Fatigue Crack Propagation and Final Fracture of High Speed

Rotating Disk under Cyclic Start and Stop*

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Cyclic start-stop test of notched disk and fatigue test of compact specimen were made for an Al-alloy (JIS 6061-T6). Effects of the symmetry of notch and the thickness of disk on the crack growth and the fatigue life were investigated. These effects were found pretty small. Relations between the crack growth rate and the cyclic range of stress intensity factors for both disk- and compact specimen-tests were found to be nearly coincident with each other. Criteria on the final fracture of disk were discussed based on the critical crack length. K_{IC} criterion predicted a too small life (or a too small critical crack length), while mean stress criterion gave a little longer life (or a little larger critical crack length). Authors' formulae derived from rigid plastic analysis showed a fairly good agreement between the calculations and the experiments.

key words : Rotating Disk, Fatigue, Crack Propagation, Fracture, Critical Crack Length, Mean Stress Criterion

1. Introduction

A knowledge of the low-cycle fatigue of high speed rotating disk under cyclic start and stop is essential to the strength-design of the high speed rotating machinery, such as ultra-centrifuges, gas-turbines, etc.

In the case of a considerably high upper speed of rotation, final fracture of the disk arises mainly from cyclic creep instability without large propagation of crack. For this case the authors have already found a method for estimating cyclic creep deformation and fatigue life of the disk⁽¹⁾, by using their cyclic creep constitutive equation under varying applied stress^{(2),(3)} and the deformation theory of plasticity, from the comparison of the experimental results between disk-tests and uniaxial push-pull simulation tests on SNCM 439 specimens. In the case of a comparatively low upper speed of rotation, cyclic creep deformation of the disk is hardly observed. Fatigue crack initiates at a

highly stressed part of the disk, and grows up stably to the critical length, at which final fracture of the disk occurs. On this practically important case, few experimental works have been made so far since they require much time and tedious work.

This paper intends to find a fracture criterion, based on the crack growth, for the low cycle fatigue strength of rotating disk. Cyclic start-stop test of a notched disk and fatigue test of a compact specimen are made for an Al-alloy (JIS 6061-T6). Crack growth characteristics in these two kinds of tests are compared with each other. Criterion of the final fracture of the disk is discussed based on the critical crack length.

2. Experimental Method

An extruded Al-alloy (JIS 6061-T6) bar of 270 mm outside diameter for ultra-centrifuge rotor was used as a testing material. Chemical components and mechanical properties of the material are given in Table 1 and Table 2, respectively. The standard disk-specimen was a symmetrical notched hollow disk (with two notches) of uniform thickness of 5 mm, as shown in Fig.1. Besides, symmetrical notched disks of thicknesses of 10, 20, and 30 mm, and asymmetrical notched disks (with single

Table 1. Chemical components of the specimen material

Al	Cu	Cr	Mg	Si
97.9	0.25	0.25	1.0	0.6

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notch) of thickness of 5 mm were made for investigating the effects of the thickness of disk and of the symmetry of the notch distribution on the crack growth and the fatigue life. The disks were tested by a cyclic spin tester shown in Fig.2. The upper and lower rotating speeds of the disk in the cyclic start-stop test were controlled by a two points digital control frequency meter within the fluctuations of ± 80 rpm. The upper speeds were chosen as 30,000, 27,000, 25,000, 23,000, and 21,000 rpm, while the lower speed was held constant at 2,500 rpm. The disk was stopped every one to one hundred cycles (according to the fatigue crack growth rate), and the length of growing crack from the notch root of the disk was measured by a reading microscope. Tensile test and low cycle fatigue test of the compact specimens were made to obtain fracture toughness and crack growth characteristics of the disk material. The fatigue test of the compact specimens was carried out under completely pulsating stress condition in the range of the stress intensity factors $\Delta K = 15 \sim 40 \text{ MPa}\sqrt{\text{m}}$ corresponding to that in the disk-test. The thickness of the compact specimen was set as the same values of 5, 10, 20, and 30 mm as those of the disk thickness. Fracture toughness K_{IC} was determined by 5% secant method from the load-deformation curve of a specimen with thickness of 5 mm.

