Unsteady Flow Near a Circular Cylinder Oscillated Sinusoidally in Uniform Flow*

(1st Report, Vortex-shedding Mechanism in Synchronization Phenomena)

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The flow past a circular cylinder undergoing transverse oscillation in a uniform flow is investigated by means of flow visualization technique. The experimental conditions are $Re=1100$, $A/D=0.2$, and $St=fD/U=0.2$, where $Re$ is the Reynolds number based on the diameter $D$ of the cylinder, $A$ and $f$ the maximum amplitude and frequency of the oscillation of the cylinder, respectively, and $U$ the speed of the uniform flow. Oscillations of the separation points and the stagnation point, and the rate of vorticity fluxes through the section near the separation points were examined during a cycle. The phenomenon that the vortex sheds periodically in the same frequency as the cylinder was found to be caused by a significant change in the vorticity distribution in the vortex region, which was induced by vorticity feeding from the third separation point found behind two ordinary separation points.

Key Words: Unsteady Flows, Separated Flow, Vortex Shedding, Vorticity Flux, Synchronization, Flow Visualization

1. Introduction

The vortices are shed periodically from a cylinder in a uniform flow and arranged downstream in a Karman vortex street. When the cylinder is oscillated normal to the flow at or near the shedding frequency of the vortex for a stationary cylinder, the shedding frequency of the vortices is synchronized with the frequency of the oscillating cylinder. This phenomenon is called synchronization phenomenon or lock-in phenomenon and causes an increase of the drag and formation of a strong wake. In the field of fluid mechanics and its engineering application, there have been many studies on the flows around the oscillating cylinder since Den Hartog (1). Sarpkaya (2) has reviewed these studies on the flows. These studies have been carried out to make clear the detail of the wake far from the cylinder and the total drag, so far. However, few experimental studies on the flow near the cylinder have been reported so far, though the flow is intimately linked to the mechanism of the vortex-shedding.

Lately, Zdravkovich (3) studied the instant of the vortex shedding in the synchronization phenomenon by use of flow visualization method and he found that there are two modes of vortex-shedding. Since he visualized the flow using Al-dust which was spread on the free surface of water, he could not observe the detail of the flow near the cylinder. Sarpkaya & Shonoff (4) studied numerically the mechanism of vortex-shedding in the synchronization phenomena by use of discrete vortex filament method. In this method, however, there are many problems to be solved, concerning the mechanism of vorticity transfer. Therefore, the results obtained by this method should be verified by some experiments. Furthermore, in order to apply this method successfully, reliable experimental data concerning the vorticity transfer are required.

In this study, the flow near or immediately behind a circular cylinder, which was oscillated sinusoidally in normal direction to a uniform flow, was visualized and the movement of stagnation point and separation point, the vorticity flux through the sections near the separation points were measured and the patterns of the flow in the vortex region were observed. On the basis of these data on the near wake of the cylinder, the mechanism of vortex shedding in the synchronization phenomenon was made clear in relation to the vorticity transfer.

2. Experimental Apparatus and Method.

2.1 Apparatus

Figure 1 shows the general arrangement of the apparatus. The test cylinder (5) (diameter: $D=50\text{mm}$, length: $L=220\text{mm}$) is oscillated sinusoidally in the direction normal to a uniform flow in a recirculating water tank (width: $500\text{mm}$, depth: $220\text{mm}$). It is attached to the carriage (6), on which a camera (7) is also fixed. The carriage can be driven sinusoidally along the rails (8) by the Scotch-Yoke mechanism (9). In the following section, we describe the motion of the cylinder under the conditions: the displacement of the cylinder $\gamma=Asin\omega t$, the
velocity of the cylinder: \( U_t = f(\omega) \) (\( \omega \): angular frequency of oscillation of the cylinder, \( A_t \): amplitude of the oscillation of the cylinder, \( t \): time). The cylinder is oscillated in a plane perpendicular to the horizontal plane but we call the location of the cylinder at the phase \( \omega t = 90^\circ \) "the upper dead point" and that of the cylinder at phase \( \omega t = 270^\circ \) "the lower dead point".

