 3 is the case where the relaminarization of flow occurs in an early stage of accelerating period as shown in Fig.3.

Here the effects of three parameters A_1 , f and Re_{ta} on turbulence distribution across the pipe are made clear, and the applicability of the present turbulence model is also examined.

5.1 Effect of velocity amplitude ratio A_1

We focus on Run 1 through Run 3 and examine the effect of A_1 showing the distributions of velocities and turbulences in Run 1 and Run 2 in detail. The velocity waveforms and velocity distributions in Run 2 are shown in Figs.4 and 5, respectively.

Table 1 Experimental conditions

Run	f (Hz)	$\sqrt{\omega'}$	Re_{ta}	A_1
1	2.08	22.6	36000	0.257
2	2.08	22.6	36000	0.522
3	2.08	22.6	36000	0.802
4	2.08	22.6	58700	0.533
5	2.08	22.6	70300	0.334
6	2.08	22.6	84500	0.266
7	1.02	16.5	76000	0.264
8	1.02	16.5	70600	0.284
9	1.02	16.5	35100	0.248
10	1.02	16.5	18900	0.326
11	0.299	9.01	36500	0.262

Fig.3 Velocity waveforms in Run 3

Fig.4 Velocity waveforms in Run 2

Fig.5 Velocity distributions in Run 2 (the broken line shows the $1/n$ -th power law)

Figure 4 indicates that the flow is fully turbulent over one cycle. In Fig.5 the measured values of $\bar{u}$ at every phase are well approximated by the $1/n$ -th power law expressed by a broken line. The exponent n can be determined from

$$\bar{u}_m / \bar{u}_{cl} = 2n^2 / [(n+1)(2n+1)] \quad (12)$$

by substituting the measured values of $\bar{u}_m$ shown in Fig.7 and $\bar{u}_{cl}$ in Fig.5. Figure 6 shows the values of n , where $\omega t / (\pi/6) = 0 \sim 6$ is the decelerating period and $6 \sim 12$

Fig.6 Measured values of the exponent n in Run 2

Fig 7 Measured values of $\bar{u}_m$; — Eq.(14)

the accelerating period. The solid line is calculated from the following empirical equation⁽⁹⁾

$$n = 2 \log_{10}(Re/10) \quad (13)$$

If the change of speed in flow is very slow and gradual, the exponent n should follow the solid line. Such a case is called the quasi-steady state. But the present measured value of n is smaller than Eq.(13) in the decelerating period and much higher in the accelerating period except for the latest stage. This means that the flow in the accelerating period is more liable to be influenced by the acceleration than in the decelerating period. The turbulent velocity distributions in all runs are found to be well approximated by the $1/n$ -th power velocity distribution.

Figure 7 shows the cross-sectional mean velocity in Run 1 through Run 3. The solid lines indicate the finite Fourier approximation expressed by

$$\bar{u}_m = \bar{u}_{m,ta} + |\bar{u}_{m,os,1}| \cos \omega t \quad (14)$$

Experimental values of every run are approximated by Eq.(14) within a scatter of $\pm 3\%$ and hence the variation in $\bar{u}_m$ is regarded as sinusoidal. Variations in pressure gradient $\Delta \bar{p}/L$ and wall shear stress $\bar{\tau}_w$ are illustrated in Figs.8 and 9. From Figs. 7 through 9, it is evident that $\Delta \bar{p}/L$ pulsates before $\bar{u}_m$ with a phase angle of about

Fig.8 Measured values of pressure gradient $\Delta \bar{p}/L$ in Run 1 through Run 3

Fig.9 Measured values of wall shear stress $\bar{\tau}_w$ in Run 1 through Run 3

70° and $\bar{\tau}_w$ pulsates almost in phase with $\bar{u}_m$ (except in Run 3).

The measured values of $\bar{\tau}_w$ in Run 1 and Run 2 agree well with their individual turbulent quasi-steady values

$$\left. \begin{aligned} \bar{\tau}_w &= \lambda_{s,t} \rho \bar{u}_m^2 / 8 \\ \lambda_{s,t} &= 0.3164 / Re^{1/4} \end{aligned} \right\} \quad (15)$$

The negative value of Run 3 appearing in the latest stage of decelerating period is attributable to an inherent error accompanying the present evaluation method. This error has its maximum near the latest stage of decelerating period. Judging from the instantaneous velocity distribution which is not shown here, we can conclude that a flow reversing does not occur.

