Prediction of Turbulent Flow in Two-Dimensional Channel with Turbulence Promoters*

(2nd Report, Numerical Prediction by $k-\epsilon$ Model
and Comparison with Experimental Results)

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Fully developed incompressible turbulent flow in a two-dimensional channel with turbulence promoters has been simulated numerically. A $k-\epsilon$ model, which consists of two model transport equations for turbulence energy and energy dissipation rate besides the mean velocity equation, is applied. The Reynolds number is set at 1.1×10^4 and only 4000 grid points are distributed. A detailed comparison between the calculated results by $k-\epsilon$ model and large eddy simulation is made. The distributions of mean velocities, turbulence energies and turbulence shear stresses by both methods are in good agreement near the upper wall without turbulence promoter, but quite different behind the turbulence promoters. The calculated results are also compared with the experimental results. The large eddy simulation method can describe these flow fields well. Calculations were carried out on the HITAC M 200/280H system computer of University of Tokyo.

Key Words : Turbulence, Channel Flow, Turbulence Promotes, Numerical Prediction, $k-\epsilon$ Model, Large Eddy Simulation, Hot-Wire Measurement, Eddy Viscosity, Turbulence Energy, Turbulence Dissipation

1. Introduction

As means to simulate the turbulent flow around a body or in a straight pipe/plane channel, zero, one and two equation models, Reynolds stress closure models and large eddy simulation have been proposed. It is said that the large eddy simulation (abbreviated as LES) can represent a one-dimensional channel flow more precisely than the experiment⁽¹⁾. The LES computation, however, requires a great deal of memory capacity and computation time. Simulations using $k-\epsilon$ model are rather popular at present and the validity of the model is examined for several types of flows. For example, Johns et al.⁽²⁾ applied this model to the laminarization in a plane channel flow and computed the friction coefficients, which agreed well with the experimental values. Murakami et al.⁽³⁾ applied it to the airflow inside of a three-dimensional room and showed that it can predict well the mean velocity and turbulence energy profiles. Elghobashi et al.⁽⁴⁾ succeeded in predicting

time-mean velocities for a turbulent pipe flow with a reverse and concentric jet.

Turbulence models in general, including $k-\epsilon$ model, are based on experimental facts for a equilibrium turbulence. However, the availability of these models for computing a non-equilibrium turbulent flow such as a separated flow has scarcely been examined, except in Leschziner's work concerning the mixing of two parallel jets⁽⁵⁾, among others.

In this study, a turbulent flow in a plane channel with turbulence promoters is calculated numerically using LES and $k-\epsilon$ model and the feasibility of applying these models to turbulent separated flows is discussed. In the previous report⁽⁶⁾ the authors described a LES approach and the typical computation results. In the present paper, computation results by LES, by $k-\epsilon$ model and experimental results by hot-wire measurements are compared and characteristics of both models are also contrasted.

2. Basic equation for $k-\epsilon$ model

Basic equations for $k-\epsilon$ model are composed of a continuity equation, momentum equations and two additional transport equations for the turbulence energy and the turbulence dissipation rate. The flow field in this study is the inside of two parallel flat walls as shown in Fig.1. Rectangular cylinders with square cross section, which are called turbulent promoters, are installed on the lower wall at equal intervals. The following are assumed for the computation: (1) flow is

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incompressible and two-dimensional, (2) turbulence is isotropic in the greater part of the flow field, and (3) the flow conditions are the same at inlet and outlet sections of the computational region.

An orthogonal coordinate system, with x_1 -axis in streamwise direction and x_2 -axis in the direction normal to two parallel planes, is used and velocity u_i , coordinate x_i , pressure p and time t are non-dimensionalized by mean velocity U in x_1 direction at section III, double length L of the channel width, ρU^2 (ρ : fluid density) and L/U , respectively. In general, u_i is divided into the following two parts,

$$u_i = \bar{u}_i + u_i' \quad (1)$$

, where u_i is x_i component of time mean velocity and u_i' is x_i component of velocity fluctuation. Basic equations are written as,

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu_T + \frac{1}{Re} \right) \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad (2)$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (3)$$