2.2 Flow visualization and photography

The flow around the cylinder was visualized by the electrolysis method and the hydrogen bubble technique in the plane 30mm under the free surface of water. The electrolysis method was used in the same way as described in previous papers. In this method, the surface of the cylinder was coated with an electrically conductive paint used as an electrode for generating a white colloidal dye. By the simultaneous use of this method and hydrogen bubble technique, we can easily visualize the streamlines as well as the pathline and time-line. An example of the photographs taken by these methods is shown in Fig. 2. In this figure, the white colloidal dye stretching from the cylinder indicates the streamlines which consists of the fluid particles departing from the surface of the cylinder. The outline of the distribution of vortexes convected from the cylinder can be known by observing the streamlines.

A piece of bubble generating-wire was set upstream of the cylinder and five pieces of generating wire were stretched in radial direction from the surface of the cylinder as described in the previous paper. The bubbles generating pulsatively from these wires are shed by the flow and they indicate clearly timelines as well as pathlines as seen in Fig. 2. By use of the same method as in a previous paper, we can determine the flow velocity by using the relation between the length of pathline and the exposure time. Furthermore, we can detect the onset of a reversed flow near the surface of the cylinder from the relative location between the time-line and the generating wire.

The photographs were taken synchronized with the motion of the cylinder by the motor-driven camera. The phases of oscillation of the cylinder and the exposure instants were determined by the records of the signals from the X-contact of the camera and the photo-switch to detect the motion of the cylinder.

3. The Synchronization Range
and Experimental Condition

The experiment was carried out at Reynolds number \( Re = Ud/\nu \sim 10^3 \) (\( U \): velocity of the uniform flow, \( \nu \): kinematic viscosity), because it was easy to visualize the flow clearly and cause the turbulence to appear in the vortex region. No experimental study concerning the synchronization phenomenon at \( Re \sim 10^3 \) has been reported in literature. First, we confirmed the appearance of synchronization phenomenon and measured the range of synchronization frequencies when \( Re \) was 600 to 2500, and \( A/D \) was 0.20.

The shedding frequency of a vortex was measured by a wind tunnel experiment. A test cylinder (\( D=100 \)mm or \( 200 \)mm) was set horizontally in the central plane of the wind tunnel (300mm in width, 200mm in height) and the cylinder was oscillated vertically by a crank mechanism. The shedding frequency of the vortex was determined from the Lissajous figure of the velocity fluctuation in the wake which was measured by a hot-wire anemometer.

Figure 3 shows the relation between
the frequency $f_r$ of the vortex and the oscillating frequency $f_c$ of the cylinder. The ratios of the frequency $Su/S_c$ ($S_r=\pi D/U, S_c=\pi D/U$) are plotted against the non-dimensional oscillating frequency $S_c$ of the cylinder. In the figure, the synchronization range can be recognized as the range in which the ratio of $Su/S_c$ is equal to unity. As seen in the figure, the experimental values of $Su/S_c$ at $Re=600-2500$ are also unity in the range of $S_c=0.1-0.2$, which is similar to the results obtained by Taneda at $Re=100$, that is, the synchronization phenomenon was recognized to occur in this range. Referring to the above results, we set the value of the ratio $Su/S_c$ at 0.2 (the upper region of the synchronization range) in the present experiment because, at $Su/S_c=0.2$, there happens only one of the two modes of the vortex shedding reported by Zdravkovich.

4. Experimental Results and Discussion

4.1 Movement of stagnation point

Figure 4 shows an example of photographs of the flow near the front stagnation point visualized by the hydrogen bubble technique. The angular location $\theta_n$ of the stagnation point was measured from the photographs and plotted against the phase $\omega t$ of the oscillating cylinder in Fig.5. The value of $\theta_n$ shows the angular position measured from the stagnation point of a stationary cylinder in a uniform flow. A dash-dotted line in the figure indicates the variation of the angle of incidence $\alpha$ of a uniform flow to the cylinder. As seen in the figure, the angular positions $\theta_n$ are well approximated by a sinusoidal curve similar to the curve of the incidence angle $\alpha$ with a small phase lag (about $10^\circ$) behind the oscillation of the cylinder.

