Relation between Suspension and Dispersion of Coarse

Spherical Particles in Horizontal Pipe Flow*

By Kyokai OKUDA** and Hideaki YAMAGISHI***

In order to express the particle behaviour in a pipe flow, the vertical- and longitudinal-probability density functions of particles should be adopted. The method using those functions to express the particle behaviour in a pipe flow was already adopted by Binnie⁽¹⁾, Barnard⁽²⁾ and Sharp⁽³⁾. They supposed that the particle movements were Markov's process and that suspension and dispersion of particles were caused by turbulent diffusion of a pipe flow.

The authors have analysed and tested the particle movements on the basis of assumption that suspension and dispersion of particles have been caused mainly by lift of shear-flow and particle saltation at the pipe wall, especially with coarse particles, rather than by a turbulent diffusion of the pipe flow.

The test apparatus has a 22 m closed conduit pipe of 52 mm diameter with a transparent test section. The particles used are 6 mm diameter nylon spheres.

Key Words : Hydraulic Transport System, Pipe Flow, Liquid-Solid Flow, Suspension Flow, Multiphase Flow

1. Introduction

In 1950s to 1970s, the solid particle movements in the hydraulic transport pipe were investigated by Binnie, Barnard and Sharp using a model of coarse spherical particles in a carrier water of a pipe on the basis of assumptions that particle movements were Markov's process and that suspension and dispersion of particles were caused by a turbulent diffusion of the pipe flow.⁽¹⁾⁻⁽³⁾

For the latter assumption in the above the authors have supposed that suspension and dispersion of particles are caused mainly by lift of shear-flow and particle saltation at a pipe wall, especially with coarse particles, rather than by a turbulent diffusion of the pipe flow. The particle behaviour in a pipe is expressed by the vertical- and longitudinal-probability density functions of particle passages in a pipe, in the same way as adopted by the above authors. In the case of longitudinal-probability density functions, the particle behaviour is treated by Lagrangian method as used in fluid mechanics.

In this paper, the dispersion of coarse solid particles in a pipe flow and the relation between suspension and dispersion of particles are treated under the condition of low particle concentrations.

The test apparatus has a 22 m closed conduit pipe of 52 mm diameter with a transparent test section enclosed with a

square transparent pipe. The particles used are 6 mm diameter nylon spheres of 1.15 specific gravity.

Nomenclature

- A : area of horizontal layer with distance $\bar{y}$ from pipe center, (Fig. 2(b))
- a : pipe radius
- D : suspension coefficient of spheres (vertical diffusivity of spheres)
- D_0 : dimensionless suspension coefficient of spheres,
 $D_0 = \epsilon_1(\ell/\gamma) + \epsilon_2$
- f_1 : shear-lift coefficient of spheres
- f_2 : saltation coefficient of spheres
- i : subscript ordinal number of area of A
- K : longitudinal dispersion coefficient of spheres
- K_0 : dimensionless longitudinal dispersion coefficient of spheres
- $l = (z/a)^2 = a^2$
- N : total number of spheres passing through A
- n : number of spheres passing through A
- P : probability density of sphere passages in a pipe
- r_s : sphere radius
- s : gradient of probability density of sphere passages in a pipe (Fig. 3)
- t : time
- u : mean horizontal velocity of carrier water or spheres at horizontal layer A, (Fig. 2(b))
- U : mean velocity of carrier water throughout a pipe section
- V : terminal velocity of a sphere in water at rest
- x : coordinate of flow direction
- x_0 : dummy parameter (Fig. 5, Eqs. (15) and (16))
- $\hat{x} = x - Ut$
- y : vertical coordinate in a pipe cross-section

* Received 10th October, 1983.