3. Test Results and Discussion

3.1 Fatigue crack growth

Figure 3 shows the relation between the total crack length a (including the initial notch length of 1 mm) and the number of cycles N for symmetrical notched disks with thickness of 5 mm under five upper rotating speeds given in the figure. The curves move to the left with an increase of the upper rotating speed, but they have approximately similar shapes. Figure 4 shows the effect of the thickness t in the symmetrical notched disks on the a - N relation. The effect is found to be pretty small except for the disk of thickness of 30 mm. However, a further study must be made about this effect by using much thicker disks, as the result for the disk of thickness of 30 mm seems to be contradictory to the "thickness-effect". Figure 5 shows the experimental results on the effect of the symmetry of the notch distribution on the a - N relation. In the case of the upper rotating speed of 23,000 rpm, a - N curve for a symmetrical notched disk lies on the left of one for an asymmetrical notched disk, while those for both disks are in good coincidence with each other in the case of the upper speed of 27,000 rpm. These results show that the effect of the symmetry of the notch distribution becomes smaller with an increase of the severity of the loading condition. Figure 6 col-

Table 2. Mechanical properties of the specimen material

Yield stress (MPa)	Tensile strength (MPa)	Hardness (HB)	Elongation (%)
275	304	95	12

Fig.1 Shape of the disk (in mm)

Fig.2 Cyclic spin tester

lectively shows the relations between the cyclic range of the stress intensity factors ΔK and the crack growth rate da/dN for all the disks and compact specimen tested. The stress intensity factor K of the disk was evaluated from the following equations derived by Murakami and Nishitani⁽⁴⁾,

$$\left. \begin{aligned} K &= \sigma_0 \sqrt{\pi r_i} \frac{1}{F(\lambda)} \\ \sigma_0 &= \frac{3+\nu}{8} \rho (r_0 \omega)^2 \\ \lambda &= a/r_i \end{aligned} \right\} \dots\dots\dots (1)$$

where ν and ρ are Poisson's ratio and the density of the disk material ω is the angular velocity of the disk, r_0 and r_i are inside and outside radii of the disk, and $F(\lambda)$ is a given function of λ . No significant difference in the $\Delta K \sim da/dN$ relation is found from the figure among the various tests. Hence the effects of the thickness of the disk and the symmetry of the notch distribution on the $\Delta K \sim da/dN$ relation in the disk are almost negligible, and the relation in the disk can be estimated from that in the compact tension test. The relation is given by the following equation,

$$\frac{da}{dN} = C(\Delta K)^m \dots\dots\dots (2)$$

where ΔK and da/dN are expressed in $\text{MPa}\sqrt{\text{m}}$ and m/cycle , respectively, and material constants C and m are as follows:

$$C = 1.18 \times 10^{-11}, \quad m = 3.85$$

3.2 Discussion on the final fracture criteria

3.2.1 Mean stress criterion and K_{IC} criterion.

Mean stress criterion assumes that the final fracture of a disk will occur when the mean value of the elastic tangential stress at its critical section reaches the tensile strength σ_B of the disk material. According to this criterion, the critical crack length a_c to the final fracture is given by the following equation,

$$a_c = (r_0 - r_i) - \frac{1}{3} \frac{\rho}{\sigma_B} \omega^2 (r_0^3 - r_i^3) \dots\dots\dots (3)$$

K_{IC} criterion in the Linear Elastic Fracture Mechanics is expressed as follows:

$$K = K_{IC} \dots\dots\dots (4)$$

Experimental values of the critical crack length a_c were estimated at 58 mm and 41 mm for the upper rotating speeds of 23,000 rpm and 27,000 rpm, respectively, from the observations of the crack growth curves immediately before the final fracture and the final fracture surfaces in the symmetrical notched disks of thickness of 5 mm. These experimental values of a_c were compared with their calculated values from various fracture criteria. Mean stress criterion gave a_c values of 75 mm and 62 mm for the rotating speeds of 23,000 rpm and 27,000 rpm, respectively, while K_{IC} criterion gave a_c values of

Fig.3 $a \sim N$ relation for various upper rotating speeds of disk

Fig.4 Effect of the thickness of disk on $a \sim N$ relation

Fig.5 Effect of the symmetry of notch distribution on $a \sim N$ relation

4.3 mm and 1.9 mm, respectively, where K_{IC} was taken as $21.6 \text{ MPa}\sqrt{\text{m}}$. It is clear from the comparison that the mean stress criterion gives a little longer life (or a little longer critical crack length), while K_{IC} criterion predicts a too small life (or a too small critical crack length).