4.2 Movement of separation point

4.2.1 Criterion of separation

As is well known, in a steady flow the boundary layer separates from the surface at the point where the wall shear stress $\tau$ vanishes, that is,

$$\tau = \mu \frac{d\upsilon}{dy} |_{y=0} \tag{1}$$

where $\mu$ is the coefficient of viscosity, $\upsilon$ the velocity, and $y$ the distance normal to the wall. A reversed flow appears downstream of this separation point. Therefore, the appearance of the reversed flow in the boundary layer can be taken as the definition of the separation in a steady flow. In an unsteady flow, however, vanishing of the wall shear stress does not connect with separation of the boundary layer. Many problems concerning the separation in unsteady flows remain unsolved. Even the definition of separation in unsteady flows is still uncertain.

In the present study, the definition of separation proposed by Taneda was adopted as the criterion for separation; the separation point is defined as "the point at which the streakline departs from the surface of the wall" (2)

By the electrolysis method, the point satisfying the condition (2) can be detected as a point at which the colloidal dye separates from the surface of the cylinder. Moreover, since the streakline visualized with the dye is invariant for different reference frames, the condition (2) is reasonably considered the criterion of separation in unsteady flows.

4.2.2 Movement of separation point visualized by electrolysis method

Figure 6 shows the time sequence of the flow behind the cylinder. The order of arrangement of the photographs (I) and (II) corresponds to the movement of the cylinder, that is, the upper dead point and the lower one of the cylinder are arranged at the top and the bottom of the figure, respectively, and we can follow the time sequence of the flow, seeing these photographs in the clock-wise direction, that is, seeing the left column from bottom to top, the right column from top to bottom.

Photo (I) in Fig.6 shows the flow around the cylinder visualized by the electrolysis method and the hydrogen bubble technique. In these photographs the dye generated from the surface of the cylinder can be seen clearly. It is remarkable that the third separation point, which is not found in the steady flow around a stationary cylinder, appears at a location on the rear surface of the oscillating cylinder between two usual separation points.

In Fig.7, the first separation point $\theta_n$ (on the side of lower dead point) and the third separation point $\theta_n$ (cf. Fig.8) are plotted against the phase $\omega t$ of the oscillating cylinder. The solid line in the
The figure shows the sinusoidal curve fit to the measured values of \( \theta_1 \) and \( \theta_2 \) and the dashed line indicates the location of the separation point when the cylinder is stationary, that is, \( \delta_c = 0 \). The separation point \( \theta_1 \) on the oscillating cylinder moves sinusoidally around the neutral location \( 90^\circ \) which is situated slightly downstream of the separation point \( 84^\circ \) on the stationary cylinder, and its amplitude is slightly smaller than that of the stagnation point. Since the separation points \( \theta_1 \) and \( \theta_2 \) (cf. Fig. 6) move sinusoidally with a phase difference of \( 180^\circ \), the value of \( \theta_2 - \delta_i \) remains about \( 150^\circ \) throughout the period of the oscillation.

The third separation point moves sinusoidally around the neutral location of \( 180^\circ \) in phase with the oscillating velocity \( U_c \) of the cylinder (\( U_c = A \omega \cos \omega t \)). The amplitude of \( \theta_3 \) is about two times as large as that of \( \theta_1 \), \( \theta_1 \), or \( \theta_2 \). The movement of the third separation point is linked to the mechanism of the appearance of the third separation point itself. When the cylinder is stationary, the dead water region is formed downstream of the two separation points. On the other hand, when the cylinder moves normal to the uniform flow, the relative velocity of the flow to the cylinder develops a boundary layer on the rear surface of the cylinder. This boundary layer separates from the third separation point \( \theta_3 \). As a result, the third separation point \( \theta_3 \) oscillates in phase with the movement of the cylinder with a large amplitude in comparison with \( \theta_1 \) and \( \theta_2 \). It should be noticed that the sense of vorticity shed from this separation point \( \theta_3 \) changes corresponding to the direction of the motion of the cylinder. This fact is important for the mechanism of the shedding of a vortex, because the sense of vorticity fed to the vortex

Fig. 6(1)  Flow around the circular cylinder oscillating in a uniform flow
( \( R_e = 1100, A/D = 0.2, S_c = 0.2 \) )
region has a significant influence on the behaviour of the vortex flow.

