Relaminarization of Pulsatile Turbulent Flow in a Rectangular Duct*

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Experiments on the behaviour of turbulent slugs and relaminarization in pulsatile flows are made using four ducts with different aspect ratios. When acceleration is relatively low, the propagation characteristics of a slug is qualitatively similar to that of a steady duct flow. For example, the propagation speed of the leading edge of the slug is larger than that of the trailing edge. In such a case relaminarization of flow is controlled mainly by viscous dissipation. Meanwhile, when acceleration is relatively high, the disappearance and the following appearance of turbulent slugs are seen at fixed phases in a cycle and these phases hardly change in the downstream direction. This phenomenon is not inferable from the knowledge of steady duct flows. The relaminarization under the influence of such an acceleration is controlled mainly by acceleration itself and the limit under which relaminarization occurs is described by an acceleration parameter K first introduced about steady accelerating boundary layer flows. Its critical value agrees well with that obtained for steady accelerating flows.

Key Words : Unsteady Flow, Pulsatile Flow, Rectangular Duct, Turbulence, Relaminarization, Acceleration Parameter

1. Introduction

It is well known that a strong spatial acceleration causes a relaminarization, i.e. retransition of steady turbulent flows. Such a phenomenon attracts much attention of investigators interested in the generation and damping of turbulence⁽¹⁾⁻⁽³⁾. From an industrial point of view, the heat transfer rate, for example, drastically decreases when relaminarization occurs, and a large number of studies have been done mainly in this field. Here the relaminarization is defined as a process by which an initially turbulent flow is rendered effectively laminar after cessation of a wall-layer bursting. Bursting is now known to be a primary mechanism for the generation of turbulent energy and to consist of the so-called ejection, sweep, wall-ward and outward interactions. In brief, bursting is a periodically sequential event during which a retarded fluid existing near the wall is swept by a fluid from upstream with a higher axial velocity. The bursting rate decreases monotonically in the course of relaminarization and the burst measurement is considerably complex. Therefore, it is difficult to identify a point at which a bursting ceases, so that simple substitute par-

ametric criterions are used. In particular, $\Delta\tau$ and K are well known. Patel and Head⁽⁴⁾ proposed the acceleration parameter $\Delta\tau$ considering the gradient of shear stress near the wall. The other acceleration parameter K was first derived by Moretti and Kays⁽⁵⁾. They suggested that relaminarization in accelerating steady flows is associated with the cessation of a turbulent bursting and proposed a critical value of 3.5×10^{-6} for K . However, it is difficult to select the best criterion at present time.

The authors found out that a pulsatile turbulent flow is relaminarized in an accelerating period when the pulsation frequency is relatively high even if the minimum Reynolds number is much higher than the critical Reynolds number of steady flow⁽⁶⁾. Here the critical Reynolds number is defined as the value at which the intermittency factor becomes unity. The results are summarized as follows: The velocity waveforms at two different axial positions whose distance is 20 diameters are recorded on oscillograph paper. Both waveforms were found to coincide with each other when the upstream measuring position is farther than about 50 diameters from an inlet, and the phases at which a bursting ceases and occurs are almost the same on both velocity waveforms. Therefore, the relaminarization occurs and then a turbulence generates at fixed phases in any axial position which is farther than 50 diameters from the inlet.

This fact means that a fluid element flows downstream while experiencing alternately a laminar and a turbulent state, and that relaminarization is not controlled by viscous dissipation but by temporal acceleration when pulsation frequency is relatively high. Such a relaminarization is

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very similar to the relaminarization observed in steady accelerating turbulent flows. So the authors introduced a parametric criterion K for pulsatile turbulent flows in a pipe and examined its applicability⁽⁷⁾. The critical value of K was found to agree well with that reported for steady accelerating flows.

