Self-excited Oscillation of a Hollow Rotating
Shaft Partially Filled with a Liquid
(1st Report, Instability Based on the Fluid
Force Obtained from Boundary Layer theory)*

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A self-excited oscillation of a hollow rotating shaft partially filled with a liquid is investigated both theoretically and experimentally.

In this report, the theoretical explanation about the mechanism of a self-excited oscillation is given, the validity of which is confirmed by experiments using a model with one degree of freedom.

The results show that the fluid force acting on the wall of a hollow rotating shaft consists of two components, one of which is out of phase by 90 degrees with the displacement of the shaft and that this component makes the field of rotating resistance with resonant characteristics which causes the self-excited oscillation.

Key Words : Vibration of Rotating Body, Self-excited Vibration, Surface Waves, Resonance, Boundary Layer

1. Introduction

It is well known that a self-excited nonsynchronous whirl occurs when a hollow rotating shaft is partially filled with a liquid. Recently some reports on this problem have been presented. The following theoretical investigations are reported:
(1) Analyses by an inviscid theory.⁽¹⁾⁻⁽³⁾
(2) Analyses by a viscid theory.⁽⁴⁾⁻⁽⁹⁾
(3) Calculations by finite difference method taking viscosity into consideration.⁽¹⁰⁾⁻⁽¹⁴⁾

As experimental studies, investigations by Wolf⁽²⁾, Lichtenberg⁽³⁾ and Saito⁽¹⁵⁾ are reported. However there are few reports on investigations about the theoretical explanation of the excitation mechanism. Moreover no investigations have been made about the effects of the viscosity of a liquid and an external damping on the stability of the rotating shaft.

In this report, in order to give a clear explanation for the excitation mechanism of a self-excited nonsynchronous whirl, the stability of a hollow rotating shaft is analyzed based on the fluid force obtained from boundary layer theory.⁽¹⁶⁾ The validity of the analysis is confirmed by the experiments taking the amount of a liquid, viscosity and an external damping as parameters.

Nomenclature

a: inner radius of a hollow rotating shaft
b: inner radius of a liquid at the equilibrium state
c: external damping factor of a support system
E: Ekman number = $\nu/(a^2\Omega)$
2h: longitudinal length of a hollow rotating shaft
k: spring constant of a support system
m: mass of a hollow rotating shaft
o-xyz: orthogonal coordinate system fixed on a hollow rotating shaft
O-XYZ: fixed coordinate system
R: non-dimensional liquid depth = b/a
t: time
 ϵ_x : amplitude in the direction of X
 ζ : non-dimensional damping factor when a hollow rotating shaft is partially filled with a liquid
 ζ_e : non-dimensional damping factor when a hollow rotating shaft is not filled with a liquid
 ν : coefficient of kinetic viscosity
 ρ : density of a liquid
 ω : angular frequency of a hollow rotating shaft system
 $\bar{\omega}$: non dimensional angular frequency of a hollow rotating shaft system = ω/Ω
 Ω : rotating angular velocity of a hollow rotating shaft
 Ω_0 : undamped natural angular frequency when a hollow rotating shaft is not filled with a liquid
 Ω_R : undamped natural angular frequency when a hollow rotating shaft is partially filled with a liquid
 $\bar{\omega} = \omega/\Omega_0$
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2. Analysis

The system to be analyzed is shown in Fig.1. A rigid rotating shaft is restricted to move only in the direction of X . The support system is composed of a linear external damping and a linear spring. Here the effects of gravitational force and the unbalance of a rotating shaft on the stability are considered small enough to be neglected.