4.2.3 Separation point and the appearance of a reversed flow

In order to make clear the relation between the separation point and the appearance of a reversed flow, we observed a flow near the surface of the cylinder visualized by the hydrogen bubble technique and the electrolysis method.

The photographs in Fig. 6 (II) show the flow near the separation point. A piece of bubble-generating wire is set across a uniform flow and five pieces of generating wire are stretched from the cylinder in the radial direction as described in the previous paper. The angular location of each wire is shown by $\theta_w$ in the figure. As seen in these photographs, the occurrence of a reversed flow can be detected from the relative locations between bubbles intermittently shed from the wire and the generating wire, that is, the relative location of the time-line to the generating wire. The separation point can also be detected from the dye separation point.

When the cylinder moves from the lower dead point toward the upper dead point, as seen in Fig. 6 (II) (g), (h), (a) and (b), the reversed flow can be seen clearly behind the separation point which is visualized by dye filament. On the other hand, when the cylinder moves from the upper dead point toward the lower one, as seen in Fig. 6 (II) (c), (d) and (e), no reversed flow can be seen behind the separation point. Such phenomena do not occur in the flow around the stationary cylinder.

Figure 9 shows the range of the phases in which the reversed flow occurs behind the separation point. The ordinate shows the angular location $\theta$ and the abscissa shows the phase $\omega t$ of the oscillation of

Fig. 6 (I) Flow around the circular cylinder oscillating in a uniform flow ($R_e=1100, A/D=0.20, S=0.20, \theta_w=75^\circ, 90^\circ, 105^\circ, 118^\circ, 133^\circ$)
the cylinder. The existence of the reversed flow is judged from the thickness $S$ of the reversed-flow layer (illustrated in the figure). Black circles in the figure indicate the existence of the reversed flow ($S>0$) and a broken line indicates the location of the separation point $\theta_n$ which is determined from the observation of the dye filament departing from the surface of the cylinder. It is found that the phase $\omega$ of the oscillation of the cylinder without a reversed flow ranges from about $90^\circ$ to $210^\circ$, this range being illustrated as the region with the mark $\mathcal{E}=0.3$ in the figure corresponding to one third of the period of oscillation of the cylinder. In the other range of the phases, the separation point almost corresponds to the point of appearance of a reversed flow ($S=0$) in the boundary layer.

The reason why the reversed flow does not occur behind the separation point over the phase $\omega$ ranging from about $90^\circ$ to $210^\circ$, is explained by the relative velocity of the cylinder. In other words, since the velocity opposite to the motion of the cylinder is superposed on the whole flow field, a forward flow is superposed on the flow behind the separation point on the side of the direction of the motion of the cylinder and the reversed flow may vanish. Furthermore, since the superposed flow forces the separated shear layer to move closer to the cylinder, the region between the shear layer and the rear surface of the cylinder becomes narrower. Then, as the cylinder approaches the lower dead point ($\omega=270^\circ$), the superposed velocity becomes smaller and the induced velocity by the vorticity concentrated in the shear layer becomes dominant. Therefore, the range of the phases without a reversed flow is considered to be confined to the range $\omega=90^\circ$ to $210^\circ$. Moreover, the third separation point $\theta_3$ mentioned above is a point where the superposed flow separates from the rear surface of the cylinder.

On the other hand, since the flow behind the separation point on the side opposite to the direction of the motion of the cylinder is decelerated by the superposed flow, a forward flow cannot exist behind the separation point. The motion of the cylinder causes the separated shear layer to move far away from the cylinder.

