Development of Digitally Controlled Magnetic Bearing*

(1st Report, Single Axis Control Method)

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The use of digital control techniques for active magnetic bearings has, in comparison with analogue controls, the following advantages: (1) more simplified hardware for the electric circuit (2) universality of electrical hardware (3) more sophisticated control techniques are applicable (4) software flexibility (5) extension of the system using additional CPU. Using a simple experimental apparatus of vertical single axis control, digital controls with a microcomputer were tested together with the analogue technique, as a first step to multi-axes control with high speed rotation. In impulse tests the digital control was almost equal in performance to the analogue control. Especially, the first-order digital PID control gave satisfactory results. However, noise of half the sampling frequency is amplified and harms control performance in some cases.

Key words : Control Theory, Magnetic Bearing, Digital Control, PID Control, Stability, Impulse Response

1. Introduction

Magnetic bearings, which suspend rotors without physical contact by controlling magnetic attractive force, are now being intensively developed and gradually put into actual use in fields such as momentum wheels for artificial satellites, turbomolecular pumps, spindles for high-speed machining, and fly-wheels for energy-storage.

In addition to the main advantage of little mechanical loss, magnetic bearings have many other excellent qualities such as extremely high stiffness in some specific control method, no inherent limit of peripheral speed, no need for lubricating systems, being suitable for use in special environments such as ultra-low temperature and high vacuum, low levels of vibration and noise, no wear or corrosion and no contamination of lubricant into the process gas. Although problems of high cost and limit of load capacity still have to be solved, their range of applications is being extended mainly to rotors of high speed and low weight. They will certainly be applied to general turbomachinery of turbines and compressors in the near future for energy saving purposes.

Control of the magnetic attractive force in the bearing is needed in order to stabilize the system. The rotor

displacement detected is fed back into the electric circuit, which controls the current of electro-magnetic coils, and therefore, the magnetic attractive force. Usually analogue circuits with operational amplifiers are used for this purpose and PID control is usually performed.

In contrast with this, owing to the recent development of microelectronics, easy digital treatment of signals has been realized without using large computer systems. Therefore the digital control of magnetic bearings has become fully possible.

The merits of the digital control when compared with the analogue control are as follows:

- (1) Simplification of control circuits (hardware).
When the interference of each control axis is considered, the complexity of analogue circuits is proportional to N^2 , where N is the number of control axes, and the complexity of digital circuits is proportional to N . Therefore in case of practical multi-axes control, the hardware of digital control circuit is simpler.
- (2) Universality of electrical hardware.
For different control apparatus, the same hardware can be used by changing the control method and control constants of the software.
- (3) Possibility of more sophisticated control techniques.
Higher levels of control other than PID action can easily be realized due to the flexibility of the software.

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(4) System extension.

It is possible to use another CPU as a supervisor, and monitor the states of the rotor system such as rotational speed, vibration, eccentricity, and bearing loads. Thus it can interrupt the work-horse CPU run and modify the control constants for stable and optimum control.

The disadvantages of the digital control are:

(1) Necessity for high-speed chips.

A sampling frequency at least double the oscillating force frequency is necessary for the satisfactory control. Therefore, a CPU chip of higher operational speed and an A/D converter of shorter conversion time are required for multi-axes control and higher rotational speed.

(2) High cost.

It is more expensive than the analogue circuit of the same complexity.

(3) Signal saturation and digitizing error.

As the saturation value of signals and the digitizing error are proportional to each other, it is impossible to make the former larger and the latter smaller at the same time.

In this paper, test results of digital control with a microcomputer together with those of analogue control are reported using a simple experimental apparatus of vertical single-axis control as the first step to multi-axes control with high-speed rotation.