In the equilibrium state, a hollow shaft is rotating at a constant angular velocity of Ω with the inner radius of a filling liquid b . The equation describing the motion of a hollow rotating shaft with the inner radius a and longitudinal length $2h$ is

$$m\ddot{X} + c\dot{X} + kX = F_x \quad (1)$$

where denotes differentiation by t and F_x denotes the fluid force acting on the hollow rotating shaft. According to the previous investigation⁽¹⁶⁾, when a hollow rotating shaft is undergoing a sinusoidal one-dimensional motion described by the equation of $\epsilon_x \cos \omega t$, the fluid force acting on the shaft can be evaluated by the following equation.

$$F_x = \rho a^2 h \pi \epsilon \omega^2 (f_1 \cos \omega t + f_2 \sin \omega t) \quad (2)$$

where f_1 and f_2 , which are functions of non-dimensional liquid depth R , non-dimensional angular frequency $\bar{\omega}$ and Ekman number E , are defined as follows.

$$f_1 = \frac{2Rg_1/K}{g_1^2 + g_2^2} + \frac{1}{2} - \text{sign}(K) \frac{\lambda ER(g_1 - g_2)}{g_1^2 + g_2^2} \quad (3)$$

$$f_2 = \frac{-Rg_2/K}{g_1^2 + g_2^2} - \frac{\lambda ER(g_1 + g_2)}{g_1^2 + g_2^2} \quad (4)$$

where $g_1 = f + g(-1 + 1/\lambda)$, $g_2 = g(-1 + 1/\lambda)/\lambda$, $f = R(2 - 2/K - K/2)$, $g = (2 + 2/K - K/2)/R$, $K = 2(1 - \bar{\omega})$, $\lambda = (|K|E)^{-1/2}$

Putting $\epsilon_x \cos \omega t = X$, we obtain $\dot{X} = -\epsilon_x \omega \sin \omega t$, $\ddot{X} = -\epsilon_x \omega^2 \cos \omega t$. By using these relations, Eq.(2) can be rewritten as follows.

$$F_x = \rho a^2 h \pi (-f_1 \ddot{X} - f_2 \omega \dot{X}) \quad (5)$$

From this equation, we can see that the fluid force acting in the direction of

Fig.1 Shaft support system and coordinate system

X consists of two components, one of which is proportional to the acceleration of the shaft and the other is to the velocity. Substituting Eq.(5) into Eq.(1), we obtain

$$\ddot{X} + 2\zeta \Omega_R \dot{X} + \Omega_R^2 X = 0 \quad (6)$$

where Ω_R , ζ are defined as follows, which respectively represent undamped natural angular frequency and non-dimensional damping factor when a hollow rotating shaft is partially filled with a liquid.

$$\Omega_R = [k/(m + \rho a^2 h \pi f_1)]^{1/2} \quad (7)$$

$$\zeta = (c + \rho a^2 h \pi f_2 \omega) \Omega_R / (2k) \quad (8)$$

Meanwhile the equation describing the motion of a hollow rotating shaft not filled with a liquid can be written as follows.

$$\ddot{X} + 2\zeta_e \Omega_0 \dot{X} + \Omega_0^2 X = 0 \quad (9)$$

where Ω_0 , ζ_e are defined as follows, which respectively represent undamped natural angular frequency and non-dimensional damping factor when a hollow rotating shaft is not filled with a liquid.

$$\Omega_0 = (k/m)^{1/2} \quad (10)$$

$$\zeta_e = c \Omega_0 / (2k) \quad (11)$$

From Eq.(7) and Eq.(10), we obtain

$$\Omega_R / \Omega_0 = [m / (m + \rho a^2 h \pi / 2)]^{1/2} \quad (12)$$

Here, based on the results obtained in our investigation⁽¹⁶⁾, we put $1/2$ into f_1 . This equation represents the added mass effect owing to the existence of a liquid.

Now we can determine the stability from the following equation. Here we put Ω_R into ω , for ω is considered to be

(1) support (2) joint (3) spring
(4) connecting arm (5) lamp
(6) front panel (7) rotor

Fig.2 Experimental apparatus

nearly equal to Ω_R when a self-excited oscillation occurs.

$$\zeta = (c + \rho a^2 h \pi f_2 \Omega_R) \Omega_R / (2k) \dots (13)$$

When we compare the experimental results with the theoretical ones, we take Ω_R/Ω as $\bar{\omega}$ in the abscissa.