4.3 Vorticity fluxes through the sections near the separation points

The vortex region is formed by concentration of the vorticity which is produced on the surface of the cylinder and convected downstream passing the separation points. Vorticity fluxes through the sections near the above three separation points $\theta_1$, $\theta_2$, and $\theta_3$ are considered to have a significant influence on the mechanism of vortex formation.

When the separation point does not move, as in steady flows, the nondimensional vorticity flux $K$ is expressed approximately by

$$K = \int_0^1 \omega \, dx / U^3 = \frac{1}{2} U^3 / U^3$$

(3)

where $\omega$ denotes the vorticity, $U$ the velocity, $x$ the distance normal to the wall, and $U^3$ the velocity at the outer edge of the boundary layer. In the present study, since the separation points move with the motion of the cylinder, according to Sears, the actual vorticity flux $K'$ in this case should be expressed by

$$K' = \int_0^1 \frac{\omega (U - U_h)}{U^3} \, dx = K - \frac{U_h U_h}{U^3}$$

(4)

where $U_h$ denotes the speed of the separation point.

In order to calculate the values of $K$ and $K'$, we measured the velocity at the outer edge of the boundary layer near the three separation points $\theta_1$, $\theta_2$, and $\theta_3$. The relation between the vorticity fluxes and the phase $\omega$ of the oscillation is shown in Fig. 10. In the figure, the range of the phases without a reversed flow be-
hind the separation point is denoted by the mark \( \Delta \) and the location of the stagnation point is also plotted.

As seen in the figure, values of the actual vorticity fluxes, i.e., \( K_1 \), \( K_2 \), and \( K_3 \), vary sinusoidally. It is remarkable that the values of \(- (K'_1 + K'_2)\) indicated by the triangles \( \Delta \) in the figure are almost in good agreement with the values of \( K' \) indicated by black circles \( \bullet \). This fact means that the total amount of the vorticity flux, that is, \( K'_1 + K'_2 + K'_3 \), is nearly zero over a period of the oscillation of the cylinder and the extended Kutta condition is also satisfied in this case. This means that those measured values are considered reliable and the rate of vorticity fed to the vortex region can be evaluated approximately by the values of \( K'_1 \) and \( K'_2 \).

Furthermore, the value of \( K'_3 \), which denotes the vorticity flux through the section near the third separation point \( \theta_3 \), is not negligibly small in comparison with those of \( K'_1 \) and \( K'_2 \) and it varies not only in its magnitude but also in its sign. Therefore, the vorticity shedding from the third separation point is considered to have a significant influence on the mechanism of vortex shedding.

4.4 Mechanism of vortex-shedding

Figure 11 is a schematic diagram illustrating the mechanism of vortex-shedding. In the figure, the main results described above and sketches of the shear layer are contained;

Column 1: Shear layer visualized by the dye generated from the surface of the cylinder

Column 2: Movement of the separation points \( \theta_1 \) and \( \theta_2 \), and the stagnation point \( \theta_0 \)

Column 3: Variation of the vorticity fluxes of \( K_1 \) and \( K_2 \)

Column 4: Existence of the reverse flow toward the separation point \( \theta_0 \).

In the following, we will discuss the mechanism of vortex-shedding in the synchronization phenomenon with reference to this figure.

4.4.1 Configuration of the separated shear layer

We will discuss the flow in the lower vortex region in each sketch of column 1 in Fig. 11.

During the period when the cylinder moves from the lower dead point to the upper one (\( \omega t \): from 270° to 90°), the vortex behind the cylinder is growing and does not shed, as is shown with the sketches in the first column 1 on the left hand of Fig. 11. Then, when the cylinder begins to move from the upper dead point to the lower one, a kink appears as shown by an arrow in the sketch (c) in the dye filament which indicates a separated shear layer and subsequently the vortex region becomes turbulent. The kink of the dye filament becomes more remarkable as the cylinder approaches the phase of 180°, and the flow in the wake becomes more turbulent. In this state, the mass of fluid with vorticity seems to be separated into two parts as shown in the sketch (d) and (e). Then, as the cylinder reaches the lower dead point, the dye filament with the kink curls up clearly into the near wake of the cylinder and a discrete vortex appears there. The irrotational fluid surrounding the lower attached vortex is entrained into the upper vortex region and as a result, the center of this upper vortex shifts farther downstream of the cylinder.

