Study on an Oil Damper with Variable

Damping Mechanism*

(1st Report, Damping Characteristics of the Damper
with a Piston Having Cylindrical Orifices)

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The purpose of this work is how to design the variable oil damper whose damping is adjustable by varying the area of orifice in the piston. In this report, we deal with an oil damper having a few cylindrical orifices in order to study the fundamental characteristics of the variable oil damper. Two new method for analyze the damper are proposed. The first method of analysis is based on the assumption that the diameter of the piston is equal to the inner diameter of the cylinder but no friction occurs between the contact surfaces. However the assumption is far from being satisfactory in the actual oil damper. The second can be applied to the oil damper whose piston is smaller than the cylinder (in other words, the oil flows through the annular clearance and the cylindrical orifices). In these analyses, the flow through the channels is assumed to be incompressible, unsteady and laminar. Consequently, it is made clear that the magnitude of the damping in the latter type of oil damper is heavily subject to variation of the frequency of the oscillating piston.

Key Words : Vibration, Vibration Isolation, Oil Damper, Fluid Vibration, Variable Damping, Cylindrical Orifice

1. Introduction

It is drawing special interest recently that damping plays a great part in isolation of mechanical vibrations, and dampers of varied types are proposed by many authors. It goes without saying that these dampers produce a desirable degree of damping. Moreover, it is earnestly hoped that they are able to change the damping freely according to operating condition of machines. Under the title "Fundamental Investigation on an Oil Damper", we have been researching on a simple structured oil damper which has an annular clearance between the cylinder and the piston as a flow channel of oil.⁽¹⁾⁻⁽⁴⁾ As the result of this study, we could derive an equation for calculating the damping precisely under the assumption that the flow through the annular channel is a one-dimensional and unsteady flow. In this simple type of oil damper, however, the magnitude of damping cannot be changed in a simple manner. Therefore we propose a new oil damper with variable damping mechanism by changing the area of holes through the

piston. We name it a variable oil damper. This report deals with the damper having a few cylindrical orifices of same diameter in the piston in order to make clear the fundamental characteristics of the variable oil damper.

Two general approaches to the analysis of this oil damper are discussed. One is based on the assumption that the clearance between the cylinder and the piston is too small to pass the oil; and that no friction occurs between the sliding surfaces. In practice, however, there is a frictional force on the surfaces and it is difficult to predict the magnitude of the force. Another approach is to consider the oil flows through the clearance and the circular holes. This approach teaches that the oil damper having two different forms of channels is more susceptible to the frequency than the one having only one channel either annular or circular. Therefore the analysis based on the unsteady flow assumption proves most effective for this type of oil damper.

Comparing the theoretical value with the experimental one, we found that they agree well with each other. Hence we were ready to give a theoretical foundation to the variable oil damper.

2. Theoretical Analysis of an Oil Damper Having a Circular Channel

In this chapter, we discuss an oil damper in which the oil flows not through the clearance between the cylinder and the

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Table 1 Theoretical analysis of an oil damper having only a circular channel

Item	Steady flow	Unsteady flow
Equation of continuity	$Ac v_{crms} - Ap v_{prms} = 2n\pi \int_0^r v(r) dr$ [1.1]	$Ac v_c \cos(\omega t - \phi_1) - Ap v_p \cos(\omega t - \phi_2) = 2n\pi \int_0^r v(r, t) dr$ [1.2]
Equation of motion	$\frac{dp}{dy} = \mu \left(\frac{d^2 v}{dr^2} + \frac{1}{r} \frac{dv}{dr} \right)$ [1.3]	$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right)$ [1.4]
Boundary condition	$v(0) = \text{finite}, v(r) = v_{prms}$ [1.5]	$v(0, t) = \text{finite}, v(r, t) = v_p \cos(\omega t - \phi_2)$ [1.6]
Solution	$v(r) = 2(A+1)[(r/r_1)^2 - 1] v_{prms}$ [1.7]	$v(r, t) = E_1 v_p \cos(\omega t - \phi_2) + E_2 v_p \sin(\omega t - \phi_2) + (E_1 - 1) P v_r \sin(\omega t + \epsilon) - E_2 P v_r \cos(\omega t + \epsilon)$ [1.8]
Pressure drag	$D_{ps2} = 8n\pi\mu l(A+1) v_{prms}$ $D_{ps1} = -8n\pi\mu l(A+1)^2 v_{prms}$ [1.9]	$D_{p2}(t) = 2n\pi\mu l \Omega^2 A v_r P \cos(\omega t + \epsilon)$ $D_{p1}(t) = -2n\pi\mu l \Omega^2 (A+1) v_r P \cos(\omega t + \epsilon)$ [1.10]
Friction drag	$D_{fs2} = 8n\pi\mu l(A+1) v_{prms}$ $D_{fs1} = 0$ [1.11]	$D_{f2}(t) = 2n\pi\mu l v_r F \cos(\omega t + \delta)$ $D_{f1}(t) = 0$ [1.12]
Damping coefficient	$C_s = 8n\pi\mu l(A+1)^2$ [1.13]	$c = 2n\pi\mu l \Omega^2 (A+1) P_1$ [1.14]
Added mass	$m_{As} = \frac{m_0}{3}(4A^2 + 2A + 1)$ [1.15]	$m_A = m_0[(A+1)P_2 - A] : m_0 = n\pi r_1^2 \rho l$ [1.16]