In the present paper we concentrate on the relaminarization of pulsatile flows in rectangular ducts of various aspect ratios. First we investigate the behaviour of turbulent slugs, i.e. the propagation speeds of leading and trailing edges, and then examine the applicability of the parametric criterion K .

2. Nomenclature

- $2a, 2b$: width and height of duct
 D_h : hydraulic diameter = $4ab/(a+b)$
 f : pulsation frequency
 K : acceleration parameter
 $\Delta p/L$: pressure gradient
 Re : instantaneous Reynolds number
 $= u_m D_h / \nu$
 Re_{os} : Reynolds number based on velocity amplitude = $|u_{m,os,1}| D_h / \nu$
 Re_{ta} : time-averaged Reynolds number
 $= u_{m,ta} D_h / \nu$
 t : time
 u : axial velocity component
 u_{cl} : velocity on the center line
 u_m : cross-sectional mean velocity
 x, y, z : coordinate shown in Fig.1 ($y' = y/a, z' = z/b$)
 γ : intermittency factor
 $\lambda_{q,t}$: turbulent quasi-steady friction factor
 ν : kinematic viscosity
 ρ : density
 τ_w : wall shear stress
 ω : angular frequency of pulsation = $2\pi f$
 ω' : dimensionless angular frequency
 $= (D_h/2)^2 \omega / \nu$
 Subscripts and others
 c : critical value
 os, ta : oscillatory and time-averaged components
 1 : fundamental wave in the finite Fourier series
 $|, \angle$: amplitude and phase angle

3. Experimental Apparatus and Procedure

The experimental apparatus using air as the working fluid is schematically shown in Fig.1. Four kinds of ducts are used. The test section of 40 mm x 40 mm duct is made of vinyl chloride and those of remaining ducts (20 mm x 20 mm, 90 mm x 18 mm, 120 mm x 12 mm) are made of acrylic resin. The thickness of each duct is 10 mm. Pulsation frequencies are 0.139, 0.300, 1.027, and 2.096 Hz and the piston stroke can be varied from 20 mm to 168 mm at about 20 mm intervals. In this apparatus the rate of the amplitude of the first harmonic ($n=2$) to the fundamental amplitude is less than 8% and those of higher harmonics ($n=3, 4, \dots$) are negligibly small. Such a rate was already found not to significantly affect the relaminarization⁽⁶⁾. Each test

Fig.1 A schematic of the experimental apparatus (A and B denote measuring positions)

section was followed by a circular pipe having inner diameter of 40.3 mm. The cross-sectional mean velocity in the test section was evaluated from the measured value obtained at the measuring position A by using a hot wire anemometer. The time-averaged value $u_{m,ta}$ and amplitude $|u_{m,os,1}|$ and phase angle $\angle u_{m,os,1}$ of the fundamental component were obtained by approximating 12 measured values of u_m in a cycle by a finite Fourier series.

For each duct the propagation speeds of the leading and trailing edges of a slug were measured, and on the basis of this result the position of observing velocity waveforms of both steady and pulsatile flows was determined and fixed to B in Fig.1. The distance from the entrance to the position B is about 110 times the hydraulic diameter D_h for ducts of 20 mm x 20 mm, 90 mm x 18 mm, 120 mm x 12 mm and 67.5 D_h for 40 mm x 40 mm duct as shown in Table in Fig.1. At the cross-section B of each duct a hot wire probe was inserted and placed at a point of $y'=0$ and $z'=0.75$. An output signal of the hot wire anemometer was recorded on photo oscillograph paper, and we judged whether or not the flow is fully turbulent observing the velocity waveforms. For example, velocity waveforms shown in Fig.2 were obtained by increasing the time-averaged Reynolds number for a fixed piston stroke. In this case the limit between a fully turbulent flow and a flow accompanied by relaminarization is the case of $Re_{ta} = 10700$ and $Re_{os} = 7740$.