3. Experiment

3.1 Experimental apparatus

The experimental apparatus is shown in Fig.2. In this apparatus, though the rotating shaft is supported by a pin-joint mechanism, the amplitude of which is so small that we may regard the motion of the shaft as one-dimensional. When we want to change the value of external damping factor, an oil damper is attached to the connecting arm. In Table 1, the specification of a hollow rotating shaft system is shown. In Table 2, coefficient of viscosity and specific gravity of liquids used in this experiment are listed.

Table 1 Specification of a hollow rotating system

$a = 66\text{mm}$	$h = 33\text{mm}$
$m = 3.91\text{kg}$	$k = 3.0\text{kg/mm}$
$\zeta_e = 6.7 \times 10^{-3}$	

Table 2 Coefficients of viscosity and specific gravity of liquids used in this experiment

working liquid	temp. $^{\circ}\text{C}$	coefficient of viscosity $\text{Pa}\cdot\text{s (cP)}$	specific gravity
water	20.0	0.001 (1)	1.0
spindle oil	26.3	0.016 (16)	0.901
compressor oil	22.0	0.068 (68)	0.856
turbine oil	21.1	0.183 (183)	0.875
mobile oil	21.5	0.450 (450)	0.905
glycerine	21.4	0.794 (794)	1.275

Fig.3 Amplitudes and frequencies of the shaft displacement (working liquid:water, parameter:liquid depth)

Fig.4 Wave forms of the shaft displacement when a self-excited oscillation occurs. (working liquid:water, liquid depth=20mm)

3.2 Experimental instrumentation

In order to examine a hysteresis phenomenon, we performed experiments with both increasing and decreasing rotating angular velocities. In the following graphs, experimental values for an increased rotating angular velocity (○) and for a decreased rotating angular velocity (●) are shown. Amplitudes of the shaft displacement were measured after a long time had passed since the rotating angular velocity was set at a certain value. Frequencies of the shaft displacement were determined by averaging the frequencies of typical five wave forms of the shaft displacement. Finally, damping factors of the shaft displacement were determined from the wave forms of a transient response.

4. Experimental Results

4.1 Amplitudes and frequencies of the shaft displacement

Amplitudes and frequencies of the shaft displacement for three values of liquid depth are shown in Fig.3, wherein water is used as a working liquid. The upper graphs correspond to the lower graphs. In the

graphs, amplitudes (dimension: mm) and the angular frequency non-dimensionalized by the undamped natural frequency when a hollow rotating shaft is not filled with a liquid (Ω_0) are shown against the rotating angular velocity non-dimensionalized by Ω_0 . These graphs suggest the following: (a) a hysteresis phenomenon appears when a liquid with a small coefficient of kinetic viscosity such as water is used as a working liquid; (b) the width of the unstable region is almost independent of liquid depth; and (c) the frequency of the shaft when a self-excited unstable whirl occurs is almost constant independently of liquid depth. In Fig.4, wave forms of stationary response of the shaft displacement when a self-excited unstable whirl occurs are shown. The numbers used in this graph correspond to ones in Fig.3.

In Fig.5, amplitudes and frequencies of the shaft displacement for three kinds of coefficients of kinetic viscosity are shown, wherein liquid depth is kept at 10mm. From these graphs, it is found that when we use a liquid with a large coefficient of kinetic viscosity, the width of the unstable region becomes wider and a hysteresis

Fig.5 Amplitudes and frequencies of the shaft displacement (liquid depth=10mm, parameter:coefficient of kinetic viscosity)

Fig.6 Wave forms of the shaft displacement when a self-excited oscillation occurs. (working liquid:mobile oil, liquid depth=10mm)

esis phenomenon disappears. In Fig.6, wave forms of a stationary response of the shaft displacement are shown, wherein mobile oil with a coefficient of kinetic viscosity 500 times larger than that of water is used as a working liquid. The numbers used in this graph correspond to ones in Fig.5. When mobile oil is used as a working liquid, a transition region was observed.