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j} (k \bar{u}_j) = \frac{\partial}{\partial x_j} \left(\left(\frac{\nu_T}{\sigma_k} + \frac{1}{Re} \right) \frac{\partial k}{\partial x_j} \right) + \nu_T \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - \epsilon \quad (4)$$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x_j} (\epsilon \bar{u}_j) = \frac{\partial}{\partial x_j} \left(\left(\frac{\nu_T}{\sigma_\epsilon} + \frac{1}{Re} \right) \frac{\partial \epsilon}{\partial x_j} \right) + C_1 \frac{\epsilon}{k} \nu_T \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - C_2 \frac{k \epsilon}{\nu_T} \quad (5)$$

$$\nu_T = C_\mu k^2 / \epsilon \quad (6)$$

, where $i, j = 1, 2$, $\bar{P} = \bar{p} + 2k/3$. k and ϵ denote the turbulence energy and the turbulence dissipation rate, and are normalized by U^2 and U^3/L , respectively. $Re = UL/\nu$ (ν : kinematic viscosity) is the Reynolds number and the eddy viscosity ν_T is introduced along with the Prandtl-Kolmogorov assumption.

3. Computational procedure and boundary conditions

Equations (2)~(6) are transformed into a series of finite difference equations and solved simultaneously by using SMAC method with the central difference scheme for the partial derivatives in space and the Adams-Bashforth scheme in time, similarly to the

Fig.1 Model flow field

previous report. The outline of computational procedure is shown in Fig.2. A staggered grid system is used, where u_i is defined at the center of grid surface, and p, k, ϵ and ν_T are at the center of grid volume. The computational region is confined to one cycle of turbulent promoters, i.e. a region surrounded with sections I, IV, upper and lower walls (see Fig.1). The geometrical dimensions of the computational region are $d_1 = d_2 = d$, $Pi/d = 5$, and $L/(2d) = 3.2$ in Fig.1. Computational grid points are 80 in the x_1 direction and 50 in x_2 , that is, grid intervals are 0.01 in both directions.

The boundary conditions for velocities are as follows: no-slip condition is adopted at wall boundaries and cyclic condition is imposed at sections I and IV, that is

$$\begin{aligned} \bar{u}_i &= 0 & (\text{at walls}) \\ (\bar{u}_i)_I &= (\bar{u}_i)_{IV} & (\text{at inlet and outlet sections}) \end{aligned} \quad (7)$$

The quadratic profile near walls and cyclic conditions at sections I and IV are assumed for k and ϵ .

$$\begin{aligned} \frac{k}{k_0} &= \frac{x}{x_0} \left\{ a + (1-a) \frac{x}{x_0} \right\} & (0 \leq x \leq x_0) \\ (k)_I &= (k)_{IV} & (\text{at inlet and outlet sections}) \\ \frac{\epsilon}{\epsilon_0} &= \frac{x}{x_0} \left\{ a + (1-a) \frac{x}{x_0} \right\} & (0 \leq x \leq x_0) \\ (\epsilon)_I &= (\epsilon)_{IV} & (\text{at inlet and outlet sections}) \end{aligned} \quad (8)$$

, where x is the distance from the nearest

Fig.2 Outline of computational procedure

wall, x_0 is the length which is representative of the predominant non-isotropy of the turbulence. k_0 and ε_0 are values of k and ε at $x=x_0$, respectively, and a is a constant. In this paper, as an example, $x_0=5h/2$ (h : grid size) and $a=7/4$ are adopted. Eddy viscosity ν_T is estimated from Eq.(6), except in the neighborhood of the wall, where Eq.(6) seems to be unreasonable. The linear function is assumed for ν_T near walls that is,

$$\frac{\nu_T}{\nu_{T0}} = \frac{x}{x_0} \dots \dots \dots (9)$$

As additional conditions, mean values of k and ε are required at inlet section. In this paper $((k))_I = 0.055$ and $((\varepsilon))_I = 0.166$ are chosen so that the profile of k near the upper wall may be closer to the experimental one in a plane channel flow⁽⁷⁾.

For initial conditions, the results of LES computation are applied. Convergence condition in computation is:

$$\text{Max} \left| \left[\text{dif} \left(\frac{\partial \bar{u}_i}{\partial x_i} \right) \right]_N \right| < 0.0005 \dots \dots \dots (10)$$

where $\text{dif}(\partial \bar{u}_i / \partial x_i)$ denotes a difference formula of $(\partial \bar{u}_i / \partial x_i)$, N is number of iterations (see Fig.2). In this paper, non-dimensional time step δt is 1/2000 and non-dimensional grid width h equals 1/100. The relation between h and δt is so determined as to satisfy the numerical stability condition⁽⁸⁾. The main computation conditions are shown in Table 1. Model constants, appearing in Eqs.(4)~(6), are chosen from the computation example⁽⁹⁾ as follows:

$$\sigma_1 = 1.0, \sigma_2 = 1.3, C_D = 0.09, C_1 = 1.59, C_2 = 0.18$$

Reynolds number in computation of this paper is 1.1×10^4 .