** Professor, Muroran Institute of Technology, Mizumotocho, Muroran, Hokkaido.

*** Assistant, ibid.

$$\bar{y} = y/a$$

$$\alpha = u/a$$

$$\gamma = V/U$$

ϵ_1 = dimensionless factor of f_1

ϵ_2 = dimensionless factor of f_2

ϕ = probability density of spheres in vertical direction

ψ = probability density of spheres in flow direction

2. Analysis

The movement of relatively coarse spheres that are carried by water in a pipe is supposed to be Markov's process, then the probability densities of sphere center passages are described in the diffusion equation as follows:

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} - V \frac{\partial P}{\partial y} = K \frac{\partial^2 P}{\partial x^2} + D \frac{\partial^2 P}{\partial y^2} \quad (1)$$

where P is probability density of sphere center passages, u carrier water velocity, V terminal velocity of a sphere, K longitudinal dispersion coefficient of spheres and D suspension coefficient of spheres. If the concentration of spheres is very low, it is assumed that collisions of spheres in a pipe flow can be neglected and P is separable as follows:

$$P(x, y, t) = \psi(x, t)\phi(y) \quad (2)$$

Substitute Eq.(2) into Eq.(1) and put U , mean carrier water velocity, for u , then the following equations are obtained:

$$\phi \frac{\partial \psi}{\partial t} + U \phi \frac{\partial \psi}{\partial x} - K \phi \frac{\partial^2 \psi}{\partial x^2} = V \psi \frac{d\phi}{dy} + D \psi \frac{d^2 \phi}{dy^2} = k \quad (3)$$

Supposing $k=0$, Eq.(3) would be reduced to the following two equations:

$$V \frac{d\phi}{dy} + D \frac{d^2 \phi}{dy^2} = 0 \quad (4)$$

$$\frac{\partial \psi}{\partial t} + U \frac{\partial \psi}{\partial x} - K \frac{\partial^2 \psi}{\partial x^2} = 0 \quad (5)$$

where Eq.(4) expresses the suspension of spheres (in the vertical direction) in a pipe cross-section and Eq.(5) the dispersion of spheres in the flow direction.

Following the analysis of the preceding paper of the authors, the solution of Eq.(4) was derived as follows (cf. Appendix):

$$\phi(\bar{y}) = \phi_0 \exp(-\bar{y}/D_0) \quad (6)$$

where the dimensionless suspension coefficient of spheres is

$$D_0 = \epsilon_1(\ell/\gamma) + \epsilon_2 \quad (7)$$

By a normal solving method for differential equations, the solution of Eq.(5) is obtained as follows:

$$\psi(x, t) = \frac{1}{\sqrt{4\pi Kt}} \exp\left\{-\frac{(x-Ut)^2}{4Kt}\right\} \quad (8)$$

Set the coordinate origin that is moving with velocity U together with carrier water, at the middle of the sphere cloud, that is

$$\bar{x} = x - Ut \quad (9)$$

then Eq.(8) is reduced to

$$\psi(\bar{x}, t) = \frac{1}{\sqrt{4\pi Kt}} \exp\left(-\frac{\bar{x}^2}{4Kt}\right) \quad (10)$$

Equation (10) describes the probability density of spheres according to $\bar{x}$ of which origin is moving with velocity of carrier water U .

After obtaining logarithms of both sides of Eq.(10) and rearranging, $d\bar{x}/dt$ is given as follows:

$$\frac{d\bar{x}}{dt} = -\frac{K(B + \ln K)}{\bar{x}} \quad (11)$$

where

$$B = 1 + \ln\{(n(\bar{x}, t)/N)^2 \cdot 4\pi t\} \quad (12)$$

$$\psi(\bar{x}, t) = n(\bar{x}, t)/N \quad (13)$$

Put $d\bar{x}/dt = c$, then we get

$$B + \ln K = -c\bar{x}/K \quad (14)$$

In the above equation, introducing the dummy parameter x_0 , we get

$$x_0 = \ln K + B \quad (15)$$

$$x_0 = -c\bar{x}/K \quad (16)$$

By graphic solution method, the value of K at the point at which two locus lines of Eqs.(15) and (16) intersect is a desired one of K .