The time-averaged value $\bar{u}_{ta}$, the amplitude $|\bar{u}_{os,1}|$ and the phase angle $\angle \bar{u}_{os,1}$ of the fundamental component of $\bar{u}$ are compared with their corresponding values of $\bar{u}_m$ in Figs.10 and 11. For Run 1 and Run 2, the distributions of $\bar{u}_{ta}/\bar{u}_{m,ta}$ agree favourably with the steady flow distribution having the same Reynolds number and are well approximated by the $1/7$ -th power law. The amplitude ratio $|\bar{u}_{os,1}|/|\bar{u}_{m,os,1}|$ is approximated by the $1/7$ -th power law and the phase difference $\angle \bar{u}_{os,1} - \angle \bar{u}_{m,os,1}$ is almost zero all over the cross-section. The difference between Run 1 and Run 2 is negligibly small in Figs.10 and 11. Therefore, we can con-

Fig.10 Distributions of $\bar{u}_{ta}/\bar{u}_{m,ta}$ in Run 1 through Run 3

Fig.11 Distributions of $|\bar{u}_{os,1}|/|\bar{u}_{m,os,1}|$ in Run 1 through Run 3

clude that the effect of velocity amplitude ratio A_1 on the velocity distribution is negligible. Besides, the experimental results of Run 3 agree well with those of Run 1 and Run 2. This is because the rate of relaminarized period to T is very small and because the measured relaminarized distribution of $\bar{u}$ does not differ so much from the expected turbulent velocity distribution. Accordingly, it can be emphasized that the data arrangement in Figs.10 and 11 can no longer reveal a fine feature such as the above.

Figure 12 shows u'_{rms} distributions at 12 phases in a cycle. The left hand side of the figure shows the data in the decelerating period and the right does the data in the accelerating period. As a whole, u'_{rms} is larger in the decelerating period than in the accelerating period. Figures 13 and 14 are drawn by using the finite Fourier approximation of u'_{rms} and $\bar{u}_m$ of Run 1 through Run 3. Since both the time-averaged and the oscillatory components of Run 1 and Run 2 agree well with each other, it can be concluded that the velocity amplitude ratio A_1 has no effect on the turbulence distribution. The time-averaged values of Run 1 and Run 2 in Fig.13 agree well with steady flow values. Such an agreement is also observed in pulsatile turbulent boundary layers on

Fig.12 Distributions of u'_{rms} in Run 2

Fig.13 Time-averaged values of turbulence intensity in Run 1 through Run 3 (The Reynolds number of steady flow is 36200.)

Fig.14 Oscillatory component of turbulence intensity in Run 1 through Run 3

a flat plate⁽¹⁰⁾. The same results were obtained for Run 4 through Run 11. This fact proves the applicability of the present turbulence model under the experimental conditions examined here. The solid lines in Fig.14 represent the result about the present model. The calculation procedure is explained as follows: First for Run 1 and Run 2 we draw a solid line using the mean of the measured values of phase difference $\angle u'_{rms,os,1} - \angle \bar{u}_{m,os,1}$, and then calculate $\omega\theta$ from Eq.(11) using the value of solid line. Finally the amplitude ratio $|u'_{rms,os,1}|/|\bar{u}_{m,os,1}|$ can be calculated from Eq.(10), where k is evaluated from the solid line shown in Fig. 13. It is clear that the present model is applicable for the range of $r' \lesssim 0.8$.

Mizushima, et al. show that the propagation of the turbulence generated from the vicinity of the wall is not a monotonically diffusive process but is one affected by some discrete events occurring in sequence⁽²⁾. The process is made up of three stages: (1) the turbulence slowly propagates to $y/R \approx 0.1$ ($y^+ \approx 31.4$); (2) it propagates quickly to $y/R \approx 0.3$ ($y^+ \approx 94.3$) and then (3) it propagates to $y/R = 1.0$ with nearly constant propagation speed. It is difficult to find out such a process in the present experimental results because the scattering of data is relatively large.

The root mean square of turbulence near the wall will be considered in detail. Figure 15 shows a semi-logarithmic plot of u'_{rms} . The solid line is drawn such as to best fit the measured values at each phase. From a later stage of accelerating period to a later stage of decelerating period u'_{rms} reaches a peak value at $y/R \approx 0.15$. Such a distribution of u'_{rms} also is observed in the corresponding period of other runs, but the reason cannot be given here. The measured value of u'_{rms} at every phase in the remaining period increases monotonically toward the wall in the region between $r/R = 0$ and 0.975.

5.2 Effect of pulsation frequency

Figure 16 shows measured values of θ for Runs 1, 2, 3, 9, and 11 having almost the same time-averaged Reynolds number of 36000. As can clearly be seen from Fig.17, measured values of $\angle u'_{rms,os,1} - \angle \bar{u}_{m,os,1}$ become smaller and finally vanish all over the cross-section as the pulsation frequency decreases. The degree of data scattering seems to be independent of pulsation frequency. Therefore, the scattering of θ calculated from Eq.(11) increases as ω decreases. Considering this point we can conclude that the difference in θ values between Runs 1, 2, 3, 9, and 11 is very small and hence the effect of pulsation frequency f on θ is negligible. This conclusion matches the findings of Mizushima, et al.⁽²⁾ about the propagation of a generated turbulence. They found that the propagation time of turbulence is independent of pulsation frequency. Unfortunately, they didn't consider the amplitude of turbulence intensity. The measured values of amplitude ratio $|u'_{rms,os,1}|/|\bar{u}_{m,os,1}|$ for Run 9 and Run 11 are well approximated by the present turbulence model when $r' \lesssim 0.8$ in Fig.17.