$P \cos \epsilon = \frac{v_p}{v_r} (P_2 \sin \phi_2 + P_1 \cos \phi_2) - \frac{v_c}{v_r} [(P_2 + 1) \sin \phi_1 + P_1 \cos \phi_1]$ $P \sin \epsilon = \frac{v_p}{v_r} (P_2 \cos \phi_2 - P_1 \sin \phi_2) - \frac{v_c}{v_r} [(P_2 + 1) \cos \phi_1 - P_1 \sin \phi_1]$ $F \cos \delta = \frac{v_p}{v_r} (F_2 \sin \phi_2 + F_1 \cos \phi_2) - \frac{v_c}{v_r} (F_2 \sin \phi_1 + F_1 \cos \phi_1)$ $F \sin \delta = \frac{v_p}{v_r} (F_2 \cos \phi_2 - F_1 \sin \phi_2) - \frac{v_c}{v_r} (F_2 \cos \phi_1 - F_1 \sin \phi_1)$ [1.17]	$P_1 = \frac{(A+1)B_1}{B_1^2 + (B_2+1)^2}$ $P_2 = \frac{(A+1)(B_2+1)}{B_1^2 + (B_2+1)^2} - 1$ $F_1 = \Omega^2 P_1$ $F_2 = \Omega^2 (P_2 - A)$ [1.18]
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$E_1 = \frac{\text{ber} \eta_1 \text{ber} \eta + \text{bei} \eta_1 \text{bei} \eta}{\text{ber}^2 \eta_1 + \text{bei}^2 \eta_1}, E_2 = \frac{\text{ber} \eta_1 \text{ber} \eta - \text{ber} \eta_1 \text{bei} \eta}{\text{ber}^2 \eta_1 + \text{bei}^2 \eta_1}$ $B_1 = \frac{2}{\eta_1} \frac{\text{ber} \eta_1 \text{ber} \eta + \text{bei} \eta_1 \text{bei} \eta}{\text{ber}^2 \eta_1 + \text{bei}^2 \eta_1} = \frac{2}{\eta_1} \frac{dE_1}{d\eta} = \frac{2}{\eta_1} \int_0^\eta E_2 d\eta$ $B_2 = \frac{2}{\eta_1} \frac{\text{ber} \eta_1 \text{ber} \eta - \text{ber} \eta_1 \text{bei} \eta}{\text{ber}^2 \eta_1 + \text{bei}^2 \eta_1} = \frac{2}{\eta_1} \frac{dE_2}{d\eta} = -\frac{2}{\eta_1} \int_0^\eta E_1 d\eta$ [1.19]
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$r_p, A_p = \pi r_p^2 - n\pi r_1^2, l$: Radius, area and length of piston
 $r_c, A_c = \pi r_c^2$: Radius and area of cylinder
 $r_1, A_0 = n\pi r_1^2, n$: Radius, area and number of circular holes
 ρ, μ, ν : Density, viscosity and kinematic viscosity of fluid
 $v_{prms} = v_{prms} - v_{crms}, v_r = \sqrt{v_p^2 + v_c^2} - 2v_p v_c \cos(\phi_1 - \phi_2)$
 $A = A_p/A_0, \eta = r\sqrt{\omega/\nu}, \eta_1 = r_1\sqrt{\omega/\nu}, \Omega = \eta_1/\sqrt{2}$ [1.20]

piston but through the cylindrical orifices of same diameter in the piston. The analytical procedure and result are summarized in Table 1. A cross-sectional view of the oil damper is shown in [1.21] in Table 1. Assuming that the radii r_c and r_p of the cylinder and the piston are equal but no friction occurs between their walls, and that the number of circular holes is n and they are of the same radius r_1 , we have then the area of the flow channel $A_0 = n\pi r_1^2$ and the piston area $A_p = \pi r_p^2 - n\pi r_1^2$. Let the ratio A_p/A_0 be A , then the ratio of the cylinder area A_c to A_0 is represented as $A+1$.

Suppose the motion of a piston and a cylinder are $v_p \cos(\omega t - \phi_1)$ and $v_c \cos(\omega t - \phi_2)$ respectively, and the flow through the circular channel is incompressible and one-dimensional. Then the equation of motion of the fluid is represented as a partial differential equation [1.4] in Table 1 by using the cylindrical coordinates r - y whose origin is located at the center of the channel. Since the velocity of flow, v , is dependent on time t , the analysis is named *unsteady flow analysis*. On the other hand, if the piston and the cylinder move at constant velocities v_{prms} and v_{crms} respectively, the equation of motion is expressed as [1.3]. Since the velocity v is independent of t , the analysis is named *steady flow analysis*.

Fig.1 Two-degree-of-freedom system equipped with an oil damper

Fig.2 Effect of unsteadiness on damping coefficient of the oil damper

Fig.3 Effect of unsteadiness on added mass of the oil damper

To solve the partial differential equation [1.4], we assume as a tentative solution the product $v(r, t) = X(r)e^{i\omega t}$ and a uniform pressure gradient with phase difference $\partial p/\partial y = (\Delta P_r/l)e^{i(\omega t + \epsilon)}$. Hence the equation is transformed into an ordinary differential equation

$$\frac{d^2 X}{dr^2} + \frac{1}{r} \frac{dX}{dr} - i \frac{\omega}{\nu} X = \frac{\Delta P_r}{\mu l} e^{i\epsilon} \dots \dots \dots (1)$$

where ΔP_r is a pressure difference between the front and the rear walls of the piston caused by fluid friction in the channel, l is the length of the piston, and μ and ν are viscosity and kinematic viscosity of the fluid, respectively. Using the Kelvin function, a general solution of Eq.(1) can be written as

$$X(r) = A_1(\text{ber} \eta + i \text{bei} \eta) + A_2(\text{ker} \eta + i \text{kei} \eta) + i P v_r e^{i\epsilon} \dots \dots \dots (2)$$

In this equation, $P \equiv \Delta P_r/(\omega \rho l v_r)$ means a dimensionless pressure gradient and $\eta = r\sqrt{\omega/\nu}$ a dimensionless frequency, where ω is circular frequency, ρ is density of the fluid and v_r is relative velocity of the piston and the cylinder. The integration constants A_1 and A_2 are determined by the boundary condition [1.6] in Table 1. Finally, taking the real part from the solution $v(r, t)$

expressed in complex notation, the solution [1.8] in Table 1 is obtained. The value of P is decided by an equation of continuity [1.2], which means that the amount of flow through the circular channel equals the volume excluded by the motions of the piston and the cylinder. The result is shown in [1.17] and [1.18] of Table 1.