3. Experimental Apparatus and Method

General view of experimental apparatus is shown in Fig.1. The apparatus consists of a horizontal, 22 m long, closed conduit pipe of diameter 52 mm which is made of transparent methacryl plastics. The particles used are 6 mm diameter nylon spheres of 1.15 specific gravity. The sphere feeder is provided near the suction pipe of the pump, which is "Vortex Pump" designed specially for non-breaking of particles in the pump casing. A Venturi tube is provided near the first corner of conduit for measuring the flow rate of a water-particle mixture. The test-section is located 9.5 m distant from the second

- ① Pump
- ② Venturi tube
- ③ Metacryl pipe
- ④ Light source
- ⑤ Square pipe
- ⑥ Photoelectric transducers

Fig. 1 Experimental apparatus

corner of conduit, and is free from the effect of the second corner vortex in pipe flow. The pipe of test-section is enclosed with a square, transparent pipe and filled with water between them to avoid the distortion of particle images for horizontal, parallel light beams, as shown in Fig.2(a). The light source is set at one side of the test-section and photoelectric transducers are provided across a slit box at the opposite side of the test-section, as shown in the figure.

We use the apparatus for observation of sphere suspension, and count the total number of spheres passing through each of four horizontal hatched layers, decreasing the light to the transducers, as shown in Fig.2(a). For observation of sphere dispersion (in flow direction), we set many transducers close to each other vertically at the slit box and are able to catch the sphere images passing through any horizontal layer. We repeat the test by circulating the mixture many times in the closed conduit apparatus. A "Personal Computer" with A/D converter is used for data-sampling, storing and processing of the varying voltages from the photoelectric transducers with sampling time intervals from 1,000 to 3,000 Hz.

Fig. 2 (a) Test section apparatus

Fig. 2 (b) Test section

Fig. 3 Probability density gradients of spheres

4. Experimental Results and Discussion

4.1 Dimensionless suspension coefficient D_0

We count the number of spheres passing through each hatched layer, obtain the distribution of $n(\bar{y}_i)$ and calculate the gradients of probability density of spheres by the method as shown in Ref.(5), with parameter of mean water velocity U . The calculated gradients are shown in Fig.3. Mean water velocities are chosen in four steps: — 0.75, 1.04, 1.36, and 1.62 m/s. Reynolds numbers of the flow are in the range from 3.3×10^4 to 7.3×10^4 , and we suppose that velocity distributions across the pipe section may be approximately those of one-seventh power law for turbulent pipe flow. This law is applied to calculate the velocities of water passing through the horizontal, hatched layers as shown in Fig.2(b) and those of spheres passing through the same layers. We suppose that the velocities of spheres would be approximately equal to those of water in each layer. The spherical particles used have the terminal velocity $V = 0.14$ m/s in water at rest. In the experiment, the concentrations of particles in volume are less than 0.2 %, so we have confirmed that mutual collisions of particles may be few, compared with the test results about a single sphere movements in a pipe flow.

The probability densities of spheres, shown in Fig.3, are calculated from the following equations:

$$N = \int_{-1}^{1-a} n(\bar{y}) d\bar{y} = N \int_{-1}^{1-a} \phi(\bar{y}) A(\bar{y}) u(\bar{y}) d\bar{y} \\ \approx N \sum_{i=1}^4 \phi(\bar{y}_i) A(\bar{y}_i) u(\bar{y}_i) \quad (17)$$

where the values of $\phi(\bar{y}_i)$ are normalized as

$$\sum_{i=1}^4 \phi(\bar{y}_i) A(\bar{y}_i) = 1 \quad (18)$$

Table 1 Specifications and results of tests

layer	1	2	3	4	
$\bar{y}_i$	0.789	0.365	-0.058	-0.48	
$A(\bar{y}_i)$ mm ²	1599	2420	2596	2280	
$u(\bar{y}_i)$	$\frac{U}{U_{sec}}$	0.75	0.711	0.800	0.839
		1.04	0.966	1.087	1.141
		1.36	1.211	1.362	1.430
		1.62	1.529	1.721	1.806
$n(\bar{y}_i)$	$\frac{n}{U_{sec}}$	0.75	1985	35	1
		1.04	1957	150	6
		1.36	1707	282	20
		1.62	1458	538	56
$\phi(\bar{y}_i)$	$\frac{\phi}{U_{sec}}$	0.75	1600	0.017	4.2×10^{-4}
		1.04	1519	0.068	2.5×10^{-3}
		1.36	1393	0.144	9.1×10^{-3}
		1.62	1191	0.258	0.024
U	0.75	1.04	1.36	1.62	
L/γ	0.075	0.104	0.136	0.162	
D_0	0.102	0.132	0.161	0.214	
$D \times 10^3$	1.31	1.78	2.21	2.79	
ϵ_1	1.202				
ϵ_2	7.69×10^{-3}				