Recently, the rotating strength to instable deformation of a notched disk

made of ductile material under monotonously increasing speed has been discussed based on the modified mean stress criterion or on the J -integral in the disk and R -curve obtained from the compact tension specimens, and the calculated values of the strength by these two criteria have agreed fairly well with the observed values (5). However, the modified mean stress criterion cannot reasonably be applied to the present case where plastic deformation is restricted within narrow area adjacent to the crack tip, contradicting to the assumption of the overall plastic deformation of a disk made by this criterion. On the other hand, J -integral method may reasonably be applied to the present case, but it will need tedious work and much time.

3.2.2 Rigid plastic analysis

(a) Rigid plastic analysis of the notched disk based on the notched plate model.

Let us simulate a rotating notched hollow disk with a single edge notched plate which is subjected to a linearly distributed axial stress at its ends, as shown in Fig.7, where the geometry of the notch and the width of the ligament in the plate are the same as those in the disk. The linearly distributed stress σ_θ applied to the plate is assumed to satisfy the following conditions:

(i) The integral of σ_θ^* with respect to its width W is equal to the integral of the centrifugal tangential stress σ_θ , with respect to its radial width, along the unnotched diametrical section,

(ii) The minimum stress $(\sigma_\theta)^*_{r=r_0}$ of σ_θ^* at the outer edge of the plate is equal to the minimum stress $(\sigma_\theta)_{r=r_0}$ of σ_θ at the outer circumference of the disk.

From the above conditions, the stress $(\sigma_\theta)^*_{r=r_i}$ at the inner edge of the plate is given as follows:

$$(\sigma_\theta)^*_{r=r_i} = \frac{\rho\omega^2}{12} \{ (5+3\nu)r_0^2 + 8r_0r_i - (1+3\nu)r_i^2 \} \quad (5)$$

Assuming that the linearly distributed plate load is applied at a long distance, the effects of the load on the notched section of the plate can be replaced by those of the following tensile axial force T applied in the axial direction through the center of the ligament of the notched section and the bending moment M applied about the ligament center,

$$T = \int_{r_i}^{r_0} \sigma_\theta dr = \frac{\rho\omega^2}{3} (r_0^3 - r_i^3) \quad (6)$$

and

$$M = eT \quad (7)$$

where T and M are defined in unit thickness of the plate, and e is the distance between the center of the ligament and the centroid C of the trapezoid

Fig.6 Relation between ΔK and da/dN

Fig.7 Simulation of a disk with a plate

that expresses the integral of the linearly distributed load σ_θ as shown in Fig.7. The value of e is given in terms of the thickness of the plate W and the length of the ligament $b(W-a)$ as follows:

$$e = W\xi - (b/2) \quad (8)$$

where ξ is a constant given by the equation

$$\xi = \frac{(13+3\nu)r_0^2 + 16r_0r_i + (7-3\nu)r_i^2}{24(r_0^2 + r_0r_i + r_i^2)} \quad \text{.....(9)}$$

The tensile yield force T_0 and the fully plastic bending moment M_0 of a smooth plate of unit thickness having the same width b as that of the ligament of the notched plate are given, respectively, by

$$T_0 = 2kb, \quad M_0 = \frac{1}{2}kb^2 \quad \text{.....(10)}$$

(for plane strain condition)

or

$$T_0 = \sqrt{3}kb, \quad M_0 = \frac{\sqrt{3}}{4}kb^2 \quad \text{.....(11)}$$

(for plane stress condition)

where k is the shearing yield stress, and Mises' yield condition is adopted for plane stress condition.

Now denote the non-dimensional values of the tensile axial force and the bending moment as

$$T_1 = \frac{T}{2kb}, \quad M_1 = \frac{M}{(1/2)kb^2} \quad \text{.....(12)}$$

(for plane strain condition)

or

$$T_1 = \frac{T}{\sqrt{3}kb}, \quad M_1 = \frac{M}{(\sqrt{3}/4)kb^2} \quad \text{.....(13)}$$

(for plane stress condition)

Then T_1 and M_1 represent the plastic restriction factors of the axial tension and the bending under combined loading, respectively. Substituting Eq.(12) or Eq.(13) into Eq.(7), the relation between M_1 and T_1 becomes

$$M_1 = \frac{4e}{b} T_1 \quad \text{.....(14)}$$

(for both plane strain and plane stress conditions)