Fig.2 Velocity waveforms with and without relaminarization (20 mm x 20 mm duct, $\sqrt{\omega'} = 3.54$, $\alpha_p = 49$ mm, $x/D = 120$, $y/a = 0$, $z/b = 0.75$, α_p : half the piston stroke)

As will be stated later in detail a bursting ceases almost at the same time in a duct when the pulsation frequency is highly increased, and then turbulent eddies flow downstream while decaying. It is well known that an eddy with higher frequency decays faster than one with lower frequency. Take up a phase after the cessation of a bursting and assume the phase angle between this phase and the phase where the bursting ceases is constant for any pulsation frequency. Under this assumption the time interval between these phases becomes smaller as the pulsation frequency increases. Therefore, the frequency of a turbulent eddy remaining at the later phase we mentioned becomes higher with an increase in pulsation frequency. And it becomes difficult to distinguish between the decay and generation of a turbulence for a higher pulsation frequency. This is a fatal defect of the present method. This method cannot be applied when the pulsation frequency is much increased beyond the present experimental range. In such a case the judgement of the limit between a fully turbulent flow and a flow accompanied by relaminarization must be done after turbulence intensity measurements.

4. Propagation of Turbulent Slug

It is necessary for full understanding of the condition under which relaminarization occurs to investigate the behaviour of turbulent slugs. Here the propagation speeds of the leading and trailing edges of slugs are considered.

4.1 Steady flow

Since the generation of a turbulent slug in the transitional Reynolds number range occurs randomly in time and space, it is difficult to measure the propagation

speeds by a hot wire probe, so that the measurement is not made here. From the previous experimental results it is evident that the propagation speed of the leading edge is higher than the cross-sectional mean velocity u_m and that of the trailing edge is lower than $u_m^{(8)-(10)}$. This fact means that the slug grows in the downstream direction. Accordingly, the intermittency factor γ increases downstream and becomes unity at a certain position if the pipe is long enough to be seen from the experimental result of Rotta⁽¹¹⁾. As is well known, the critical Reynolds number giving the limit of $\gamma = 1$ is dependent on the axial position of a monitor probe as well as the condition of an incoming flow. This critical Reynolds number corresponds to the limit between fully turbulent flows and flows accompanied by relaminarization.

4.2 Pulsatile Flow

Turbulence intensity $u'_{rms}/u_{m,ta}$ at the entrance of every duct is almost 1.5 %. Since the exit size of the inlet nozzle is designed slightly larger than the size of the test section for each duct, a flow is tripped at the entrance. Velocity measurements were mainly made at $x/D_h \geq 42.5$. Two typical behaviours were observed under the present experimental conditions according as the pulsation frequency is low or high. For a low frequency a slug grows in the downstream direction in a similar manner to one in a steady flow. For a high frequency a slug neither grows nor decays when $x/D_h \geq 42.5$. The occurrence of a turbulent bursting and the onset of relaminarization are fixed in a cycle and do not change throughout the test section mentioned here. The former case is shown in Fig.3. In drawing the velocity waveforms at several axial points which were obtained independently, the ordinate of each curve was displaced vertically by equal distance. Here it should be emphasized that the phases of appearance and disappearance of a turbulence on the velocity waveforms do not change in every cycle.

At $x/D_h = 42.5$ in Fig.3(a) the flow is laminar from $\omega t/(\pi/6) = 3.5$ to 5. This period becomes shorter and the appearance of laminar flow lags as x/D_h increases. If we define the intermittency factor γ of a pulsatile flow in terms of the ratio of a period where the flow is turbulent to the pulsation period T , the value of γ increases downstream and approaches unity. In Fig.3 (c) open circles denote the occurrence of a turbulent bursting and shaded circles do the onset of relaminarization. A solid line runs from the open circle at $x/D_h = 42.5$ toward downstream, the gradient being a reciprocal of the time-averaged cross-sectional mean velocity $u_{m,ta}$. If the measured value denoted by open circle comes over this line, the propagation speed of the leading edge is higher than $u_{m,ta}$. In Fig.3 the propagation speed of the leading edge is almost the same as $u_{m,ta}$ and that of the trailing edge is about $0.4u_{m,ta}$. That is, the propagation speeds of a slug's leading and trailing edges are different from each other. Such a slug's behaviour is similar to that of a steady flow.