2. Nomenclature

F	: magnetic attractive force	N
f_s	: sampling frequency ($= 1/T$)	Hz
g	: acceleration rate of gravity	m/s ²
h	: drop height of weight in impulse test	mm
i	: electric current circulating in magnetic coil	A
$k_i = (\partial F / \partial i)_u - (\partial F / \partial i)_l$		N/A
$k_x = (\partial F / \partial x)_u - (\partial F / \partial x)_l$		N/mm
$k_{0u}, k_{11}, k_{12}, k_{13}, k_2$	: control constants in Fig. 2	
K	: overall gain $= k_i k_{0u} k_{12} / (mL)$ (Eq. (1) & Eq. (11))	
L	: inductance of coil	H
m, m_w	: rotor mass and weight mass respectively	Kg
R	: coil resistance	ohm
T	: sampling period	s
T_c	: effective time constant of coil	s
X	: displacement of rotor upward from equilibrium point	s
ω_{DS}	: upper limit of effective angular frequency of differential circuit	rad/s

ω_l : lower limit of effective angular frequency of integral circuit

$$\omega_m = \sqrt{k_x / m} \quad \text{rad/s}$$

$$\omega_r = (R + k_{13}) / L \quad \text{rad/s}$$

Subscripts

u : upper coil

l : lower coil

3. Experimental Apparatus

Fig. 1 shows the experimental apparatus. A rotor with mass of about 1 kg having two pole pieces ⑤ of pure iron at both ends is attracted and controlled by two electro-magnets ④ (nominal 24V, 100 ohms) both separated by gaps of 1 mm. A circular steel plate ⑧ is attached to the aluminum shaft ⑥ and its displacement in the direction of vertical axis is detected by two displacement sensors ⑦ of eddy current type with the sensitivity of 1 V/mm, two displacement signals being averaged. A fan blade ⑨ is installed for rotation drive.

The upper magnetic coil is mounted on the plate ③ movable vertically with the aid of a mechanism of a ball-spline ① and a ball-screw ②, which enables easy and arbitrary setting of the gaps. Therefore, this mechanism is also used to measure accurately the magnetic attractive force as a function of the magnet current and gap.

Fig. 1 Experimental apparatus of magnetic bearing

4. Overall Control System

Fig. 2 shows the block diagram of the overall control system for this experimental apparatus. The rotor system having a negative spring constant is stabilized by compensation circuits of phase-lead PID. A negative feedback of electric current is performed in order to decrease the effective time constant of the coil.

Fig. 2 Block diagram of overall control system

In Fig. 2 k_x and k_i are gradients of the characteristic curve of the attractive force at the equilibrium point with regard to the gap and the current respectively, indicating constants instead of the inherently non-linear characteristic. Fig. 3 shows the characteristic curve of the electromagnetic coil. It is measured using a loadcell, the displacement sensors mounted on the movable plate and a current meter of the coil.

Fig. 3 Measured performance of attractive force

5. Analogue Control

The characteristic equation of the rotor system is shown as the following expression of fifth degree:

$$(s^2 - \omega_m^2)(s + \omega_T)(s + \omega_{DS})(s + \omega_{LF}) + k\omega_{DS}(B_1 s^2 + B_2 s + B_3) = 0 \quad \dots(1)$$

where

$$\begin{cases} B_1 = 1 + (k_{11}/k_{12})/\omega_{DS} \\ B_2 = \omega_{LF} + (k_{11}/k_{12})(\omega_{LF}/\omega_{DS} + 1) \\ B_3 = k_2/k_{12} + (k_{11}/k_{12})\omega_{LF} \end{cases} \quad \dots(2)$$

ω_{DS} indicates the upper limit of the angular frequencies at which the differential circuit of the operational amplifier works effectively, and ω_{LF} the lower limit of the angular frequencies at which the integral circuit works effectively. The integral circuit works as a proportional one at frequencies below ω_{LF} .

Fig. 4 Root locus of analogue control

In Fig. 4 the root locus is drawn with the overall gain k as a parameter, showing the permissible range of the gains for the stability of the system. Outside this range, the system becomes unstable due to a low frequency (~ 10 Hz) with a smaller gain and a high frequency (~ 50 Hz) with a larger gain. The range of the gains treated here is up to 100 Hz as only the levitation was tested without high-speed rotation.

6. Digital Control

The simple structure of the digital circuit is shown also in Fig. 2. Both A/D and D/A converters are of 12 bits. The conversion time of A/D is $25 \mu s$. A CPU of 16 bits is installed on a one-board microcomputer of general use, with 1K-word RAM, having ports used for both input and output. It runs with a clock of 2 MHz. Input keys mounted on the board enables the programming with mnemonic codes.