4. Computational results by k- ε model and comparisons with LES

Table 1 Computation conditions

	Number of Grids X_1, X_2, X_3	δt	h	Convergent Condition
k- ε Model	80, 50, -	1/2000	1/100	0.0005
LES	40, 25, 9	1/1000	1/50	0.001

Table 2 Correspondence of symbols

	k-E Model	LES		k-E Model	LES
Mean Velocity	$[\bar{u}_i]$	$[\langle \bar{u}_i \rangle]$	Turbulence Shear Stress	$\overline{u_i' u_j'}$	$\langle \bar{u}_i' \bar{u}_j' \rangle + \bar{u}_i' \bar{u}_j'$
Stream Function	ψ	$\langle \psi \rangle$	Eddy Viscosity	ν_T	—
Turbulence	$\sqrt{\overline{u_i'^2}}$	$\sqrt{\langle \bar{u}_i'^2 + \bar{u}_i'^2 \rangle}$	Turbulence Dissipation Rate	ε	—
Turbulence Energy	k	$\frac{\langle \bar{u}_i'^2 + \bar{u}_i'^2 \rangle}{2}$	SGS Eddy Coefficient	—	K

Physical quantities obtained from the present computation using k- ε model are an ensemble averaged value $\bar{f}$ and the mean square value $\bar{f'^2}$, which are functions of x_1, x_2 and t . In this case, it is not necessary to average in x_3 direction, and $\langle \langle f \rangle \rangle$ and $[\bar{f}]$ represent the spatial average in x_1 direction and the time average in a certain period, respectively. On the contrary, instantaneous value f is calculated from LES, which is a function of x_1 and x_2, x_3 and t , and in the previous report⁽⁶⁾, $\langle f \rangle$, $\langle \langle f \rangle \rangle$ and $[\langle f \rangle]$ denote the spatial average in x_3 direction, the spatial average in $x_1 - x_3$ plane and the time average in a certain period, respectively. Symbols used in k- ε model computation and in LES computation are shown in Table 2. 1000 time steps of integration are chosen, after which the influences of initial conditions vanish.

4.1 Time dependence of stream line and termination of computation

The extinction of the influences of the initial conditions and the establishment of the steady state solution can be judged by observing a stream function at a certain fixed point. The instantaneous stream function at an arbitrary point H is defined as,

$$\psi_H = \int_0^{x_2} \bar{u}_1 dx_2 \dots \dots \dots (11)$$

Figure 3 shows a time-dependence of ψ_H , where point H locates behind a promoter ($x_1 = 0.650, x_2 = 0.091$, see Fig.1). The figure shows that ψ_H by k- ε model becomes constant in a relatively short time, whereas ψ_H by LES changes periodically with time. It seems to be impossible in this study to describe a large-scale vortex motion by k- ε model.

Fig. 3 Variations of stream function with time

For reference the computation time per one time step using HITAC M200/280H system computer is shown in Table 3.

4.2 Mean velocity profiles

Figure 4 shows the computed profiles of mean velocities $[\bar{u}_i]$ at sections II and III (see Fig.1). It is clear from the figure that profiles of $[\bar{u}_1]$ by k- ϵ model and by LES are quite different near and behind the promoter, and that both profiles are in relatively good agreement only near the upper wall. Figure 5 shows a detailed profile of mean velocity near the upper wall. Abscissa y' denotes the distance from the upper wall and is non-dimensionalized by $L/2-d$. In the figure, computed profile is

compared with the experimental results in a two-dimensional channel without a turbulence promoter by Laufer(7). Although the shapes of the flow fields are quite different between computation and experiment, the computed profile of $\langle\langle\bar{u}_1\rangle\rangle$ near the straight wall by k- ϵ model has an almost identical tendency with the experimental one. It is shown in Figs.4 and 5 that, only in the

Table 3 Computation time

	Number of Grids	Integration Time per One Step (s)
k- ϵ Model	4000 (80×50)	0.73
LES	9000 (40×25×9)	5.6

(a) Section II

(b) Section III

Fig.4 Mean velocity profiles

Fig.5 Detailed profiles of mean velocity near straight wall

(a) Section II

(b) Section III

Fig.6 Turbulence energy

Fig.7 Turbulence energy near straight wall

neighborhood of the straight wall, the mean velocities predicted by LES and k- ϵ model agree qualitatively, but in the region, to which the influence of flow separation extends, the mean velocity profiles by k- ϵ model do not agree with those by LES.