Combination of T_1 and M_1 at the plastic collapse of the plate with a single notch of length a_c like Fig.7 has already been obtained by rigid-plastic analyses⁽⁶⁾⁽⁷⁾. The following are results or their approximations by the method of least squares;

[I] Lower bound method
(for plane strain condition)

$$M_1 + T_1^2 = 1 \quad \text{.....(15)}$$

[II] Slip line field analysis
(for plane strain condition)

$$M_1 + 0.739 T_1^2 - 0.521 T_1 = 1.261 \quad \text{.....(16)}$$

which is effective for

$$0 \leq T_1 \leq 0.551 \quad \text{.....(17)}$$

[III] Upper bound method
(for plane strain condition)

$$M_1^2 = \alpha_0 + \alpha_1 T_1 + \alpha_2 T_1^2 + \alpha_3 T_1^3 + \alpha_4 T_1^4 \quad \text{.....(18)}$$

where values of the coefficients α_i ($i=0\sim4$) in Eq.(18) are given in the left column of Table 3.

Maximum error of Eq.(18) is 2.8 %

for $0 \leq T_1 \leq 0.994$.

[IV] Lower bound method
(for plane stress condition)

$$M_1 = \beta_0 + \beta_1 T_1 + \beta_2 T_1^2 + \beta_3 T_1^3 + \beta_4 T_1^4 \quad \text{.....(19)}$$

which is effective for $a/W > 0.220$

where values of the coefficients β_i ($i=0\sim4$) are given in the right column of Table 3. Maximum error of Eq.(19) is 2.0 % for $0 \leq T_1 \leq 0.96$.

Calculated results of the relation between T_1 and M_1 from Eqs.(15), (16), (18) and (19) are shown in Fig.8. In Fig.8, the relation in Eq.(14) would be given as a straight line inclined at an angle of $\tan^{-1}(4e/b)$ to the ordinate axis. As the eccentricity e depends on W , b and ξ , the slope of the straight line of Eq.(14) is determined by the non-dimensional ligament b/W and the constant ξ .

The intersection of the straight line represented by Eq.(14) and a fully plastic yield curve shown in Fig.8 gives a combination of the plastic restriction factors T_1 and M_1 at the plastic collapse of the notched plate. Hence the fully plastic axial yield load T is given as a

Fig.8 Fully plastic yield curves

Table 3. Coefficients in Eqs.(18) and (19)

$\alpha_0=1.88788$	$\beta_0=1.06749$
$\alpha_1=0.753188$	$\beta_1=0.294225$
$\alpha_2=-5.65160$	$\beta_2=-1.94345$
$\alpha_3=11.8451$	$\beta_3=2.34455$
$\alpha_4=-8.82817$	$\beta_4=-1.75539$

function of b/W and ξ from the first of Eq.(12) or Eq.(13). Combining this result with Eq.(6), the critical crack length a_c is obtained as follows:

[I] Lower bound method

(for plane strain condition)

$$a_c = (r_0 - r_i) \left\{ 1 + \frac{1}{6} \eta \left(1 - \sqrt{2 + \frac{24\xi}{\eta}} \right) \right\} \quad \dots (20)$$

where

$$\eta = \frac{\rho \omega^2}{k} (r_0^2 + r_0 r_i + r_i^2) \quad \dots (21)$$

[II] Slip line field analysis

(for plane strain condition)

$$a_c = (r_0 - r_i) \left\{ 1 + \frac{1}{6} \eta \left(1 - 0.891 \times \sqrt{2 + \frac{24\xi}{\eta}} \right) \right\} \quad \dots (22)$$

which is effective for

$$1 > \frac{a_c}{r_0 - r_i} > 1 - 0.909 \xi \quad \dots (23)$$

[III] Upper bound method

(for plane strain condition)

$$a_c = (r_0 - r_i) \left[1 + \alpha_0^* \eta + \frac{1}{2} \left(A - \frac{2}{3} B \right)^{1/2} - \left\{ -\frac{1}{4} A - \frac{1}{3} B + \frac{1}{2} C \left(A - \frac{2}{3} B \right)^{-1/2} \right\}^{1/2} \right] \quad \dots (24)$$