Control methods tested are as follows:

- (1) The most basic method replacing analogue PID. Against the input displacement signal e , the output control signal w is determined as;

$$w = k_{0v}(k_{11}e + k_{12}de/dt + k_2 \int e dt) \quad \dots(3)$$

e_j is the latest displacement data. e_{j-1} and e_{j-2} are the data T and $2T$ earlier respectively. Extrapolating the proportional and differential values $1.5T$ later by a parabola approximation using these data, Eq.(3) is approximated as follows;

$$w_j = k_{ov} \{ (15k_{11}/8 + 2k_{12}/T)e_{j-2} - (42k_{11}/8 + 5k_{12}/T)e_{j-1} + (35k_{11}/8 + 3k_{12}/T)e_j + k_2 T \sum_{i=-\infty}^j e_i \} \dots (4)$$

Here $1.5T$ means the average time when the output w_j $1T$ later than the acquisition time of the latest data e_j , works effectively up to the next output time $2T$ later. While the equation above has an accuracy of second order, only the integral term was set with first order in order to decrease the operational time using as simple a calculation method as possible. In this case T became $681 \mu s$. The values of control constants of k_{ov} etc. correspond to those of analogue control.

- (2) PD method ($T = 440 \mu s$).
The integral term being omitted with $k_2 = 0$, T was decreased. Other control constants are the same.
- (3) PD method ($T = 681 \mu s$).
 T was increased intentionally from that of (2) to reveal its effect.
- (4) Z-transformation method (PID with 1st order, $T = 461 \mu s$)
As the digital control should be treated inherently by the sampling control theory, the closed-loop transfer function was calculated using z-transformation, and the next equation was obtained as the z-transform of the displacement x against the impulsive disturbance of unit strength.

$$X^*(z) = \frac{z(e^{\omega_m T} - e^{-\omega_m T})}{2m\omega_m(z - e^{\omega_m T})(z - e^{-\omega_m T})} \left\{ 1 + \frac{k_i(z-1)}{mL\omega_m^3} \left(\frac{A}{z - e^{\omega_m T}} + \frac{B}{z - e^{-\omega_m T}} \right) + \frac{C}{z - e^{-\omega_m T}} + \frac{D}{z-1} \right\} G^*(z) \dots (5)$$

where

$$\begin{cases} A = \frac{1}{2(\omega_T/\omega_m - 1)}, B = \frac{1}{2(\omega_T/\omega_m + 1)} \\ C = \frac{-1}{(\omega_T/\omega_m)((\omega_T/\omega_m)^2 - 1)}, D = \frac{-1}{\omega_T/\omega_m} \end{cases} \dots (6)$$

$G^*(z)$ is the transfer function of the compensation circuit, and the following expression is adopted here.

$$G^*(z) = \frac{a_2 z^2 + a_1 z + a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - z^{-1}} \dots (7)$$

This corresponds with the next relation between input e and output w :

$$w_j = a_{-2}e_{j+2} + a_{-1}e_{j+1} + a_0e_j + a_1e_{j-1} + a_2e_{j-2} + w_{j-1} \dots (8)$$

where e_{j+1} and e_{j+2} indicate the predicted values of e of T and $2T$ later respectively, and are actually calculated extrapolating by the parabola approximation using three data of e_j , e_{j-1} , and e_{j-2} : therefore

$$w_j = (6a_{-2} + 3a_{-1} + a_0)e_j + (-8a_{-2} - 3 \times a_{-1} + a_1)e_{j-1} + (3a_{-2} + a_{-1} + a_2) \times e_{j-2} + w_{j-1} \dots (9)$$

Compared with the calculation method (1), this method gives smaller operation time due to the lack of multiplication of the integral term. Now this equation is a kind of recurrence formula and is transformed, taking the initial value of w as zero, into the next equation.

$$w_j = (5a_{-2} + 2a_{-1} - a_1 - a_2)e_j - (3a_{-2} + a_{-1} + a_2)e_{j-1} + \sum_{k=-2}^2 a_k \cdot \sum_{i=-\infty}^j e_i \dots (10)$$

Therefore, although derived from z-transformation method, as long as the compensation formula of Eq.(7) is adopted this method is essentially the PID control of 1st order, which calculates proportional and differential values from two data.