As shown in Fig. 11, it is difficult to indicate the precise moment of the shedding of vortex, because the shedding of each vortex does not occur suddenly and, moreover, it is accompanied with the onset of a turbulence. If we dare to indicate the instant of vortex-shedding, it will be the moment when the actual vorticity flux \( K'_1 \) indicates the minimum. The value of \( K'_1 \) indicates the minimum at the phase \( \omega t = 120° \), which corresponds to the moment when the kink of the dye filament becomes significant (cf. Fig. 10 and 11).

4.4.2 Mechanism of vortex-shedding

As described in the above section, the shedding of the vortex is interpreted as a process in which the shear layer begins to kink and subsequently is rolled up into the vortex region immediately behind the cylinder and the vortex region is divided into two parts of the mass of fluid with vorticity. In other words, the flow becomes unable to keep the vorticity distribution with a single peak at the center of the vortex and the vorticity distribution turns into one with two peaks. The main causes of the change of the vorticity distribution are considered as follows:

(1) self-induced velocity of the vorticity,
(2) variation of the vorticity fluxes through the sections near the separation

![Fig. 10 Time-variation of the vorticity fluxes and separation points](image-url)
points and (3) cancellation of vorticity by coalescence of the fluid particles with the vorticity of opposite sign to each other. These causes (1), (2) and (3) are dependent on each other in relation to the mechanism of vortex-shedding. In this paper, we will discuss the mechanism of vortex-shedding concerning the causes (2) and (3).

The cause (2) has been discussed already in the above section. That is, it was found that the vortex is seen to shed when the actual vorticity flux $K^1$ indicates minimum (at $t = 120\degree$, cf. Fig.10). In other words, the decrease of the flux of vorticity with the same direction as the rotation of the vortex is closely related to the vortex-shedding.

The cause (3) is important for the mechanism of vortex-shedding; when a vorticity opposite in sense to the rotating direction of the vortex flow is introduced, not only cancellation of the vorticity occurs in the vortex region but also the flow becomes unstable there, and there is a possibility of the transition to turbulence in the vortex region being promoted.

With respect to the lower vortex in each sketch of column $\theta_2$ in Fig.11, the vorticity opposite in sense to the rotating direction of the lower vortex is shed from the upper separation point $\theta_2$ (cf. Fig.8) as well as the third separation point $\theta_3$. (cf. Fig.8). The rate of feeding of the vorticity into the vortex region was measured at the section near the separation points $\theta_2$ and $\theta_3$, and these non-dimensional values are plotted as the values $K_2$ and $K_3$ in Fig.10. Since the value $K_2$ corresponds to the non-dimensional vorticity flux in the upper vortex region through the section near the separation point $\theta_3$, the flux $K_2$ scarcely affects the behaviour of the flow in the lower vortex region. Meanwhile, the flux $K_2$ is important to understand the mechanism of vortex-shedding, because its magnitude is not only negligibly small in comparison with the magnitudes of $K_2$ and $K_3$ but also its sign changes alternately with time. In the following, we will discuss the role of the vorticity flux $K_2$ in the vortex-shedding referring to the configuration of a separated shear layer.

The period when the value $K_2$ becomes opposite in sense to the rotation of the lower vortex (cf. Fig.8) is limited to the period for the sketch (f) to (a). And the magnitude $K_2$ becomes maximum at the phase $\omega t = 320\degree$ corresponding to the sketch (h). In this period (sketch (f) to (a) in column $\theta_2$), however, the vorticity fed from $\theta_2$ at the rate of $K_2$ is not connected toward the shear layer stretching from the separation point $\theta_3$ (cf. Fig.8) and the lower vortex in the figure keeps growing without shedding. At an instant in the period corresponding to the sketch (b) to (c), the vorticity (opposite in sense to the rotation of the lower vortex) shed from $\theta_3$ in the period (f) to (a) at last begins to approach the shear layer from $\theta_2$ (cf. Fig.8). This instant is not determined definitely because of the onset of a turbulence. At the phase $\omega t = 100\degree - 120\degree$ (after the phase shown by the sketch (c)) the convection of vorticity toward the lower shear layer is significant, and the shear layer is undulated. Subsequently, the vortex-region is seen to be divided into two parts. Furthermore, since the third separation point is oscillating in phase with the oscillation of the cylinder, the vorticity shed from the separation point $\theta_3$ is considered the main cause of the synchronization for the vortex-shedding.