where

$$A = \left\{ -\frac{q}{2} + \frac{1}{2} (q^2 + 4p^3)^{1/2} \right\}^{1/3} + \left\{ -\frac{q}{2} - \frac{1}{2} (q^2 + 4p^3)^{1/2} \right\}^{1/3}$$

$$B = \alpha_1^* \eta^2, C = \alpha_2^* \eta^3 + \alpha_3^* \eta^2 \xi$$

$$p = \alpha_4^* \eta^4 + \alpha_5^* \eta^3 \xi + \alpha_6^* \eta^2 \xi^2$$

$$q = \alpha_7^* \eta^6 + \alpha_8^* \eta^5 \xi + \alpha_9^* \eta^4 \xi^2$$

and values of the coefficients α_i^* ($i=0\sim9$) are given in the left column of Table 4.

[IV] Lower bound method

(for plane stress condition)

$$A^* = \left\{ -\frac{q^*}{2} + \frac{1}{2} (q^{*2} + 4p^{*3})^{1/2} \right\}^{1/3} + \left\{ -\frac{q^*}{2} - \frac{1}{2} (q^{*2} + 4p^{*3})^{1/2} \right\}^{1/3} \quad \dots (25)$$

which is effective for

$$1 \geq \frac{a_c}{r_0 - r_i} > 0.220 \quad \dots (26)$$

where

$$B^* = \beta_1^* \eta^2 + \beta_2^* \eta \xi, C^* = \beta_3^* \eta^3 + \beta_4^* \eta^2 \xi$$

$$p^* = \beta_5^* \eta^4 + \beta_6^* \eta^3 \xi + \beta_7^* \eta^2 \xi^2$$

$$q^* = \beta_8^* \eta^6 + \beta_9^* \eta^5 \xi + \beta_{10}^* \eta^4 \xi^2 + \beta_{11}^* \eta^3 \xi^3$$

Table 4. Coefficients in Equ.(24) and (25)

$\alpha_0^*=0.0166233$	$\beta_0^*=0.103402$
$\alpha_1^*=-0.143669$	$\beta_1^*=-0.131580$
$\alpha_2^*=0.0338058$	$\beta_2^*=-0.721128$
$\alpha_3^*=0.235420$	$\beta_3^*=0.0384438$
$\alpha_4^*=0.00321397$	$\beta_4^*=0.149132$
$\alpha_5^*=0.00521795$	$\beta_5^*=0.00466073$
$\alpha_6^*=0.313893$	$\beta_6^*=-0.0108055$
$\alpha_7^*=0.000659316$	$\beta_7^*=-0.0577806$
$\alpha_8^*=-0.0144178$	$\beta_8^*=0.000423586$
$\alpha_9^*=0.0347711$	$\beta_9^*=0.00350991$
	$\beta_{10}^*=0.00779213$
	$\beta_{11}^*=0.0277781$

and values of the coefficients β_i^* ($i=0\sim11$) are given in the right column of Table 4.

(b) Comparison with experiments.

Table 5 shows comparison of the calculated results of a_c by Eqs.(20),(24) and (25) with their experimental results. Calculation from Eq.(22) is excluded because of the condition (23). The calculated relations by Eqs.(20),(22), (24) and (25) between the upper rotating speed n (rpm) in the cyclic start-stop test of the disk and the non-dimensional critical crack length $a_c/(r_0 - r_i)$ are compared with their experimental values in Fig.9, where $\xi=0.588$ is used to fit the calculations to the experiments. The calculated relation by the mean stress criterion (3) is also shown in Fig.9 for reference.

The results obtained are as follows:

(i) Lower bound methods both for plane strain and plane stress conditions give almost safe estimates of the critical crack length.

(ii) Between the above two methods, the plane stress condition gives a little safer estimate than the plane strain condition.

(iii) Upper bound method for plane strain condition and slip line field analysis give almost the same results as the experiment.

In the case of a disk of 30 mm thickness, the experimental value is found to lie below the calculation from the lower bound method for plane stress condition which gives the shortest value of a_c among the methods described above.

But the rigid plastic analysis proposed in this paper may be practically applicable to estimate the critical crack length, since its relatively small difference between the calculation and the experiment corresponds to only a few cycles near the final fracture.

(c) Cyclic life to final fracture.