Fig. 5 shows the root locus of z-transformation with the values of control constants of k_{ov} etc. almost the same as those of the digital control method (1). k is expressed as follows;

$$k = k_i a_{-2} / (mL\omega_m^3)$$

k_0 is the design value of k . Letting $a_1 = a_2 = 0$ the order of the characteristic equation is dropped down to fourth. Fig. 5 indicates a wide range of allowable values of the gain for stability. However, actually permissible range of the gains are rather narrow.

Fig. 5 Root locus of z-transform

7. Experimental Results

With both the analogue and digital control methods, the system worked extremely stably. However, in the radial direction the system is only passively stable with a positive but low stiffness. Therefore, at the rotational speed of more than several hundred rpm the lateral vibration increases and finally causes the rotor to touch and stick to either the upper or the lower pole piece of the stator. Hereafter, performance test was executed with the rotor suspended stably without rotation.

Fig. 6 shows the transfer function of the rotor displacement measured by an FFT analyzer against the white noise imposed on the lower coil together with the regular control signal. The calculated value in the figure is obtained by the equation below:

$$X(s) = \frac{k_k k_{it}(s + \omega_T)(s + \omega_{DS})(s + \omega_{LF})}{mL\{(L_i/L)s + (R_i/R)/T_c\}} \\ / \{(s^2 - \omega_m^2)(s + \omega_T)(s + \omega_{DS})(s + \omega_{LF}) + k\omega_{DS}(B_1 s^2 + B_2 s + B_3)\} \quad \dots(11)$$

In the analogue control the measured and calculated values correspond to each other well especially near the lower natural frequency (calculated value being 8.9 Hz). Near the higher natural frequency (calculated 66.3 Hz) the measured curve has some roughness, representing eigen-mode of vibration of the supporting structure with rather low stiffness. This is ascertained by giving mechanical impulse to the supporting structure with the rotor fixed on it, analyzing the frequency distribution of the displacement outputs and obtaining a curve with almost the same roughness as stated above. The peak of the gain near the higher natural frequency hardly appears even on the calculated curve due to the large damping. At much higher frequencies the measured value of the gain becomes about 10dB lower than the calculated one, which seems to be the effect of eddy current occurring in the pole pieces.

In the case of the digital method (1) the characteristic curves are similar to the analogue ones. Calculated and measured values also correspond well to each other. In other cases of the digital methods (2) (3) and (4) almost the same results were obtained.

A special feature in the digital method is that the phase angle at DC (zero frequency) is 90°, which is clearly different from 180° in the analogue case. This indicates that in the digital control integral action works effectively from DC with $\omega_{LF} = 0$ in Eq.(11).

Fig 7(a) shows the responses of control signal and rotor displacement in the analogue control when a mechanical impulse was imposed on the base plate of

Fig. 6 Transfer function
(upper: analog lower: digital (1))

the structure. The superposition of waves of 54 Hz and 13 Hz is the result of imposition of damped external vibration generated by the eigen-mode of the vibration of the supporting structure as stated before. It can be seen that the phase of the control signal leads the displacement signal owing to the phase-lead compensation circuit of PID.

In the digital response of Fig. 7(b) the wave form of the displacement is almost the same as that of analogue case, while the control signal displays rather large noises even in the steady state. The details of this noise are shown in Fig. 8. The control signal or D/A output represents a stepwise form, thus the frequency of $f_s/2 = 1.1$ KHz is amplified greatly as the noise.

In both analogue and digital cases the displacement signal contains noise of about 5 mV of peak-to-peak which seems to be the thermal noise of operational amplifiers. In the analogue case, however, such high frequencies cannot be amplified due to the decrease of the gain of the differential action. In the digital case, on the contrary, given the input data $e_{j-2} = 5$ mV, $e_{j-1} = 0$ and $e_j = 5$ mV to Eq.(4) and being amplified about 120 times, the output becomes $\Delta w = 0.6$ V, and turns into a noise of $f_s/2$ with the order of 1V, as shown in Fig. 7(b) and Fig. 8.