Table 1 shows the test specifications and the calculated results. Dimensionless suspension coefficients D_s are calculated from Eq.(6) with the values of probability density gradients shown in Fig.3. The relation between D_s and $1/\gamma$ is shown in Fig.4, which suggests that they may be in linear correlation, as implied in Eq.(7). Though the authors' assumption of Eq.(7) is a simple model of the sphere suspension flow, it is confirmed that the assumption is appropriate (cf.Ref.(5)).

4.2 Dimensionless dispersion coefficient K_s

The observation for the sphere dispersion in flow direction is carried out in such manner that a sphere cloud (number of spheres used is 150) is transported in the conduit and observed for dispersed distances of the spheres when the cloud is forced to circulate within the closed conduit at the first round ($x_1 = 20.42$ m), at the second round ($x_2 = 42.16$ m) and at the third round ($x_3 = 63.90$ m) respectively. The tests are repeated 15 to 20 times. Table 2 shows the test specifications and the calculated results about the sphere dispersion tests. In the table, the column of the first, the second and the third round gives the results about one example of the tests, and the other columns give mean values of all tests. The columns of $2\hat{x}_{d1}$, $2\hat{x}_{d2}$ and $2\hat{x}_{d3}$ show the dispersed distances of the sphere cloud from the leading sphere to the trailing one, observed in the paths of x_1 , x_2 and x_3 respectively. The distances of $2(\hat{x}_2 - \hat{x}_1)$ and $2(\hat{x}_3 - \hat{x}_2)$ are the dispersed ones of sphere clouds travelling the same paths of 21.74 m from x_1 to x_2 and from x_2 to x_3 respectively.

The velocity of any particle departing from the center of the particle cloud may be expressed as follows :

$$c = d\hat{x}/dt \approx (\hat{x}_{i+1} - \hat{x}_i)/\Delta t \quad (19)$$

where subscript $i = 1, 2$. Equation (19) may be applied to any particle in the cloud. The leading or trailing sphere in the cloud can be distinguished from the others in observation. Putting subscript "d" to these spheres, then Eq.(19) reads

$$c_d = d\hat{x}_d/dt \approx (\hat{x}_{d,i+1} - \hat{x}_{d,i})/\Delta t \quad (20)$$

and Eq.(16) is reduced to

$$x_0 = -c_d \hat{x}_d / K \quad (21)$$

The value of B in Eq.(15) and that of c_d in Eq.(21) are obtainable from the experiments. Let ordinate be x_0 and abscissa K , then the value of the point at which two lines expressing Eq.(15) and Eq.(21) intersect is the desired one of K , as shown in Fig.5, that is, the graphic solution for K is demonstrated, for example. In Fig.6, the relation between obtained K and velocity of carrier water U is illustrated. The figure shows that the value of K may be greater as the velocity U increases.

To non-dimensionalize the dispersion

coefficient of spheres K_s , the authors choose the velocity gradient of a pipe flow (U/a) and the sectional area of a pipe (πa^2) as the factors affecting K_s . Then K_s , which has the dimension of (m^2/s), divided by the product of U/a and πa^2 may be non-dimensionalized as follows :

