Investigations of Mach Reflection of a Shock Wave*

(3rd Report, Strength of a Reflected Shock Wave)

By Kazuyasu MATSUO**, Toshiyuki AOKI***, Hiroyuki HIRAHARA[†]
Nobuaki KONDOH^{††} and Kousei KURIWAKI^{†††}

The reflection phenomena of a plane moving shock wave on a wedge surface have been investigated by many researchers, and four different types of reflection have been observed in previous shock tube experiments. The authors have reported on the domains and boundaries of these four types of reflection and the shape of a reflected shock wave in the previous reports.

In the present paper, the distribution of the strengths of a reflected shock wave along its wave front has been calculated from its shape obtained by optical observations, and the relation between the strength and the shape of a reflected shock wave has been discussed in detail. Furthermore, the flow properties around the triple point in the case of Mach reflection have been calculated using the experimental data, and the validity of the three-shock theory has been examined.

Key Words : Compressible Flow, Unsteady Flow, Shock Wave, Mach Reflection, Shock Tube, Experimental Study, Reflected Shock

1. Introduction

When a plane shock wave moves into a gas at rest and encounters a wedge inclined to the shock, it is reflected from the wedge surface. In these reflection problems, four different types of reflection have been observed in previous shock tube experiments⁽¹⁾. They are regular reflection(RR), simple Mach reflection(SMR), complex Mach reflection(CMR) and double Mach reflection(DMR). The authors reported on the domains and boundaries of these four types of reflection and the shape of a reflected shock wave in the previous papers.^{(2),(3)}

As for the strength of a reflected shock wave, only the value near a triple point has been discussed theoretically so far, using the three shock theory^{(4),(5)}. On the other hand, the density distributions in the region between the reflected shock and the wedge surface were measured by means of an interferometric technique^{(6),(7)} and an electron beam method⁽⁸⁾. However the variation of the strength of a reflected shock wave along its wave front is not yet resolved. Furthermore, in order to solve

the so-called von Neumann paradox⁽⁹⁾, a modified three-shock theory⁽⁹⁾ was presented in the case of a weak incident shock wave, but the discrepancy between the experimental and theoretical results about the strength of a reflected shock wave is still quite large.

In this report, a simple method to calculate the strength of a reflected shock wave along its wave front from a given shape of reflected shock is presented, and the relation between the strength and the shape of a reflected shock wave is discussed in detail. Furthermore, the strength of a reflected shock wave near a triple point has been calculated by using the present experimental data, and the validity of the three-shock theory has been examined.

2. Relation between the Strength and the Shape of a Reflected Shock

2.1 Basic equations

As is well known, the reflection phenomenon of a plane moving shock wave from a wedge is self-similar with time,⁽²⁾ so a set of conservative equations in an unsteady, two-dimensional, compressible, inviscid flow in this case are described as follows in terms of independent variables $\xi = x/t$ and $\eta = y/t$, instead of x, y and t ⁽¹⁰⁾. In this notation, x and y are Cartesian coordinates and t is the time measured from the moment when an incident shock wave strikes the vertex of the wedge.

$$\frac{\partial}{\partial \xi}(\rho U) + \frac{\partial}{\partial \eta}(\rho V) + 2\rho = 0 \quad \dots\dots\dots(1)$$

$$U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} + U = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} \quad \dots\dots\dots(2)$$

$$U \frac{\partial V}{\partial \xi} + V \frac{\partial V}{\partial \eta} + V = -\frac{1}{\rho} \frac{\partial p}{\partial \eta} \quad \dots\dots\dots(3)$$

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** Professor, Graduate School of Engineering, Sciences, Kyushu University, Kasuga, Fukuoka.

*** Associate Professor, Graduate School of Engineering Sciences, Kyushu University, Kasuga, Fukuoka.

[†] Post-graduate student, Graduate School of Engineering Sciences, Kyushu University, Kasuga, Fukuoka.^{††} Research Engineer, Graduate School of Engineering Sciences, Kyushu University, Kasuga, Fukuoka.^{†††} Research fellow, Kawasaki Steel Corp., Chiba.

$$U \frac{\partial s}{\partial \xi} + V \frac{\partial s}{\partial \eta} = 0 \quad (4)$$

where ρ , p and s are density, pressure and entropy, respectively. U and V are the velocity components in ξ and η directions and are defined as $U=u-\xi$ and $V=v-\eta$ in terms of u and v , respectively, where u and v are the components of velocity in the Cartesian coordinate. The vector (U, V) represents the velocity of the gas moving with the velocity (u, v) relative to the point moving with the constant velocity (ξ, η) . In the following, we call the ξ - η coordinate a ξ - η plane in this paper.