4.3 Profiles of turbulence intensities

Figure 6 shows time mean profiles of k and Fig. 7 shows its enlargement in the neighborhood of the upper wall. These figures show that in the neighborhood of the upper wall the profiles by k- ϵ model are in good agreement with those by LES and also with Laufer's experiment(7). This is partially due to the adjustment of sectional averages $((k))_I$ and $((\epsilon))_I$ at section I. In the rest of the flow region, especially behind the promoter, profiles by k- ϵ model and by LES show different tendencies.

Figure 8 shows the profiles of turbulence shear stress. Turbulence shear stress in k- ϵ model is defined as follows:

$$\overline{u'_1 u'_2} = -\nu_T \left(\frac{\partial \overline{u_2}}{\partial x_1} + \frac{\partial \overline{u_1}}{\partial x_2} \right) \dots \dots \dots (12)$$

As seen in Fig. 8, the computed profiles by k- ϵ model are different from those by LES. Z in the figure denotes the point of zero turbulence shear stress in section III and does not coincide with the point M of maximum mean velocity (see Fig. 4). This means that the production of turbulence energy becomes negative between the two points M and Z . This kind of phenomenon is observed experimentally in the asymmetric flow field and is described in the detailed experiment of Hanjalic(11). LES seems to represent this negative production of turbulence

(a) Section II

(b) Section III

Fig. 8 Turbulence shear stress

energy, but in k- ϵ model both points coincide with each other.

4.4 Eddy viscosity ν_T and turbulence dissipation rate ϵ

Figures 9 and 10 show the profiles of ν_T calculated from Eq.(6) and of ϵ from Eq.(5). The molecular viscosity $\nu/(UL)$ as well as the local distribution of SGS eddy viscosities K in LES are also shown in Fig. 9 for reference. It should be noted that K denotes an eddy viscosity in the subgrid-scale flow field, but that ν_T contains a large-scale and a subgrid-scale turbulence. These figures indicate that ν_T and ϵ have larger values near the promoter and that ν_T is much larger than the molecular viscosity and K .

5. Experimental apparatus and procedure

(a) k- ϵ model

(b) LES

Fig. 9 Eddy viscosity

Fig. 10 Turbulence dissipation rate

In this study, the flow field is considered to be composed of two parallel walls, on the lower one of which turbulence promoters with a square cross section are installed infinitely and at equal intervals. Figure 11 shows the experimental apparatus. In the figure, the channel width ($L/2$) is 96 mm, the spanwise width of channel (b) is 600 mm, the length of one side of square promoter (d) is 30 mm, an interval of promoters (p) is 150 mm and twelve promoters are installed on one side of the walls. Coordinate axis x_1 is defined in streamwise direction, x_2 in the direction normal to upper wall and x_3 in spanwise direction of the square promoters. Parameters in experiments are shown in Table 4. The flow measurements are performed by setting a constant-temperature-type hot wire anemometer (DISA 55 M series) between seventh and eighth promoters. The I-type tungsten sensor is used, the diameter and the effective length of which are 5 μ m and 1.25 mm, respectively. The sensor is held parallel to the axis of the promoter and is traversed in the x_2 direction, as shown in Fig. 12. Referring to Fig. 13 and assuming that the velocity fluctuations are small as compared

with the velocity of a uniform flow, the relations between the linearized output signals e ($= \bar{e} + e'$) and the velocity components u_i ($= \bar{u}_i + u_i'$) are expressed as follows (12):

$$\bar{e}^2/G^2 = (\cos^2 \alpha + k_3^2 \sin^2 \alpha) \bar{u}_1^2 - (1 - k_3^2) \sin 2\alpha \times \bar{u}_1 \bar{u}_3 + (\sin^2 \alpha + k_3^2 \cos^2 \alpha) \bar{u}_3^2 + k_2^2 \bar{u}_2^2 \quad (13)$$

$$\bar{e}'^2/G^2 = (\cos^2 \alpha + k_3^2 \sin^2 \alpha) \bar{u}_1'^2 - (1 - k_3^2) \sin 2\alpha \times \bar{u}_1' \bar{u}_3' + (\sin^2 \alpha + k_3^2 \cos^2 \alpha) \bar{u}_3'^2 + k_2^2 \bar{u}_2'^2 \quad (14)$$

where α : angle between the $x_1 - x_2$ plane

Fig. 13 Velocity vector resolution

Table 4 Experimental parameters

Experiment	L (mm)	dxd2 (mm)	Number of Turbulence Promoters	Mean Velocity (m/s)	Re number
1	192	30x30	12	3.0	3.8×10^4
2	192	30x30	12	4.9	6.3×10^4
3	192	30x30	12	8.6	1.1×10^5
4	192	30x30	12	13.0	1.6×10^5
5	192	30x30	5	11.9	1.4×10^5