The pressure loss ΔP_r caused by the fluid friction within the channel acts on the piston and the cylinder as pressure drags $D_{p2}(t)$ and $D_{p1}(t)$, respectively. These can be calculated as Eq.[1.10] in Table 1 by using the dimensionless pressure gradient P . In addition to the pressure drag, the piston receives a friction drag $D_{f2}(t)$ due to the shearing stress acting on the wall of the circular channel, whereas the cylinder is free from the friction drag ($D_{f1}(t) = 0$). Putting a dimensionless velocity gradient $(r_1/v_r)\partial v/\partial r|_{r=r_1} = (\eta_1/v_r)\partial v/\partial \eta|_{\eta=\eta_1}$ as $F \cos(\omega t + \delta)$, $D_{f2}(t)$ is expressed as Eq.[1.12], where $\eta_1 = r_1 \sqrt{\omega/\nu}$. We shall consider the drag forces positive if they act in the direction of flow.

In Eq.[1.10], the symbol β refers to $\eta_1/\sqrt{2} = r_1 \sqrt{\omega/(2\nu)}$; ⁽⁵⁾ it corresponds to $\lambda = h \sqrt{\omega/(2\nu)} = (r_c - r_p) \sqrt{\omega/(2\nu)}$, which is the dimensionless frequency used for the oil damper having an annular clearance as a flow channel of oil. The reason is that wetted perimeters are represented as $\pi(r_c^2 - r_p^2)/[2\pi(r_c + r_p)] = (r_c - r_p)/2 = h/2$ in the annular channel and $\pi r_1^2/(2\pi r_1) = r_1/2$ in the circular channel, so h is equivalent to r_1 .

The pressure and the friction drags in the steady flow analysis are given by Eqs. [1.9] and [1.11]; these will carry the subscript s in order to distinguish them from the drags [1.10] and [1.12] obtained by the unsteady flow analysis. There is the following connection between the drag forces;

$$D_{ps2} + D_{ps1} + D_{fs2} + D_{fs1} = 0 \quad (3)$$

$$m_o[A\ddot{x}_2 - (A+1)\ddot{x}_1] = -[D_{p2}(t) + D_{p1}(t) + D_{f2}(t) + D_{f1}(t)] \quad (4)$$

$$m_o = n\pi r_1^2 \rho l \quad (5)$$

The right-hand side of Eq.(4) means a total external force exerted on the fluid. In the left-hand side, m_o is the mass of a fluid within the channel, and $\ddot{x}_2 = -\omega v_p \sin(\omega t - \phi_2)$ and $\ddot{x}_1 = -\omega v_c \sin(\omega t - \phi_1)$ are accelerations of the piston and the cylinder, respectively, so the terms as a whole in the brackets mean acceleration of the fluid. Therefore Eq.(4) is a representation of Newton's second law of motion applied to the fluid in the channel regarded as a rigid body. Equation (3) means that when a fluid flows at a constant velocity, the external force acting on the fluid becomes zero.

There are two general methods for obtaining the damping coefficient and the added mass of the oil damper from the analytical results in Table 1. These are named the force method and the energy method, and they produce the same result. In the case of unsteady flow analysis, the force method is easier than the energy method.

On the assumption that the oil damper is applied to a system with two degrees of freedom, we shall consider a vibratory model shown in Fig.1. Putting $v_c = a_1 \omega$ and $v_p = a_2 \omega$, this model does not contradict the equation

of continuity [1.2] and the boundary condition [1.6] in Table 1. From a balance of forces acting on the bodies of masses m_2 and m_1 , the following equations of motion are derived:

$$m_2 \ddot{x}_2 + D_{p2}(t) + D_{f2}(t) + k(x_2 - x_1) = 0 \quad (6)$$

$$m_1 \ddot{x}_1 + D_{p1}(t) + D_{f1}(t) + k(x_1 - x_2) = 0 \quad (7)$$

Substituting Eqs.[1.10] and [1.12] of Table 1 into drag forces of these equations, applying the relationships $\dot{x}_2 = v_p \cos(\omega t - \phi_2)$ and $\dot{x}_1 = -\omega v_c \sin(\omega t - \phi_1)$, etc., and correcting the terms, we obtain

$$m_2 \ddot{x}_2 + m_A \ddot{x}_2 - (m_A + A m_o) \ddot{x}_1 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0 \quad (8)$$

$$m_1 \ddot{x}_1 + [m_A + (2A+1)m_o] \ddot{x}_1 - (m_A + A m_o) \ddot{x}_2 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) = 0 \quad (9)$$

$$m_A = m_o[(A+1)P_2 - A] \quad (10)$$

$$c = 2n\pi\mu l \beta^2 (A+1)P_1 \quad (11)$$

Equations (10) and (11) mean an added mass and the damping coefficient of the oil damper.

Damping coefficient c_s in the steady flow analysis can be obtained by using an equation $c_s = (D_{ps2} + D_{fs2})/v_{rms}$. Another method of obtaining c_s is to use the following dissipation function:

$$D_s = \frac{1}{2} n\pi\mu l \int_0^{r_1} 2\pi r \left(\frac{dv}{dr} \right)^2 dr = \frac{1}{2} c_s v_{rms}^2 \quad (12)$$

The result is shown in Eq.[1.13] of Table 1. The ratio of the damping coefficients

$$c/c_s = (\beta^2/4)B_1/[B_1^2 + (B_2+1)^2] \quad (13)$$

is a function of β alone. Figure 2 shows a plot of the ratio c/c_s against β . It may be seen from this figure that the damping coefficient increases with β , and for a sufficiently large value of β the ratio c/c_s approaches a straight line $\beta/4$ asymptotically. Furthermore the curve, as a whole, shifts to the left-hand direction as compared with the case of the oil damper having an annular channel. This means that the circular channel is more susceptible to the frequency or the degree of unsteadiness than the annular channel. This is also recognized from Fig.8(a), which shows that for same values of β and λ the velocity

Fig.4 Phase angles ξ and δ of pressure and friction drags to harmonic piston velocity

profile in the circular channel is flatter than that in the annular channel.