(a) Analogue upper: control signal
lower: displacement

(b) Digital (1)

Fig 7 Impulse response

Fig. 8 Detail of noise of digital control signal

This noise does not appear in the rotor displacement in the steady state because the overall system has enough low gain at high frequencies; the coil and the rotor itself work as a kind of low-pass filter. However, as the amplitude of the control signal increases and approaches the D/A output saturation value of +5V, the upper and lower peaks of the noise reach the saturation value and are cut off. This noise thus affects the regular control signal as the average value and harms the control performance. Adopting a CPU of higher-speed operation in order to decrease T makes this problem more serious as the noise amplitude is increased in inverse proportion to T . Therefore some countermeasure is required.

This noise amplitude is almost the same as in the digital control methods (1) (2) and (3), while about half as small in method (4). This is because the amplification factor of the linear approximation of first-order is smaller than that of the parabola extrapolation of second-order.

A weight of 71 grams is dropped on to the base plate from some specific heights giving impulses, and the maximum vibration amplitudes of transitional responses of the rotor displacement are measured by using both the analogue and digital methods with the result shown in Fig. 9. The abscissa represents the drop height of the weight h .

The strength of an impulse is proportionate to the momentum of the weight at the moment of collision, $m_w \sqrt{2gh}$. Therefore, for the analogue data (0) in Fig. 9, a curve was fitted by the least square method taking $\Delta x \propto \sqrt{h}$ into consideration. For the digital control methods (1)(2)(3) and (4) the same curves were drawn for guidance. The marks $\bigcirc$ show the average value of 5 data and the vertical lines the scatters of data.

The analogue data correspond well with the $\sqrt{h}$ curve, while the digital data show large scatters. This is considered to be due to the relation between the initial condition of the position and velocity of the rotor at the moment of the impulse and the sampling timing; the digital methods (2) and (4) with smaller values of T give less scatters.

When $h = 133 \text{ mm}$ the digital methods (1) (2) and (3) show quite large scatters of data. In cases of extremely large response of the amplitude as shown by the dotted lines in Fig. 9, the system becomes uncontrollable and the rotor touches and sticks to either the upper or lower pole piece of the stator. Contrary to this, the digital method (4) gives an anti-impulse performance as good as the analogue control. This is because it does not suffer so much from the initial condition due to small T , and output saturation value of +5V due to the small noise of $f_s/2$ for its first-order accuracy.

The output saturation value of the analogue control is its power supply voltage, +15V. This difference of the saturation values of the analogue and the digital control is not essential. The saturation problem can be avoided by expanding the output range of the D/A converter, although it increases the digitizing error at the same time.

Thus an almost equal control performance has been obtained with the digital control as with the analogue control. Although the PID control tested here for the digital control does not exceed the analogue control in performance, controls of higher grade or non-linear controls can easily be realized by using more sophisticated softwares. Digital control superior to analogue control is possible by adopting a high-speed processor and shortening the operational time.

It is clear that, in addition to the merits of simplification of hardware etc., the digital control of magnetic bearings has a great potential.

Fig. 9 Maximum amplitude of displacement in impulse test

8. Conclusions

Using an experimental apparatus of a magnetic bearing with vertical single-axis control, digital control with a microcomputer was tested together with analogue control, and the following results were obtained.

- (1) The principal characteristic of the system is well expressed with a simple model in both the analogue and the digital control.
- (2) Performance of the digital control was almost equal to that of the analogue control in impulse tests.
- (3) In the digital control, PID control of first-order accuracy gave satisfactory results and it is thus better to pursue the decrease of the sampling period with a simple operation method than to raise the accuracy with high order.
- (4) Noise of half the sampling frequency is amplified by the digital operation and becomes involved in the control signal to a large degree. This does not affect the rotor displacement in a steady state. However, when the peak value of the noise reaches the output saturation of the D/A converter transitionally, it affects the regular control signal as the average value and harms the control performance.

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