Fig. 11 Experimental apparatus

Fig. 12 Attitude of probe sensor

(a) Influence of pitch angle ($\alpha = 0$)

(b) Influence of yaw angle ($\theta = 0$)

Fig. 14 Output signals and inclined angle

Fig. 15 Directional sensitivity factor

and the velocity vector (yaw angle), θ : pitch angle, k_2 and k_3 : directional sensitivity factors(13) for pitch and yaw angles, respectively, G : linearizer gain. In this study, assuming that $\bar{u}_1 \gg \bar{u}_2, \bar{u}_3$, $k_2(\theta)$ at $\alpha = 0$ and $k_3(\alpha)$ at $\theta = 0$ are considered. Figures 14(a) and (b) show the calibration curves for finding $k_2(\theta)$ and $k_3(\alpha)$. The following relations between the ordinate $\bar{e}$ and k_2, k_3 are deduced from Eq.(13)

$$\left. \begin{aligned} \frac{\bar{e}_\theta}{\bar{e}_0} &= \sqrt{\cos^2 \theta + k_2^2 \sin^2 \theta} \\ \frac{\bar{e}_\alpha}{\bar{e}_0} &= \sqrt{\cos^2 \alpha + k_3^2 \sin^2 \alpha} \end{aligned} \right\} \dots\dots\dots (15)$$

, where $\bar{e}_0$ is a value of $\bar{e}$ at $\theta = \alpha = 0$. Figure 15 shows $k_2(\theta)$ and $k_3(\alpha)$ calculated from the calibration curves in Fig.14. It seems from the figure that k_2 is approximately unity for small value of θ and k_3 is nearly zero for small value of α . In such a case that $k_2 \approx 1$, $k_3 \approx 0$ and $\alpha = 0$, the output signals of the hotwire anemometer are reduced to the following equations

$$\left. \begin{aligned} \bar{e}^2/G^2 &= \bar{u}_1^2 + \bar{u}_2^2 \\ \bar{e}^2/G^2 &= \bar{u}_1^2 + \bar{u}_2^2 \end{aligned} \right\} \dots\dots\dots (16)$$

In the present experiment $\bar{e}$ and $\bar{e}^2$ are the average values of 50 measurements and the integration time of each measurement is 3.0 seconds. In addition, it is confirmed by the preparatory experiments that the two-dimensionality of flow is held in the x_3 direction and that the periodicity is kept between sixth and tenth promoters in the x_1 direction. The split-type fiber probe is used for the measurement of reverse flow field.

(a) Section II

(b) Section III

Fig.16 Comparison of mean velocity profiles

6. Comparison of computation results with measurements

Figure 16 shows the mean velocity profiles by computations and experiments. The ordinate is the vector sum of u_1 and u_2 , and corresponds to an output signal of the hot wire anemometer. The figure indicates that the computation results by LES coincide well with those of experiment, but that the difference between the computed profile by $k-\epsilon$ model and experimental one is considerable behind the turbulence promoter.

Figure 17 shows the comparison between the computed turbulence intensity and experimental one. In the figure, the relation of both results is almost the same as in the case of the mean velocity. Figures 16 and 17 suggest that LES has a predominant availability for predicting such a separated flow as in this study, compared with $k-\epsilon$ model.

7. Conclusions

A numerical prediction by $k-\epsilon$ model is applied to a two-dimensional channel flow with square-sectional turbulence promoters. The computation results are compared with those by LES and with those of wind tunnel experiment. The following results are obtained:

- (1) The computation results by $k-\epsilon$ model coincide with those of LES only in the neighborhood of a straight wall, but not in the region to which the influence of flow separation extends.
- (2) It is possible for LES to represent the negative production of turbulence energy in the asymmetric flow field and the large-scale vortex motion behind the promoter, but is impossible for $k-\epsilon$ model.
- (3) The computation results by LES coincide relatively well with those of the experiment, but the results predicted by $k-\epsilon$ model are different from the experimental results especially behind the promoter.
- (4) LES has a greater availability than $k-\epsilon$ model, for predicting a turbulent flow in this study.

Fig.17 Comparison of turbulence intensity

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The numerical computations have been performed using the HITAC M200/280 H system computer of the University of Tokyo.

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