4.2 Effects of external damping

Figs.7 and 8 show amplitudes of the

Fig.7 Amplitudes of the shaft displacement (parameter: external damping factor, working liquid: water)

Fig.8 Amplitudes of the shaft displacement (parameter: external damping factor, working liquid: compressor oil)

Fig.9 Resonant characteristics of a damping factor (parameter: external damping factor, working liquid: water)

shaft displacement when an external damping factor is changed, wherein water and compressor oil with liquid depth of 10mm are used as working liquids. From Fig.7, it is found that when we use a liquid with a small coefficient of kinetic viscosity under a large external damping, a hysteresis phenomenon becomes less prominent and finally disappears. However the width of the unstable region is independent of an external damping according to small perturbation theory. As an external damping becomes large, amplitudes of the shaft displacement become small up to the resonant amplitudes caused by the unbalance excitation. From Fig.8, it is

Fig.10 Resonant characteristics of a damping factor (parameter: external damping factor, working liquid: compressor oil)

Fig.11 Resonant characteristics of the damping factor (theoretical results, parameter: liquid depth)

Fig.12 Resonant characteristics of the damping factor (experimental values, parameter: liquid depth)

found that for a liquid with a coefficient of kinetic viscosity 80 times larger than that of water, as an external damping factor becomes larger, the width of the unstable region becomes narrower and finally the unstable region disappears at a certain value of the external damping factor.

In Figs.9 and 10, resonant characteristics of a damping factor are shown when the external damping factor is changed, when liquid depth is kept at 10mm and water and compressor oil are used as working liquids, respectively. Resonant characteristics were measured by increasing the rotating angular velocity (Ω) and holding the shaft oscillation at a reduced natural angular frequency (Ω_n). In these graphs, the non-dimensional damping factor (ζ) is shown against the angular frequency of the shaft non-dimensionalized by the rotating angular velocity ($\bar{\omega} = \Omega_n / \Omega$), therefore the increase of Ω corresponds to the decrease of $\bar{\omega}$. As expected, the experimental results are found to deviate in response to the external damping factor.

5. Comparison between Theoretical Results and Experimental Values

5.1 Resonant characteristics of a damping factor when liquid depth is changed

Fig.13 Wave forms of a transient response of the shaft displacement (working liquid: water, liquid depth=10mm)

In Fig.11, the theoretical results of ζ calculated from Eq.(13) for two kinds of liquid depth are shown, when water is used as a working liquid. In Fig.12, the experimental values corresponding to the theoretical results are shown. A continuous line and a broken line in Fig.11 correspond to and $\bullet$ in Fig.12 which represent results for the liquid depth of 20mm and 10mm, respectively. The theoretical results are in good agreement with the experimental values. Therefore the validity of the method of evaluating the fluid force can be verified. These graphs suggest the following: (a) the damping factor is nearly equal to the external damping factor except in the neighborhood of the resonant point; (b) a narrow unstable region exists only in the neighborhood of the resonant point with first order counter whirl mode⁽⁷⁾, the width of which is independent of liquid depth; (c) the damping factor becomes positive in the sub-critical region ($\bar{\omega} > 1$) and takes the maximum value at the resonant point with first order forward whirl mode⁽⁷⁾: on the contrary, it becomes negative in the over-critical region ($\bar{\omega} < 1$) and takes the minimum value at the resonant point with first order counter whirl mode; and (d) when liquid depth is changed, it contributes only to the location of the resonant point. In Fig.13, wave forms of a transient response corresponding to Fig.12

Fig.14 Resonant characteristics of the damping factor (theoretical results, parameter: coefficient of kinetic viscosity)

Fig.15 Resonant characteristics of the damping factor (experimental values, parameter: coefficients of kinetic viscosity)

are shown.