Fig.3 Propagation characteristics of turbulent slug (20 mm x 20 mm duct, $\sqrt{\omega'} = 3.53$, $\alpha_p = 10$ mm, $Re_{ta} = 5150$, $Re_{os} = 2300$)

On the other hand, when the pulsation frequency f is increased to 2.096 Hz ($\sqrt{\omega'} = 9.31$) in the same duct of 20 mm x 20 mm, the slug's behaviour changes drastically as can be seen from Fig.4, which cannot be predicted from the behaviour of a steady flow. Figure 4(a) shows that the waveform does not change in the downstream direction, and hence the intermittency factor γ is nearly constant when $x/D \geq 42.5$ [see Fig.4(b)]. In Fig.4(c) the phase where the relaminarization occurs can not be made clear because the bursting ceases gradually, so that the phase is represented by an error band (I).

In such a flow field a fluid element flows downstream, alternating between laminar and turbulent state. This phenomenon is very similar to the relaminarization observed in steady accelerating turbulent flows, and hence we can call this phenomenon a relaminarization due to temporal acceleration.

Fig.4 Propagation characteristics of a turbulent slug (20 mm x 20 mm duct, $\sqrt{\omega'} = 9.31$, $\alpha_p = 28$ mm, $Re_{ta} = 25800$, $Re_{os} = 21300$)

Meanwhile, in the example shown in Fig. 3, if a fluid element once becomes turbulent, it never reverts to laminar state while flowing downstream. According to the above terminology of relaminarization stated in Fig.4, it is not correct to call the phenomenon in Fig.3 the relaminarization. But here we define relaminarization as the case when a turbulent flow reverts to a laminar flow in a cycle at a given axial position, regardless of its cause. According to this definition the flow in Fig.3 can be regarded as a flow accompanied by relaminarization.

The magnitude of acceleration is represented by $\omega' Re_{os}$ which is readily derived by differentiating a finite Fourier approximation of u_m , Eq.(6), shown later. Therefore, it should be kept in mind that the relaminarization of pulsatile flows is controlled by the nondimensional angular frequency ω' as well as the Reynolds number based on the velocity amplitude, Re_{os} .

Though we didn't observe a change in velocity waveforms in the downstream direction in all runs, we can roughly generalize that the slug behaviour for a relatively low pulsation frequency is similar to that in Fig.3 and the behaviour for a relatively high frequency resembles that in Fig.4. Besides, if the acceleration becomes much smaller than that in Fig.3, the occurrence of a turbulent bursting and the onset of relaminarization are not fixed in a cycle and slugs appear randomly in time in a similar manner to those in steady flows.

5. Experimental Results about Relaminarization and Discussion

The critical Reynolds number giving $\gamma = 1$ in a steady flow corresponds to the limit between fully turbulent flows and flows accompanied by relaminarization. Here we first determine the critical Reynolds numbers of steady flows in four kinds of ducts and then determine the above mentioned limits of pulsatile flows listed in Table 1.

The measured values of the limit are shown in Figs.5 through 8. For each duct, regardless of ω' , the critical value of Re_{ta} approaches that of a steady flow as Re_{os} decreases. On the other hand, they distribute along a straight line $Re_{ta} = Re_{os}$ as Re_{os} increases. Since it is difficult to collect detailed information from Figs.5 through 8, the measured values of the limit are replotted in Figs.9 through 12 by replacing the ordinate Re_{ta} with $Re_{ta} - Re_{os}$, where $Re_{ta} - Re_{os}$ represents the minimum Reynolds number in a cycle. The critical value of $Re_{ta} - Re_{os}$ approaches the limit of a steady flow denoted by a solid line as Re_{os} decreases, as readily supposed from Figs.5 through 8. The condition that a steady turbulent flow in a duct of uniform cross-section reverts to a laminar flow when Reynolds number Re decreases is described as viscous dissipation⁽¹⁾. Therefore, the result obtained here means that the relaminarization of pulsatile flow is mainly controlled by viscous dissipation when the acceleration is relatively low.