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![Fig. 11 Schematic diagram of vortex-shedding](image-url)
4.4.3 Sarkarya's description of the mechanism of vortex-shedding

Sarkarya studied numerically the behaviour of a rolling up of the shear layer separated from an oscillating circular cylinder by use of the discrete vortex method and based on the calculated results, he described the process of vortex-shedding as follows.

"As soon as the cylinder begins to move upward under the action of transverse force, the lower attached-vortex sheet lags behind because of its inertia. Thus, the wake axis passing through the instantaneous center of the cylinder rotates clockwise. In the meantime, the velocity relative to the cylinder rotates a clockwise angle dictated by the velocity of the cylinder. The rotation of the velocity vector coupled with the rotation of the wake axis moves the upper separation point farther downstream and the lower separation point farther upstream relative to those on a stationary cylinder. $K_1$ at the upper separation point increases rapidly and decreases at the lower separation point $\theta_1$.

Furthermore, in his calculation, the shear layer (discrete vortex sheet) shed from the lower separation point is cut immediately after when $K_1$ reaches its maximum.

With respect to the movement of the separation points $\theta_1$ and $\theta_2$, and the behaviour of a rolling up of the shear layer, Sarkarya's description nearly corresponds to the experimental results shown in Fig.11, though he did not describe the phase of the oscillation of the cylinder in detail. In Sarkarya's description, however, the third separation is not discussed in spite of its importance to the mechanism of vortex-shedding, and only the fluxes $K_1$ and $K_2$ are discussed as the feeding sources of vorticity. As described above, the actual vorticity fluxes $K_1^*$, $K_2^*$, and $K_3^*$ defined with eq.(4) should be taken as the source of vorticity fed to the vortex region. Therefore, if the feeding mechanism of vorticity revealed in this experiment is introduced into the discrete vortex method, the method will be improved to give a more accurate solution for unsteady flow around a bluff body.

5. Conclusions

The flow around an oscillating cylinder in a uniform flow was observed and the mechanism of vortex-shedding under synchronization was made clear. Main results obtained in this study are summarized as follows:

1) The details of the flow field near the surface of the cylinder were examined with time; e.g. the angular locations of the stagnation point and the separation points, the velocity at the outer edge of the boundary layer near the separation points, the configuration of the separated shear layer etc.

2) The period in which the reversed flow behind the separation point does not exist is found out. This forward flow behind the separation point does not reach the other separation point but separates from the third separation point on the rear surface of the cylinder. The third separation point moves sinusoidally in phase with the cylinder and the vorticity fed from the third separation point promotes the shedding of a vortex synchronized with the motion of the cylinder. The vorticity $K_1^*$ fed from the third separation point is convected to the shear layer with a vorticity opposite in sense to $K_1^*$ stretching from one of the separation points $\theta_1$ or $\theta_2$ and the vortex region is divided into two parts by the local cancellation of vorticity in the shear layer due to the vorticity convected from the third separation point, so that the vortex is shed downstream.

3) The sum of the vortex fluxes $K_1^*$, $K_2^*$, and $K_3^*$ is zero, where $K_1^*$ denotes the rate of vorticity fed through the section near the separation point, even when the separation point moves with time. The suffixes 1, 2, and 3 denote the first, the second, and the third separation point, respectively. In other words, the extended condition of Kutta is satisfied in this case.

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References

(9) Tandera, S., Prog. Aerospace Sci., 17-2 (1977), 287
(10) Sears, W.R., AIAA J., 14-1 (1975), 216