The damping coefficient c is inversely proportional to the number n of cylindrical orifices, whether the flow is steady or unsteady, because the ratio $A_c/A_o = A+1$ reduces by half, if the number n becomes doubled keeping the radius r_1 constant. Until the present day an equation $c=8\pi\mu LA^2$ was used to calculate the damping coefficient of this type of oil damper.⁽⁵⁾ It should be noted that this equation differs from Eq.[1.13] of Table 1 in that a different boundary condition $v(r_1)=0$ is imposed and that the friction drag is nonexistent.

The kinetic energy T_{os} of a fluid flowing steadily through the circular channel can be written as

$$T_{os} = \frac{1}{2} \rho \omega l \int_0^{r_1} 2\pi r v^2(r) dr \quad (14)$$

The calculation of Eq.(14) gives an added mass m_{As} :

$$T_{os} = [m_{As} v_{prms}^2 + \{m_{As} + (2A+1)m_o\} v_{crms}^2 - 2(m_{As} + A m_o) v_{prms} v_{crms}] / 2 \quad (15)$$

$$m_{As} = A^2 C_{As} m_o \quad (16)$$

$$C_{As} = (4+2/A+1/A^2)/3 \quad (17)$$

Equation (17) means a magnification factor of the kinetic energy due to non-uniformly distributed velocity, and we shall call it an added mass coefficient. If the flow is unsteady, the added mass coefficient C_A may be written in the form

$$C_A = C_{Ao} + 2(C_{Ao}-1)/A + (C_{Ao}-1)/A^2 \quad (18)$$

$$C_{Ao} = (B_2+1)/[B_1^2 + (B_2+1)^2] \quad (19)$$

This can be derived from Eq.(10) by arranging the terms in order of A . The subcoefficient C_{Ao} is dependent only on β . As shown in Fig.3, C_{Ao} changes from 4/3 to 1 with an increasing β , since a parabolic velocity distribution is gradually converted into a rectangular or uniform one.

Figure 4 shows the phase angles of the pressure and the friction drags, ξ and δ , to the piston velocity versus the dimensionless frequency β when $v_c=0$ and $\phi_2=0$; in other words, when the cylinder is at rest and the piston vibrates in a harmonic function $v_p \cos \omega t$. Figure 4 is quite similar to the ξ and δ vs. λ diagram for the oil damper having an annular channel of oil.⁽²⁾

3. Theoretical Analysis of an Oil Damper Having Both Circular and Annular Channels

A mathematical model for the oil damper having both circular and annular channels is shown in [2.21] of Table 2. The velocity v_o of a fluid through the annular channel is represented as a function of the coordinate x measured radially outward from the center of the piston; the velocity v_i of a fluid through the circular channel is represented as a function of the coordinate r measured radially outward from the center of the cylindrical orifice. Put a discharge of oil from the annular channel per unit time as Q_o , and another discharge from the circular channel as Q_i . Then the equation of continuity, the equations of motion and the boundary conditions are given in Eqs.

[2.1]~[2.6] of Table 2. Solving these equations for v_o and v_i , we obtain Eqs.[2.7] and [2.8]. Since the oil damper has two kinds of channels, two new ratios $R=r_p/r_c$ and $T=r_1/r_c$ are used in these equations instead of the ratio A .

The velocity $v_o(x, t)$ of a fluid through the annular channel is expressed by symbols $E_a \sim E_r$ (see Eq.[2.23]), which are composed of the Kelvin function as shown in Eq.[2.22] of Table 2. On the other hand, the velocity $v_i(r, t)$ of a fluid through the circular channel is expressed by the symbols E_1 and E_2 , which are also composed of the Kelvin function as shown in Eq.[1.19] of Table 1. We shall use dimensionless distances $x\sqrt{\omega/\nu}$ (from the center of piston) and $r\sqrt{\omega/\nu}$ (from the center of cylindrical orifice) as a variable η of the Kelvin function. If the distance x is replaced by the radius r_c of cylinder or r_p of piston, then the variable η will carry the subscript c or p .

The discharges Q_o and Q_i can be calculated by Eq.[2.2] in Table 2, and divided by the total discharge Q_o+Q_i . These ratios of the discharge are represented by Eqs. [2.9] and [2.10]. If the flow is unsteady, the rate is averaged by time. Note that these equations are derived under a specific condition that the cylinder is at rest and only the piston vibrates sinusoidally.

In a similar way to Table 1, we can obtain the pressure drag and the friction drag as shown in Eqs.[2.11]~[2.14]. Here, $F \cos(\omega t + \delta) \equiv (-r_p/v_r) \partial v_o / \partial x|_{x=r_p}$, $F' \cos(\omega t + \delta') \equiv (r_c/v_r) \partial v_o / \partial x|_{x=r_c}$ and $F'' \cos(\omega t + \delta'') \equiv (r_1/v_r) \partial v_i / \partial r|_{r=r_1}$ mean the dimensionless velocity gradients on the piston and cylinder walls facing the annular clearance and on the wall of circular holes, respectively. The dimensionless pressure gradient $P \cos(\omega t + \xi)$ and these velocity gradients can be calculated as Eqs. [2.19] and [2.20] in Table 2.