When Re_{os} is relatively high the critical value of $Re_{ta} - Re_{os}$ becomes much larger than the steady limit and slightly

Table 1 Experimental conditions

$2a \times 2b$ mm	$\sqrt{\omega'}$	a_p mm
20 x 20	2.39	10,18,28,40,49,57,69,84
	3.54	10,18,28,40,49,57,84
	6.56	10,18,28,40
	9.31	10,18,28,40
40 x 40	4.86	10,18,28,40,49,57,69,84
	7.13	10,18,28,40,49,57,69,84
	13.25	10,18,28,40,49
	18.93	10,18,28,40,49
90 x 18	3.65	10,18,28,40,49,57,69,84
	5.37	10,18,28,40,49,57,69,84
	9.84	10,18,28,40,49
	14.15	10,18,28,40,49
120 x 12	2.61	10,18,28,40,49,57,69,84
	3.89	10,18,28,40,49,57,84
	7.23	10,18,28,40,49,57,84
	10.17	10,18,28,40,49

a_p : half the piston stroke

Fig.5 The limit between fully turbulent and relaminarizing flows in 20 mm x 20 mm duct

Fig.6 The limit between fully turbulent and relaminarizing flows in 40 mm x 40 mm duct

Fig.7 The limit between fully turbulent and relaminarizing flows in 90 mm x 18 mm duct

Fig.8 The limit between fully turbulent and relaminarizing flows in 120 mm x 12 mm duct

depends on ω' . Referring to the experimental results of propagation speed shown in the preceding chapter we may conclude that the relaminarization in such a case is controlled not by viscous dissipation but by acceleration. From this point of view, the present authors previously applied an acceleration parameter K first introduced for steady accelerating flows to a pulsatile flow in a pipe, and revealed that the relaminarization can be described by K when the acceleration is relatively high. In what follows the applicability of K to a pulsatile flow in a rectangular duct will be examined. The acceleration parameter K extended to include an unsteady flow in a duct with arbitrary cross-section is represented by⁽⁷⁾

$$K = \frac{\nu}{\rho u_{cl}^2} \frac{\Delta p}{l} \dots\dots\dots (1)$$

$$\Delta p/l = \rho du_m/dt + 4\tau_w/D_h \dots\dots\dots (2)$$

$$\tau_w = \lambda_{q,t} \rho u_m^2/8 \dots\dots\dots (3)$$

For internal flows it is convenient to replace the center line velocity u_{cl} with the cross-sectional mean velocity u_m . Therefore, we replace u_{cl} with u_m in the same manner as for a pulsatile flow in a pipe and further substitute the quasi-steady friction factor $\lambda_{q,t}$ for $\lambda_u(t)$ to get⁽⁷⁾

$$K = \frac{0.545\nu}{u_m^3} \left[\frac{du_m}{dt} + \frac{\lambda_{q,t}}{2D} u_m^2 \right] \dots\dots\dots (4)$$

$$\lambda_{q,t} = a_1 Re^{-b_1} \dots\dots\dots (5)$$

The replacement of $\lambda_u(t)$ with $\lambda_{q,t}$ is supported by measurements made under the same experimental conditions as those examined here⁽¹²⁾. The proportional constant a_1 and the exponent b_1 are listed in Table 2. Equation (4) represents the effect of viscous dissipation when du_m/dt can be neglected, whereas it does the effect of acceleration itself when $\lambda_{q,t} u_m^2/2D$ is much smaller than du_m/dt . In this experiment the cross-sectional mean velocity u_m in every run can be well expressed by the following finite Fourier approximation.