5.2 Resonant characteristics of the damping factor when the coefficient of kinetic viscosity of a liquid is changed

In Fig.14, the theoretical results of ζ calculated for three kinds of coefficients of kinetic viscosity are shown, when liquid depth is kept at 10mm. In Fig.15, the experimental values corresponding to the theoretical results are shown. A continuous line, a broken line, an alternate long short dash line in Fig.14 correspond to $\circ$, $\bullet$ and $\bullet$ in Fig.15 which represent results obtained by using spindle oil, compressor oil and mobile oil, respectively. The theoretical results are in good agreement with the experimental values which show the validity of the theory. These graphs suggest the following: (a) when we use a liquid with a large coefficient of kinetic viscosity, the rotating speed at which the damping factor takes the maximum and minimum values becomes closer to the critical speed; (b) the coefficients of kinetic viscosity contribute only to the resonant characteristics; (c) when we use a liquid with a large coefficient of kinetic viscosity, the maximum absolute value of the damping factor becomes small; and (d) the unstable limit near the critical speed is strongly dependent on the coefficient of kinetic

Fig.16 Wave forms of a transient response of the shaft displacement (working liquid:mobile oil, liquid depth=10mm)

viscosity, whereas one far from the critical speed is hardly influenced by the coefficient of kinetic viscosity. In Fig.16, wave forms of a transient response corresponding to Fig.15 are shown.

5.3 Stability diagram when liquid depth is changed

In Figs.17 and 18, stability diagrams when the liquid depth is changed under a constant external damping are shown. In these graphs, the rotating angular velocity non-dimensionalized by the undamped natural angular frequency when the shaft is not filled with a liquid ($\bar{\omega}$) is shown against the non-dimensional liquid depth (R). A continuous line shows the unstable limit theoretically determined and the region of an oblique line is an unstable region in the sense of small perturbation theory, experimentally determined. In addition, Δ shows the critical speed experimentally determined when the shaft is partially filled with a liquid and a continuous line shows the theoretical results calculated from Eq.(12). The theoretical results show good agreement with the experimental values. In Fig.17 water is used as a working liquid, wherein $\circ$ and $\square$ denote the lower and the higher unstable limit, respectively for an increased rotating speed, and $\bullet$ and $\blacksquare$

Fig.17 Stability diagram (parameter:liquid depth, working liquid:water)

Fig.18 Stability diagram (parameter:liquid depth, working liquid:spindle oil)

denote the lower and the higher unstable limit, respectively for a decreased rotating speed. For a liquid with a small coefficient of kinetic viscosity such as water, the unstable limit for an increased rotating speed is quite different from one for a decreased rotating speed. In the region enclosed with the higher unstable limit for a decreased rotating speed and one for an increased rotating speed, when a perturbation with a large amplitude is applied, a self-excited unstable whirl occurs. In Fig.18, a liquid with a coefficient of kinetic viscosity 20 times larger than that of water is used, when a hysteresis phenomenon can not be found.

5.4 Stability diagram when the external damping factor is changed

In Figs.19 and 20, stability diagrams when the external damping factor is changed under a constant liquid depth of 10mm are shown. In these graphs, $\tilde{\eta}$ is shown against the non-dimensional external damping factor (ζ_e). In Fig.19 water is used as a working liquid, when a hysteresis phenomenon is found to disappear at a large external damping factor. In addition, it is found that in the sense of small perturbation theory even for a large external damping

factor the shaft remains unstable and the width of the unstable region remains constant independently of the external damping factor. In Fig.20, a liquid with a coefficient of kinetic viscosity 80 times larger than that of water is used, wherein it is found that the unstable region becomes narrower for a large external damping factor and finally disappears.

6. Conclusions

In this report, in order to give a clear explanation for the excitation mechanism of a self-excited nonsynchronous whirl, the stability of a hollow rotating shaft was analyzed based on the fluid force obtained from boundary layer theory⁽¹⁶⁾. The validity of the analysis was confirmed by the experiments. Consequently, the mechanism of a self-excited oscillation of a hollow rotating shaft partially filled with a liquid was clarified as follows:

Owing to the existence of a boundary layer, a fluid force which includes a component out of phase by 90 degrees with the displacement of the shaft is generated. And this component makes the field of rotating resistance with resonant characteristics which causes the self-excited oscillation.

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Fig.19 Stability diagram
(parameter: external damping factor,
working liquid: water)

Fig.20 Stability diagram
(parameter: external damping factor,
working liquid: compressor oil)