$$K_0 = K / \{ (U/a) (\pi a^2) \} = K / (\pi a U) \quad (22)$$

Fig. 4 Relation between D_s and $1/\gamma$

Table 2 Specifications and results of dispersion tests

U m/sec		0.75	1.04	1.36	1.62
first	X_1	20.42 m			
round	t_1	27.2	19.6	15.0	12.6
second	X_2	42.16 m			
round	t_2	56.2	40.5	31.0	26.0
third	X_3	63.90 m			
round	t_3	85.2	61.4	47.0	39.4
$2\hat{x}_d$	$2\hat{x}_{d1}$	4.412	4.772	4.108	4.200
	$2\hat{x}_{d2}$	5.137	5.280	4.639	4.764
	$2\hat{x}_{d3}$	5.868	6.008	5.230	5.302
$2(\hat{x}_2 - \hat{x}_1)$		0.725	0.507	0.531	0.565
$2(\hat{x}_3 - \hat{x}_2)$		0.734	0.728	0.589	0.455
$2d\hat{x}_d/dt_1$		0.025	0.024	0.033	0.042
$2d\hat{x}_d/dt_2$		0.025	0.035	0.037	0.040
K_1		0.056	0.052	0.069	0.084
K_2		0.056	0.075	0.075	0.078
K		0.056	0.063	0.072	0.081
K_0		1.433	1.167	1.018	0.960

(t_1 , t_2 and t_3 in sec)

Fig. 5 Graphic solution for K

Fig. 6 Relation between K and U

Therefore, K_0 is to be understood as the dispersion coefficient of spheres per unity of pipe radius a and per unity of mean water velocity U . The relation between K_0 in ordinate and U in abscissa is shown in Fig. 7, which indicates that K_0 decreases apparently as U increases, that is, the higher the water velocity U becomes, the less dispersedly the particle cloud may be transported over a certain distance of a pipe.

4.3 Relation between suspension coefficient D_0 and dispersion coefficient K_0

In the preceding two sections, the relations of $D_0 - U$ and $K_0 - U$ have been clarified as illustrated. From the results, the relation between D_0 and K_0 is clear immediately as shown in Fig. 8. The relation can be expressed as $D_0 = n_1 K_0^{n_2}$, which is obtained by the method of least squares concerning n_1 and n_2 from the test results as follows:

$$D_0 = 0.54 K_0^{-1.69} \quad (23)$$

The above equation shows quantitatively that the higher the water velocity U becomes, the lower K_0 becomes, as shown in Fig. 7, and then the more D_0 becomes (the more suspension of spheres increases), therefore, the particle clouds are transported more effectively taking the shape of a particle group.

The authors have reported that the

Fig. 7 Relation between K_0 and U

Fig. 8 Relation between D_0 and K_0

Fig. 9 Relation between D and K

higher the water velocity U became, the less the velocity difference between carrier water and particles became, and then the velocity ratio of particles to water approached unity beyond a certain velocity range of water (Ref. (6)). In this respect, the water velocity has to be high, but the frictional loss of head of a conduit pipe increases proportionally to U^2 . This implies that there may exist an optimum water velocity to transport the solid particles in a pipe.

4.4 Diffusion equation

In the last section of this paper, we return to the diffusion equation, Eq. (1), and investigate it further briefly. The relation between the suspension coefficient of spheres with dimensions D and the dispersion coefficient K is shown in Fig. 9. The relation is also formulated from the test data as follows:

$$D = -0.00187 + 0.0573K \quad (24)$$

From the above equation, either D or K is given, and the other may be obtained immediately.

The solution of Eq. (1) can be expressed by Eqs. (6) and (8) considering Eq. (2) as follows:

$$P(x, y, t) = \phi_0 \exp\left(-\frac{y}{D_0}\right) \times \frac{1}{\sqrt{4\pi Kt}} \exp\left\{-\frac{(x - Ut)^2}{4Kt}\right\} \quad (25)$$

When particles are introduced into a conduit pipe, the probability density of the sphere existence of x - and y -coordinates in a pipe, depending on time is predictable from the above equation.

5. Conclusions

The behaviour of solid particles in the hydraulic transport pipe is treated by a statistical method when the particles are comparatively coarse so that the behaviour of particles would not be treated in terms of turbulent diffusion of a pipe flow. The data obtained in these experiments would involve naturally statistical errors which would be inevitable. Of course, we try to take as many data as possible.

Under the condition that the concentration of particles is low and the mutual collisions of spheres are so few as can be neglected, the following results are obtained:

(1) The relations are established between the dispersion coefficient of spheres K and the carrier water velocity U and between the dimensionless coefficient of spheres K_0 and U_0 .