(a) decelerating period

(b) accelerating period

Fig.15 Instantaneous distributions of u'_{rms} in Run 2

Fig.16 Characteristic time θ in Runs 1, 2, 3, 9, and 11

Fig.17 Oscillatory component of turbulence intensity in Runs 1, 9, and 11

Fig.18 Characteristic time θ in Runs 2, 4, 5, and 6

5.3 Effect of time-averaged Reynolds number

Though θ does not depend on pulsation frequency f , it is weakly dependent on the time-averaged Reynolds number as can be seen from Fig.18. The purpose of this section is to find out with what kind of reference time θ scales. In wall turbulent flows, a turbulence is generated by a process called bursting phenomenon. This process is periodic and its period is termed the bursting period. However, the bursting period is not constant but is distributed around its mean value called mean bursting period T_B . The turbulence generated near the wall propagates towards the center of pipe. Therefore, the propagation time of turbulence may be closely related to the mean bursting period T_B . We tentatively adopt T_B as a reference time.

Mean bursting period of steady flow is a function of Reynolds number. If the pulsation frequency is relatively low, it is reasonable to assume that the bursting pro-

cess of pulsatile flow follows approximately that of steady flow at every instant. So we can assume that the mean bursting period T_B is a function of the time-averaged Reynolds number Re_{ta} and it is represented by ⁽¹¹⁾

$$\omega'_B = \frac{R^2}{\nu} \frac{2\pi}{T_B} = 0.94 Re_{ta} \dots \dots \dots (16)$$

Figure 19 shows θ/T_B against y^+ at $f=2.08$ Hz and Fig.20 does the results of all runs. For the calculation of nondimensional distance $y^+ (= y \bar{u}^*/\nu)$, the friction velocity $\bar{u}^*$ is evaluated from the relation

$$\left. \begin{aligned} \bar{u}^* &= \bar{u}_{m,ta} \sqrt{\lambda_{s,t}/8} \\ \lambda_{s,t} &= 0.3164 / Re_{ta}^{1/4} \end{aligned} \right\} \dots \dots \dots (17)$$

As long as Fig.19 is concerned, θ/T_B seems to be independent of Re_{ta} when $y^+ \geq 200$ ($r' \leq 0.8$). As we already stated in Fig.16 the scattering of θ increases with a decrease of pulsation frequency. Therefore, we can draw the same conclusion from Fig.20. However, when $y^+ \leq 200$, the number of data is limited and their scattering is relatively large. Closer measurement is needed in this region. This subject is left for a future study.

Figure 21 compares the oscillatory component of Run 4 through Run 6 with the present turbulence model. When $r' \leq 0.8$ the measured values are well approximated by this model. For Runs 7, 8, and 10 the same agreement is observed, though they are not shown here.

Fig.19 Distributions of θ/T_B at $f=2.08$ Hz

Fig.20 Distributions of θ/T_B in all runs

Fig.21 Oscillatory component of turbulence intensity in Runs 1, 4, 5, and 6

6. Conclusions

(1) The time-averaged Reynolds number and pulsation frequency in Run 1 and those in Run 2 are the same, but their velocity amplitude ratios are different. There is little difference in velocity and turbulence intensity distributions between Run 1 and Run 2. Therefore, at least when $0.25 \leq A_1 \leq 0.52$, the effect of velocity amplitude ratio A_1 on these distributions is negligible.

(2) The time-averaged Reynolds numbers in Runs 1, 2, 9, and 11 are the same, but their pulsation frequencies are different. The measured values of characteristic time θ in these runs agree well with one another as can be seen from Fig.16. This means θ is independent of pulsation frequency f .

(3) The characteristic time θ is a function of the time-averaged Reynolds number Re_{θ} . Figures 19 and 20 show that θ scales with the mean bursting period T_{θ} when $y^+ \gtrsim 200$ ($r' \gtrsim 0.8$).

(4) From a later stage of accelerating period to a later stage of decelerating peri-

od, u'_{rms} increases with an increasing r' and reaches a peak at around $r' \approx 0.85$. With a further increase in r' it decreases slightly and increases again. At every phase of the remaining period, u'_{rms} increases monotonically toward the wall. However, the behaviour of u'_{rms} is not clear in the range of $0.975 < r' \leq 1$ because hot wire measurements were not made there.

(5) We propose a turbulence model considering a lag in response time. The amplitude ratio $|u'_{rms,os,1}|/|u_{m,os,1}|$ estimated from this model by using the measured values of phase difference $\angle u'_{rms,os,1} - \angle u_{m,os,1}$ shows good agreement with its measured value when $r' \leq 0.8$. Accordingly, it is possible to estimate the amplitude ratio if we evaluate θ from the preceding conclusion (3). The modelling of turbulence in the range of $0.8 \leq r' \leq 1$ must be left for a future study.

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