$$u_m = u_{m,ta} + |u_{m,os,1}| \cos(\omega t + \angle u_{m,os,1}) \dots\dots (6)$$

Substitution of Eq. (6) into Eq. (4) gives

$$K = \frac{0.545}{Re^3} [-4\omega' Re_{os} \sin(\omega t + \angle u_{m,os,1}) + \lambda_{q,t} Re^2/2] \dots\dots\dots (7)$$

where the instantaneous Reynolds number Re is represented by

$$Re = Re_{ta} + Re_{os} \cos(\omega t + \angle u_{m,os,1}) \dots\dots (8)$$

$$\left. \begin{aligned} Re &= u_m D_h / \nu, Re_{ta} = u_{m,ta} D_h / \nu \\ Re_{os} &= |u_{m,os,1}| D_h / \nu \end{aligned} \right\} \dots\dots\dots (9)$$

It is clear from Eq. (7) that the magnitude of acceleration can be expressed by $\omega' Re_{os}$.

The critical value K_c is defined as the maximum value of K during a cycle, where the measured values of Re_{ta} and Re_{os} giving the limit between a fully turbulent flow and a flow accompanied by relaminarization are substituted for Re in Eq. (8). In every run we confirmed that the relaminarization

Fig.9 The lowest Reynolds number of the flow indicating the limit between fully turbulent and relaminarizing flows in 20 mm x 20 mm duct

Fig.10 The lowest Reynolds number of the flow indicating the limit between fully turbulent and relaminarizing flows in 40 mm x 40 mm duct

Fig.11 The lowest Reynolds number of the flow indicating the limit between fully turbulent and relaminarizing flows in 90 mm x 18 mm duct

Fig.12 The lowest Reynolds number of the flow indicating the limit between fully turbulent and relaminarizing flows in 120 mm x 12 mm duct

occurred near the phase of $K = K_c$.

The dependence of K_c on Re_{os} was not recognized in the present Re_{os} range, so that we showed an ensemble averaged value of K_c for each ω' in Fig.13. Here the critical value K_c of a steady flow in every duct is shown on the line of $\sqrt{\omega'} = 1$ for convenience. It is of importance to note that K_c of a pulsatile flow approaches the steady flow value as $\sqrt{\omega'} \rightarrow 0$ and it does a constant $K_c = 4.5 \times 10^{-6}$ as $\sqrt{\omega'}$ increases for every duct. However this result may not be valid outside of the present Re_{os} range. Accordingly, the arrangement of Fig.13 is not appropriate for systematically representing the effect of acceleration on the critical value K_c .

In order to specify the effect of $\omega' Re_{os}$ on K_c we show K_c against $\omega' Re_{os}$ in Figs.14 through 17. Additionally, Fig.18 shows a result of a circular pipe. The reason why the scattering of data points increases with an increase in $\omega' Re_{os}$ can be explained as stated in Fig.2 in the preceding chapter. The solid line is drawn so as to best fit the measured values of K_c . In every duct, as can be readily supposed from Fig.13, K_c approaches a steady value as $\omega' Re_{os}$ decreases, whereas it does $K_c = 4.5 \times 10^{-6}$ as $\omega' Re_{os}$ increases. The latter critical value agrees well with that obtained for a circular pipe.