Using the force method, we obtain the damping coefficient and the added mass of the oil damper as follows:

$$m_2 \ddot{x}_2 + m_A \ddot{x}_2 - [m_A + (R^2 - nT^2)m_c] \ddot{x}_1 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0 \quad (20)$$

$$m_1 \ddot{x}_1 + [m_A + (1 + R^2 - nT^2)m_c] \ddot{x}_1 - [m_A + (R^2 - nT^2)m_c] \ddot{x}_2 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) = 0 \quad (21)$$

$$m_A = m_c [B_o - (B_a + B_c)P_1 + (B_o + B_a + 1)P_2 - (R^2 - nT^2)] \quad (22)$$

$$m_c = \pi r_c^2 \rho l \quad (23)$$

$$c = \pi \mu l \eta_c^2 [-B_a + (B_o + B_a + 1)P_1 - (B_a + B_c)P_2] \quad (24)$$

In the steady flow analysis, however, these are only obtained by the energy method. The general procedure in this method is to calculate the dissipation function and the kinetic energy within the channels as follows:

$$D_s = \frac{1}{2} \mu l \left[\int_{r_p}^{r_c} 2\pi x \left(\frac{dv_o}{dx} \right)^2 dx + n \int_0^{r_1} 2\pi r \left(\frac{dv_i}{dr} \right)^2 dr \right] = (1/2) C_s v_{prms}^2 \quad (25)$$

$$T_{os} = \frac{1}{2} \rho l \left[\int_{r_p}^{r_c} 2\pi x v_o^2(x) dx + n \int_0^{r_1} 2\pi r v_i^2(r) dr \right] = [m_{As} v_{prms}^2 + \{m_{As} + (1 + R^2 - nT^2)m_c\} v_{crms}^2 - 2\{m_{As} + (R^2 - nT^2)m_c\} v_{prms} v_{crms}] / 2 \quad (26)$$

The calculating results are shown in Eqs. [2.15] and [2.17] in Table 2.

Figure 5 shows a plot of Eq.[2.9] against the radius ratio $T=r_i/r_e$ for various values of another ratio $R=r_p/r_e$. It should be noted that there happens the negative

value of the discharge Q_0 or Q_1 , in other words, a back-flow does not occur in the annular or the circular channel. This happens when there is a remarkable difference

Table 2 Theoretical analysis of an oil damper having both circular and annular channels