For a given critical crack length a_c , the number of cycles to final fracture N_{fp} can be estimated from Eq.(2) as follows:

$$N_{fp} = \frac{1}{C} \int_{a_i}^{a_c} (\Delta K)^{-m} da + N_2 \quad \dots (27)$$

where a_c denotes the crack length at the crack initiation which is taken as $a_2 = 2\text{mm}$, for convenience' sake, because of the variation of a_2 values, and N_2 is the number of cycles to the crack initiation or crack length a_2 . Experimental values of N_2 , C and m for each disk were

Table 5. Comparisons between the prediction and experiment of the critical crack length and the cyclic life

Experimental Condition and Experimental Result	Method of Prediction	Critical Crack Length a_c (mm)	Predicted Life N_{fp} (cycle)	Error $(N_f - N_{fp})/N_f$ (%)
Symmetrical notched disk 23000 (rpm), Thickness= 5 (mm) $a_c=58$ (mm), $N_f=5614$ (cycle) $\frac{da}{dN}=1.09 \times 10^{-11} \Delta K^{4.05}$	Lower bound. (Plane strain)	39.20	6100	8.66
	Upper bound. (Plane strain)	50.62	6169	9.89
	Lower bound. (Plane stress)	38.73	6095	8.57
	Mean stress criterion	75.22	6214	10.69
	Experimental data	58.	6198	10.40
Symmetrical notched disk 27000 (rpm), Thickness= 5 (mm) $a_c=41$ (mm), $N_f=1425$ (cycle) $\frac{da}{dN}=3.05 \times 10^{-13} \Delta K^{5.08}$	Lower bound. (Plane strain)	29.05	1643	15.30
	Upper bound. (Plane strain)	42.93	1664	16.77
	Lower bound. (Plane stress)	28.58	1642	15.23
	Mean stress criterion	62.07	1672	17.33
	Experimental data	41.	1662	16.63
Symmetrical notched disk 27000 (rpm), Thickness=10 (mm) $a_c=36$ (mm), $N_f=1033$ (cycle) $\frac{da}{dN}=1.73 \times 10^{-13} \Delta K^{5.27}$	Lower bound. (Plane strain)	29.05	1426	38.04
	Upper bound. (Plane strain)	42.93	1443	39.69
	Lower bound. (Plane stress)	28.58	1426	38.04
	Mean stress criterion	62.07	1449	40.27
	Experimental data	36.	1435	38.92
Symmetrical notched disk 27000 (rpm), Thickness=30 (mm) $a_c=24$ (mm), $N_f=9246$ (cycle) $\frac{da}{dN}=1.93 \times 10^{-13} \Delta K^{4.64}$	Lower bound. (Plane strain)	29.05	9688	4.78
	Upper bound. (Plane strain)	42.93	9895	7.02
	Lower bound. (Plane stress)	28.58	9677	4.66
	Mean stress criterion	62.07	9978	7.92
	Experimental data	24.	9533	3.10

Fig.9 Relation between the upper rotating speed and the non-dimensional critical crack length

substituted into Eq.(27) to make precise comparison of the calculated values (N_{fp}) with experimental results (N_f). Results for comparison of the number of cycles to fracture are given in Table 5. The comparison leads to the following results:

(i) The estimated cyclic life is generally longer than the actual life. This may be caused by the neglect of an abrupt rise of $da/dN - \Delta K$ curve just before the final fracture in the calculation.

(ii) The rigid plastic analysis qualitatively gives a closer value to the actual life than the mean stress criterion.

In the present analysis, experimental values are used as the number of cycles to crack initiation N_2 of the disk, but in order to predict the total cyclic life of the disk exactly, a method of estimating N_2 of the disk must be studied theoretically.

4. Conclusions

From the study of the low-cycle fatigue due to start and stop of rotation the Al-alloy (JIS 6061-T6) notched disk, the following conclusions were obtained:

(1) The relation between the fatigue crack growth rate and the cyclic range of

the stress intensity factors in the disk can be predicted from that about the compact tension specimen.

(2) The effects of the symmetry of the notch distribution and the thickness of the disk on the fatigue crack growth rate are almost negligible. However further study is required for a disk of which the thickness/inner diameter ratio is larger than one.

(3) Among the criteria on the final fracture in the low cycle fatigue of rotating disk, the mean stress criterion gives a little longer (unsafe) life, while K_{IC} criterion predicts a too short (safe) life.

(4) Rather simple equations for predicting the critical crack length are derived by simulating a rotating notched hollow disk with a single edge notched plate and analyzing its unstable deformation.

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