(2) Linear correlation holds between the suspension coefficient of spheres D and the dispersion coefficient K .

(3) Between non-dimensionalized coefficients for D and K , the following expression is obtained: $D_0 = 0.54 K_0^{-1.69}$.

According to the above expression, a well-suspended sphere cloud would be transported in a less-dispersed group than in a more settled sphere cloud.

6. Acknowledgements

The authors would like to acknowledge the assistance of Mr. K. Hikita and T. Hoshino who worked on these experiments, as students, at the authors' laboratory.

7. References

- (1) Binnie, A. M. and Phillips, O. M., J. Fluid Mech., Vol.4, No.1, (1958), p.87.
- (2) Barnard, B. J. S. and Binnie, A.M., J. Fluid Mech., Vol.15, No.1, (1963), p.34.
- (3) Sharp, B. B. and O'Neil, I. C., J. Fluid Mech., Vol.45, No.3, (1971), p.575.
- (4) Yamagishi, H. and Okuda, K., Trans. Japan Soc. Mech. Engrs., (in Japanese), Vol.45, No.393, (1979), p.644.
- (5) Okuda, K., Proc. Hydrotransport 7, 7th Int. Conf. Hydraul. Transport Solids Pipes, Sendai, Japan, (1980), Cranfield, U. K., BHRA Fluid Eng., Paper G3, p.291.
- (6) Okuda, K., J. Pipelines, Vol.1, No.3, (1981), p.211.

Appendix : Solution of Eq.(4)

The solution of Eq.(4) is reproduced briefly from Ref.(5) of the authors as follows :

Integrating Eq.(4), we obtain the following equation :

$$\phi = -\frac{D}{V} \frac{d\phi}{dy} \quad (\text{a.1})$$

where ϕ depends on y only.

It is assumed in general that discrete particles of spheres in a horizontal water pipe would be carried and subjected to the following five kinds of forces :

- (1) Gravity
- (2) Lift of shear flow (shear-lift)
- (3) Effect of sphere-saltation at the pipe wall
- (4) Mutual collision impact of spheres
- (5) Turbulent diffusion of a pipe flow.

In the case of this flow model, the forces (4) and (5) would be neglected since concentration of spheres is very low and the spheres are perhaps larger than the scales of turbulence in a pipe flow. The force (1) acts downwards, the forces (2) and (3) act upwards on the spheres.

The authors define D in Eq.(a.1) as "suspension coefficient" and introduce the following equation under the above assumptions :

$$D = f_1(\epsilon_1, a, r_0, U) + f_2(\epsilon_2, a, V) \quad (\text{a.2})$$

where f_1 is defined as shear-lift coefficient of spheres, f_2 as saltation coefficient of spheres against pipe wall.

These are supposed to be given as the following formulae :

$$f_1 = \epsilon_1 \frac{r_0^2 U}{a} \quad (\text{a.3})$$

$$f_2 = \epsilon_2 a V \quad (\text{a.4})$$

where ϵ_1, ϵ_2 are dimensionless factors and U is mean velocity of carrier water, V terminal velocity of a sphere in water at rest, r_0 sphere radius and a pipe radius, respectively. Substitute Eqs.(a.3) and (a.4) into Eq.(a.2), then we obtain

$$D = \epsilon_1 \frac{r_0^2}{a} U + \epsilon_2 a V \quad (\text{a.5})$$

Furthermore, substitute Eq.(a.5) into Eq.(a.1) and integrate the equation, changing vertical coordinate to dimensionless term by putting $\bar{y} = y/a$, then we have

$$\phi = \phi_0 \exp [-(\epsilon_1(\ell/\gamma) + \epsilon_2)^{-1} \bar{y}] = \phi_0 \exp \left(-\frac{\bar{y}}{D_0} \right) \quad (6)$$

$$\text{where } D_0 = \epsilon_1(\ell/\gamma) + \epsilon_2 \quad (7)$$

$$\ell = (r_0/a)^2 \quad \text{and} \quad \gamma = V/U \quad (\text{a.6})$$