Item	Steady flow	Unsteady flow
Equation of continuity	$A_c \bar{v}_{crms} - A_p \bar{v}_{prms} = Q_0 + Q_1$ $Q_0 = 2\pi \int_{r_p}^{r_e} x \bar{v}_0(x) dx$, $Q_1 = 2n\pi \int_0^1 r \bar{v}_1(r) dr$ [2.1]	$A_c \bar{v}_c \cos(\omega t - \Phi_1) - A_p \bar{v}_p \cos(\omega t - \Phi_2) = Q_0 + Q_1$ $Q_0 = 2\pi \int_{r_p}^{r_e} x \bar{v}_0(x, t) dx$, $Q_1 = 2n\pi \int_0^1 r \bar{v}_1(r, t) dr$ [2.2]
Equation of motion	$\frac{d\bar{v}}{dy} = \mu \left(\frac{d^2 \bar{v}_0}{dx^2} + \frac{1}{x} \frac{d\bar{v}_0}{dx} \right)$, $\frac{d\bar{v}}{dy} = \mu \left(\frac{d^2 \bar{v}_1}{dr^2} + \frac{1}{r} \frac{d\bar{v}_1}{dr} \right)$ [2.3]	$\frac{\partial \bar{v}_0}{\partial t} = -\frac{1}{\eta} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 \bar{v}_0}{\partial x^2} + \frac{1}{x} \frac{\partial \bar{v}_0}{\partial x} \right)$, $\frac{\partial \bar{v}_1}{\partial t} = -\frac{1}{\eta} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 \bar{v}_1}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_1}{\partial r} \right)$ [2.4]
Boundary condition	$\bar{v}_0(r_p) = \bar{v}_{prms}$, $\bar{v}_0(r_e) = \bar{v}_{crms}$ $\bar{v}_1(r_i) = \bar{v}_{prms}$, $\bar{v}_1(0) = \text{finite}$ [2.5]	$\bar{v}_0(r_p, t) = \bar{v}_p \cos(\omega t - \Phi_2)$, $\bar{v}_0(r_e, t) = \bar{v}_c \cos(\omega t - \Phi_1)$ $\bar{v}_1(r_i, t) = \bar{v}_p \cos(\omega t - \Phi_2)$, $\bar{v}_1(0, t) = \text{finite}$ [2.6]
Solution	$\bar{v}_0(x) = -U \left[(1+R^2) \frac{nT^4}{1-R^2} \ln\left(\frac{x}{r_e}\right) + 1 - \left(\frac{x}{r_e}\right)^2 \right] \bar{v}_{rrms} + \bar{v}_{crms}$ $\bar{v}_1(r) = -U \left[T^2 - (r/r_e)^2 \right] \bar{v}_{rrms} + \bar{v}_{prms}$ where $U = -[(1+R^2) \frac{nT^4}{1-R^2} \ln R + 1 - R^2]^{-1}$ [2.7]	$\bar{v}_0(x, t) = E_a \bar{v}_p \cos(\omega t - \Phi_2) + E_b \bar{v}_p \sin(\omega t - \Phi_2) + E_c \bar{v}_c \cos(\omega t - \Phi_1)$ $+ E_d \bar{v}_c \sin(\omega t - \Phi_1) + E_e P \bar{v}_r \sin(\omega t + \xi) - E_f P \bar{v}_r \cos(\omega t + \xi)$ $\bar{v}_1(r, t) = E_1 \bar{v}_p \cos(\omega t - \Phi_2) + E_2 \bar{v}_p \sin(\omega t - \Phi_2)$ $+ (E_1 - 1) P \bar{v}_r \sin(\omega t + \xi) - E_2 P \bar{v}_r \cos(\omega t + \xi)$ [2.8]
Flow rate	$\frac{Q_0}{Q_0 + Q_1} = -\left(\frac{1}{2} nUT^4 - R^2\right) / (R^2 - nT^2)$ $\frac{Q_1}{Q_0 + Q_1} = nT^2 \left(\frac{1}{2} UT^2 - 1 \right) / (R^2 - nT^2)$ [2.9]	$\left(\frac{Q_0}{Q_0 + Q_1} \right)_{avg} = -[W_1 - (W_2 + W_4)P_1 + (W_1 + W_3 - 1 + R^2)P_2] / (R^2 - nT^2)$ $\left(\frac{Q_1}{Q_0 + Q_1} \right)_{avg} = nT^2 [B_2 + B_1P_1 + (B_2 + 1)P_2] / (R^2 - nT^2)$ [2.10]
Pressure drag	$D_{ps2} = 4\pi\mu l U (R^2 - nT^2) \bar{v}_{rrms}$ $D_{ps1} = -4\pi\mu l U \bar{v}_{rrms}$ [2.11]	$D_{p2}(t) = \pi\mu l (R^2 - nT^2) \eta_c^2 \bar{v}_r P \cos(\omega t + \xi)$ $D_{p1}(t) = -\pi\mu l \eta_c^2 \bar{v}_r P \cos(\omega t + \xi)$ [2.12]
Friction drag	$D_{fs2} = 2\pi\mu l U [1 + R^2 + nT^4 / (1 - R^2) - 2(R^2 - nT^2)] \bar{v}_{rrms}$ $D_{fs1} = 2\pi\mu l U [1 - R^2 - nT^4 / (1 - R^2)] \bar{v}_{rrms}$ [2.13]	$D_{r2}(t) = 2\pi\mu l \bar{v}_r [F \cos(\omega t + \delta) + nF \cos(\omega t + \delta')]$ $D_{r1}(t) = 2\pi\mu l \bar{v}_r F \cos(\omega t + \delta')$ [2.14]
Damping coefficient	$C_s = 2\pi\mu l U [1 + R^2 + nT^4 / (1 - R^2)]$ [2.15]	$c = \pi\mu l \eta_c^2 [-B_a + (B_b + B_d + 1)P_1 - (B_a + B_c)P_2]$ [2.16]
Added mass	$m_{As} = mc U^2 \left\{ \left[(1 + R^2) \frac{nT^4}{1 - R^2} \right]^2 \left[R^2 (1 - \ln R) \ln R + \frac{1}{2} (1 - R^2) \right] \right.$ $\left. - (1 + R^2) \frac{nT^4}{1 - R^2} \left[R^2 (2 - R^2) \ln R + \frac{1}{4} (1 - R^2) (3 - R^2) \right] \right.$ $\left. + \frac{1}{3} (1 - R^2)^3 \right\} + mc n T^2 (1 - UT^2 + \frac{1}{3} UT^4)$ [2.17]	$m_A = mc [B_b + (B_a + B_c)P_1 + (B_b + B_d + 1)P_2 - (R^2 - nT^2)]$ where $mc = \pi r_e^2 \eta l$ [2.18]
$P \cos \xi = \frac{\bar{v}_p}{\bar{v}_r} (P_2 \sin \Phi_2 + P_1 \cos \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} [(P_2 + 1) \sin \Phi_1 + P_1 \cos \Phi_1]$ $P \sin \xi = \frac{\bar{v}_p}{\bar{v}_r} (P_2 \cos \Phi_2 - P_1 \sin \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} [(P_2 + 1) \cos \Phi_1 - P_1 \sin \Phi_1]$ $F \cos \delta = \frac{\bar{v}_p}{\bar{v}_r} (F_2 \sin \Phi_2 + F_1 \cos \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2 \sin \Phi_1 + F_1 \cos \Phi_1)$ $F \sin \delta = \frac{\bar{v}_p}{\bar{v}_r} (F_2 \cos \Phi_2 - F_1 \sin \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2 \cos \Phi_1 - F_1 \sin \Phi_1)$ $F' \cos \delta' = \frac{\bar{v}_p}{\bar{v}_r} (F_2' \sin \Phi_2 + F_1' \cos \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2' \sin \Phi_1 + F_1' \cos \Phi_1)$ $F' \sin \delta' = \frac{\bar{v}_p}{\bar{v}_r} (F_2' \cos \Phi_2 - F_1' \sin \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2' \cos \Phi_1 - F_1' \sin \Phi_1)$ $F'' \cos \delta'' = \frac{\bar{v}_p}{\bar{v}_r} (F_2'' \sin \Phi_2 + F_1'' \cos \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2'' \sin \Phi_1 + F_1'' \cos \Phi_1)$ $F'' \sin \delta'' = \frac{\bar{v}_p}{\bar{v}_r} (F_2'' \cos \Phi_2 - F_1'' \sin \Phi_2) - \frac{\bar{v}_c}{\bar{v}_r} (F_2'' \cos \Phi_1 - F_1'' \sin \Phi_1)$ [2.19]		$P_1 = \frac{W_1^* (W_2^* + W_4) - W_2^* (W_1^* + W_3^*)}{(W_1^* + W_3^*)^2 + (W_2^* + W_4)^2}$ $P_2 = -\frac{W_1^* (W_1^* + W_3^*) + W_2^* (W_2^* + W_4)}{(W_1^* + W_3^*)^2 + (W_2^* + W_4)^2}$ $F_1 = -\frac{\eta_c^2}{2} [K_a - (K_b + K_d)P_1 + (K_a + K_c)P_2]$ $F_2 = \frac{\eta_c^2}{2} [K_b + (K_a + K_c)P_1 + (K_b + K_d)P_2]$ $F_1^* = -\frac{\eta_c^2}{2} [B_a - (B_b + B_d)P_1 + (B_a + B_c)P_2]$ $F_2^* = -\frac{\eta_c^2}{2} [B_b + (B_a + B_c)P_1 + (B_b + B_d)P_2]$ $F_1' = \frac{\eta_c^2}{2} [B_1(P_2 + 1) - B_2P_1]$ $F_2' = -\frac{\eta_c^2}{2} [B_1P_1 + B_2(P_2 + 1)]$ [2.20]
$N_a = \text{ber} \eta_p \text{ker} \eta_c - \text{ber} \eta_c \text{ker} \eta_p - \text{bei} \eta_p \text{kei} \eta_c + \text{bei} \eta_c \text{kei} \eta_p$ $N_b = \text{ber} \eta_p \text{kei} \eta_c - \text{ber} \eta_c \text{kei} \eta_p + \text{bei} \eta_p \text{ker} \eta_c - \text{bei} \eta_c \text{ker} \eta_p$ $N_1 = \text{ker} \eta_c \text{ber} \eta_r - \text{kei} \eta_c \text{bei} \eta_r - \text{ber} \eta_c \text{ker} \eta_r + \text{bei} \eta_c \text{kei} \eta_r$ $N_2 = -\text{kei} \eta_c \text{ber} \eta_r + \text{ker} \eta_c \text{bei} \eta_r - \text{bei} \eta_c \text{ker} \eta_r - \text{ber} \eta_c \text{kei} \eta_r$ $N_3 = -\text{ker} \eta_p \text{ber} \eta_r + \text{kei} \eta_p \text{bei} \eta_r + \text{ber} \eta_p \text{ker} \eta_r - \text{bei} \eta_p \text{kei} \eta_r$ $N_4 = -\text{kei} \eta_p \text{ber} \eta_r - \text{ker} \eta_p \text{bei} \eta_r + \text{bei} \eta_p \text{ker} \eta_r + \text{ber} \eta_p \text{kei} \eta_r$ $M_1 = N_a / (N_a^2 + N_b^2)$; $M_2 = N_b / (N_a^2 + N_b^2)$ [2.22]		$E_a = M_1 N_1 + M_2 N_2$ $E_b = M_2 N_1 - M_1 N_2$ $E_c = M_1 N_3 + M_2 N_4$ $E_d = M_2 N_3 - M_1 N_4$ $E_e = E_a + E_c - 1$ $E_f = E_b + E_d$ [2.23]
$W_1 = \frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta E_a d\eta = -B_b + R^2 K_b$; $W_1^* = W_1 + R^2 - nT^2 (B_2 + 1)$ $W_2 = \frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta E_b d\eta = B_a - R^2 K_a$; $W_2^* = W_2 + nT^2 B_1$ $W_3 = \frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta E_c d\eta = -B_d + R^2 K_d$; $W_3^* = W_3 - 1$ $W_4 = \frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta E_d d\eta = B_c - R^2 K_c$ [2.24]		$R = r_p / r_e$ $T = r_i / r_e$ $\eta = r \sqrt{\frac{\omega}{\nu}}$ $\eta_c = r_e \sqrt{\frac{\omega}{\nu}}$ $\eta_p = r_p \sqrt{\frac{\omega}{\nu}}$ [2.25]
$H_1 = \frac{2}{\eta_c} \frac{dN_1}{d\eta} \Big _{\eta=\eta_c}$; $G_1 = \frac{2}{\eta_p} \frac{dN_1}{d\eta} \Big _{\eta=\eta_p}$; $-\frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta N_2 d\eta = H_1 - R^2 G_1$ $H_2 = \frac{2}{\eta_c} \frac{dN_2}{d\eta} \Big _{\eta=\eta_c}$; $G_2 = \frac{2}{\eta_p} \frac{dN_2}{d\eta} \Big _{\eta=\eta_p}$; $\frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta N_1 d\eta = H_2 - R^2 G_2$ $H_3 = \frac{2}{\eta_c} \frac{dN_3}{d\eta} \Big _{\eta=\eta_c}$; $G_3 = \frac{2}{\eta_p} \frac{dN_3}{d\eta} \Big _{\eta=\eta_p}$; $-\frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta N_4 d\eta = H_3 - R^2 G_3$ $H_4 = \frac{2}{\eta_c} \frac{dN_4}{d\eta} \Big _{\eta=\eta_c}$; $G_4 = \frac{2}{\eta_p} \frac{dN_4}{d\eta} \Big _{\eta=\eta_p}$; $\frac{2}{\eta_c^2} \int_{\eta_p}^{\eta_c} \eta N_3 d\eta = H_4 - R^2 G_4$ [2.25]		$B_a = \frac{2}{\eta_c} \frac{dE_a}{d\eta} \Big _{\eta=\eta_c} = M_1 H_1 + M_2 H_2$; $K_a = \frac{2}{\eta_p} \frac{dE_a}{d\eta} \Big _{\eta=\eta_p} = M_1 G_1 + M_2 G_2$ $B_b = \frac{2}{\eta_c} \frac{dE_b}{d\eta} \Big _{\eta=\eta_c} = M_2 H_1 - M_1 H_2$; $K_b = \frac{2}{\eta_p} \frac{dE_b}{d\eta} \Big _{\eta=\eta_p} = M_2 G_1 - M_1 G_2$ $B_c = \frac{2}{\eta_c} \frac{dE_c}{d\eta} \Big _{\eta=\eta_c} = M_1 H_3 + M_2 H_4$; $K_c = \frac{2}{\eta_p} \frac{dE_c}{d\eta} \Big _{\eta=\eta_p} = M_1 G_3 + M_2 G_4$ $B_d = \frac{2}{\eta_c} \frac{dE_d}{d\eta} \Big _{\eta=\eta_c} = M_2 H_3 - M_1 H_4$; $K_d = \frac{2}{\eta_p} \frac{dE_d}{d\eta} \Big _{\eta=\eta_p} = M_2 G_3 - M_1 G_4$ [2.26]