Consequently, the consideration of the propagation speeds of turbulent slugs and that of K_c leads to a conclusion that the relaminarization of a pulsatile flow is controlled by viscous dissipation when the acceleration is relatively low, whereas it is caused by the acceleration itself when the acceleration is relatively high. Judging from Figs.14 through 17, the region where the relaminarization is controlled by viscous dissipation is $\omega' Re_{os} \lesssim 10^4$, and

Table 2 Coefficient a_1 and exponent b_1 of quasi-steady friction factor

$2a \times 2b$ mm	a_1	b_1
20 x 20	0.4288	0.2839
40 x 40	0.4247	0.2819
90 x 18	0.3811	0.2680
120 x 12	0.4776	0.2892

Fig.13 Relation between acceleration parameter K_c and $\sqrt{\omega'}$

Fig.14 Relation between K_c and $\omega' Re_{os}$ in 20 mm x 20 mm duct

Fig.15 Relation between K_c and $\omega' Re_{os}$ in 40 mm x 40 mm duct

Fig.16 Relation between K_c and $\omega' Re_{os}$ in 90 mm x 18 mm duct

Fig.17 Relation between K_c and $\omega' Re_{os}$ in 120 mm x 12 mm duct

the region where it is caused by acceleration is $\omega' Re_{os} \geq 10^6$. In the intermediate region $10^4 \leq \omega' Re_{os} \leq 10^6$ the relaminarization is caused by a combined effect of acceleration and viscous dissipation. Further details of relaminarization in this region are not known at present.

Although we obtained a critical value $K_c = 4.5 \times 10^{-6}$ for a relatively high acceleration under the present experimental conditions, K is not an only parameter describing the relaminarization of accelerating flows as already explained by Sreenivasan⁽³⁾ who reviewed various parametric criterions and examined their applicability in detail. Therefore, it should be kept in mind that the present critical value $K_c = 4.5 \times 10^{-6}$ is only one of the indications (criterions) describing the relaminarization of pulsatile flows. In other words, it is not clear whether or not the present critical value can be applied to accelerating flows in a duct with an arbitrary cross-section. Concerning this point further experimental work must be done.

6. Conclusions

Experiments on the behaviour of turbulent slugs and the relaminarization in pulsatile flows in four kinds of rectangular ducts were made. In this paper the relaminarization is defined as the state that a turbulent flow reverts to a laminar flow during one cycle at a given axial position, regardless of its cause. The results obtained are summarized as follows:

(1) When an acceleration is relatively low, the characteristics of a turbulent slug in a pulsatile flow qualitatively agree with those of a steady duct flow. That is, the turbulent slug grows in the downstream direction. On the other hand, the phases of occurrence and cessation of a turbulent bursting do not change downstream of $x/D_h = 42.5$ when an acceleration is relatively high. Consequently, the existence of a turbulent slug is localized within some limited phase range of a cycle. The behaviour of turbulent slugs in the region of $0 \leq x/D < 42.5$ is not investigated here.

(2) On the basis of the conclusion (1) it can be said that a fluid element which once becomes turbulent never reverts to laminar state when an acceleration is relatively low. The relaminarization under such a condition is controlled mainly by viscous dissipation. In a pulsatile flow with a relatively high acceleration a fluid element flows downstream while alternating between laminar and turbulent states. The phenomenon that a fluid element which once becomes turbulent reverts to a laminar state is the same as the relaminarization of steady accelerating flows subjected to a very strong favourable pressure gradient. The relaminarization under such a condition is caused mainly by acceleration. Therefore, the relaminarization in the intermediate region is supposed to be caused by a combined effect of viscous dissipation and acceleration.

Fig.18 Relation between K_c and $\omega' Re_{os}$ in a circular pipe

(3) A parameter K first introduced for accelerating steady flows was applied to pulsatile flows in rectangular ducts. In every duct the critical value K_c approaches a steady critical value when an acceleration is relatively low, and it approaches $K_c = 4.5 \times 10^{-6}$ when an acceleration is relatively high. The latter critical value K_c agrees well with that of a circular pipe.

(4) Judging from the variation in K_c , the region where the viscous dissipation is dominant is $\omega' Re_{os} \leq 10^4$, whereas the region where the effect of acceleration is dominant is $\omega' Re_{os} \geq 10^6$.

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