between the width $h=r_c-r_p$ of annular channel and the radius r_1 of circular channel. However, if the flow through the channels is unsteady, the back-flow may occur again, as shown in Fig.6. We shall explain the reason by using Fig.8.

Figure 8 represents various velocity distributions in the annular channel (upper portion) and the circular channel (lower portion), when only the piston vibrates in a harmonic motion $v_p \cos \omega t$ and the cylinder is at rest. Figure 8(a) contains three pairs of figures for the case of $R=0.9$, $T=0.1$ and $n=1$; they are different from each other at the dimensionless frequency λ or β of the piston. These factors are connected by an equation $\beta=T\lambda/(1-R)$. Therefore, in Fig.8(a) the equation is reduced to $\lambda=\beta$. If λ is equal to β , the oil is apt to flow through the circular channel rather than the annular channel. This is found in Fig.8(a-1) which shows that the flow through the circular channel is faster than through the annular channel. However, as the value of λ increases, the parabolic distribution is gradually converted to a rectangular one. At the same time, the velocities of flow through both channels come close to each other.

The diameter of circular hole in Fig. 8(b) is twice as large as that in (a), so the fluid flows more smoothly through the circular hole. Therefore, if λ is small, there is a great difference between velocities of flow through both channels. However, with an increasing λ , the velocities of flow come close to each other as shown in Figs.8(b-2) and 8(b-3). Figure 8(c) shows the velocity distribution on condition that the piston has two circular holes whose diameter is twice as large as the hole of the piston in Fig.8(b). In this condition ($n=2$, $T=0.4$), the discharge Q_0 is negative for the small value of λ , as shown in Fig.6. In practice, the fluid within the annular channel moves as if it were dragged by the piston wall; namely, a back-flow does not occur. As to the change of velocities of flow caused by an increasing λ , the same statement as in the case of Fig.8(a),(b) is true for this case.

A remarkable change in the flow rates Q_0 and Q_1 with an increasing λ has a considerable effect on the damping coefficient of this type of oil damper. Figure 7 shows a plot of a ratio c/c_s of damping coefficients against R for various values of T and n . Figures 7(a) and (b) are different in the value of β . The symbol "o" located at the left end of these figures means a ratio c/c_s of the oil damper having only a circular channel. The ratio is independent of T and n , and depends only on β as shown in Fig.2. Note that in the oil damper having both circular and annular channels, the ratio c/c_s does not coincide with the point "o", when the annular channel has vanished, or $R=1$. If $R=1$, then the velocity gradient of the fluid between the walls of the cylinder and the piston becomes infinite whether the flow is steady or unsteady, so that the ratio c/c_s becomes unity. As the value of R is reduced, the value of c/c_s increases at first until a certain value of R , and then decreases after reaching a maximum value. If R is further reduced, c/c_s

Fig.5 Rate of discharges Q_0 and Q_1 to the total discharge Q_0+Q_1 (steady flow)

Fig.6 Effect of unsteadiness on the rate of discharge

(a) $\beta = 5$

(b) $\beta = 2.5$

Fig.7 Effect of unsteadiness on damping coefficient of the oil damper having circular and annular channels

increases again because of an increasing λ (see a dashed-and-two-dotted line in Fig. 7 (a)). The change of the ratio c/c_s mentioned above becomes more rapid, if n and T are small and β is large. It should be noted

that the ratio c/c_s of the oil damper having two different channels can be greatly increased, in other words, this type of oil damper is more susceptible to the frequency than the oil damper having only one channel.

Fig. 8 Velocity distributions of the annular (upper portion) and the circular (lower portion) channels

4. Comparison of Experimental with Calculated Results

Experimental equipment and method for the oil damper are the same with Refs.(1) and (6), so we will only give an outlined here. Figure 9 shows the experimental equipment considered to have a single degree of freedom. A cylinder of inner diameter 36mm is filled with silicone oil of kinematic viscosity $\nu = 100 \text{ mm}^2/\text{s}$ at 23°C and density $\rho = 968 \text{ kg/m}^3$. A piston of length 20mm is attached to a weight suspended by four leaf-springs. Many pistons are provided for the experiment. The base of the equipment is excited in a harmonic motion by a vibration generator, and accelerations of the base and the supported weight are measured by piezoelectric pickups. The damping coefficient c of the oil damper is obtained by the method mentioned in Ref.(6).

The experimental results are summarized in Figs.10(a)~(c). In these figures, the damping coefficient of the oil damper having annular and circular channels is plotted against the number n of circular channels in (a), and against the radius ratio T of the circular hole to the cylinder (β is proportional to T) in (b) and (c). The solid, broken and dash-dotted lines mean theoretical curves due to Eq.[2.16] based on the unsteady flow analysis and to Eq.[2.15] based on the steady flow analysis and to the conventional

equation $c = 8\pi\mu l A^2$, respectively. The experimental data are the best fitted with the solid curve. Other curves greatly deviate from the data as β increases.

5. Conclusions

We have made an analysis and an experiment on the oil damper with a circular channel in the piston as a preparation to manufacturing an oil damper with variable damping mechanism. The results of our study can be summarized as follows:

- (1) If an oil damper has only a few cylindrical orifices (circular channel) with same diameter in the piston, the damping force is inversely proportional to the number of the orifices.
- (2) The oil damper having a circular channel is more susceptible to the frequency than that the one having an annular channel. Because the area of wall facing the circular channel is narrower than that of the walls facing the annular channel, if cross-sectional areas of them are equal.
- (3) Comparing with oil dampers having only an annular or a circular channel, the oil damper having both annular and circular channels is considerably influenced by the frequency. Because the rate of the discharge from both channels varies greatly with an increasing frequency.

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Fig.9 Experimental equipment of the oil damper

Fig.10 Comparison of the experimental with theoretical results