2.2 Method of calculation of the strength of a reflected shock

Figure 1 shows a flow field of Mach reflection in a ξ - η plane. As is shown in this figure, the gas in the region 1 ahead of the incident shock moves toward the origin O , and the gas in the region 2 behind the incident shock moves toward the point $O'(u_2, 0)$, where

$$u_2 = \frac{2}{\kappa + 1} \left(M_s - \frac{1}{M_s} \right) a_1 \quad (5)$$

In the above equation, a_1 is the sonic velocity in the region 1, and κ is the ratio of specific heats of the gas. In the figure, the head and the tail of the reflected shock are the triple point T and the point R on the wall, respectively. The flow Mach number upstream of the reflected shock TR is not uniform but varies from the point T to R . Let the velocity vector incident upon a certain point Q on the reflected shock be $\vec{V}_2$, then

$$\vec{V}_2(\xi) = (V_{2\xi}, V_{2\eta}) = (u_2 - \xi, -\eta) \quad (6)$$

In addition, since the shape of the reflected shock S is a function of ξ only, the shock angle β_s of the reflected shock is expressed as follows:

$$\beta_s(\xi) = \cos^{-1} \left(\frac{V_{2\xi} + V_{2\eta} \cdot S'}{|V_2| \cdot \sqrt{1 + S'^2}} \right) \quad (7)$$

where $S'(\xi) = dS/d\xi$. If $S=S(\xi)$ is known, then using the relations for an oblique shock wave and Eqs.(6) and (7), the properties just behind the reflected shock wave such as the pressure $p_{3s}(\xi)$ and the

Fig.1 Flow field of Mach reflection in ξ - η plane

density $\rho_{3s}(\xi)$ are calculated as functions of ξ .

2.3 Comparison between the present calculation and the experimental data of density measurement

To examine the validity of the calculating method of the strength of a reflected shock described in the previous section, the density distributions $\rho_{3s}(\xi)/\rho_1$ along the reflected wave front calculated by the present method are compared with the experimental values⁽⁶⁾ measured by an interferometric technique in Fig.2, where the points of $\xi/U_s=0$ and 1.0 represent the positions of the vertex of the wedge O and of the triple point T , respectively. In the case of $M_s=2.61$ and $\theta_w=10^\circ$, where a simple Mach reflection occurs, the values calculated by the present method agree well with the experimental results. On the other hand, when $M_s=3.76$ and $\theta_w=40^\circ$, where a double Mach reflection occurs, the calculated results are quite different from the experimental ones. The calculated values in the range of $-0.1 < \xi < 0.76$ are about 40 percent larger than the experimental results, and there is a discontinuous change at $\xi/U_s=0.76$ in the calculation, although it is not observed in the experiment. The point of $\xi/U_s=0.76$ corresponds to the kink point on the reflected shock wave, and it seems to be reasonable to think that the strength of the reflected shock changes discontinuously at this point. The fact that there is no discontinuity in the distribution in the experiment may be due to the indistinct interferometric fringes. As mentioned above, the strength of a reflected shock can be calculated easily and with good accuracy by the present method.

3. Experimental Procedures

The shock tube and the instruments used in the present study are the same as those in the previous reports.⁽²⁾⁽³⁾ Carbon-dioxide, whose gasdynamic and thermodynamic properties are well known, was used as a test gas. The experiments were

Fig.2 Density distribution along reflected shock (N_2)

carried out in the range of incident shock Mach numbers M_s from 1 to 6, and the angles of the inclination of the wall of 20, 30, 40 and 50°.

4. Experimental Results and Discussion

4.1 Distribution of the strengths of a reflected shock wave along its wave front

Typical shadowgraphs obtained in the present experiment are shown in Fig.3. As the shape of the reflected shock wave is known from these photographs, the distribution of the strengths of the reflected shock wave along its wave front can be calculated by the method described previously. The pressure ratio $p_{3s}(\xi)/p_2$ thus obtained is shown in Figs.4(a) and (b) in the cases of $\theta_w=20^\circ$ and 40° , respectively, where p_2 is the pressure in the region 2 in Fig.1. In Figs.4(a) and (b), the extreme right one ($\xi/U_s=1.0$) of these curves corresponds to the head of the reflected shock, and the extreme left to the tail of it.

In the case of SMR in Fig.4(a), $p_{3s}(\xi)/p_2$ varies smoothly, while it varies discontinuously at the point near $\xi/U_s=0.83$ in the case of CMR. As mentioned before, this point corresponds to the kink point. In the cases of $M_s=4.19$ and 5.00, the pressure ratio $p_{3s}(\xi)/p_2$ is kept constant in the region between $\xi/U_s=0$ and 0.2. This is due to the fact that this part of the reflected shock is straight as shown in Fig.3(c). The broken line in Fig.4 which connects the point N on each curve will be explained in the next section.

In Fig.4(b), $p_{3s}(\xi)/p_2$ decreases monotonously from the head to the tail of the reflected shock in cases of RR and SMR. On the other hand, a discontinuous change appears in the cases of CMR and DMR, as was pointed out in Fig.4(a), and the change in pressure at the discontinuity point increases with an increasing M_s . In

the cases of $M_s=4.43$ and 5.84, the curve between the kink point and the tail of the reflected shock is nearly symmetric with respect to the axis of $\xi/U_s=0.4$.

4.2 The minimum value of the pressure ratio along the reflected shock front

As was described in the previous section, a point where $p_{3s}(\xi)/p_2$ becomes minimum exists between the head and the tail of the reflected shock in the cases of RR and SMR, and the minimum point exists between the kink point and the tail of the reflected shock in the cases of CMR and DMR. The reason why such a minimum point exists may be explained as follows.

Figure 5 shows the shape of the reflected shock $S(\xi)$ in the case of $\theta_w=40^\circ$ and $M_s=4.43$ in a $\xi-\eta$ plane. Let the intersection of the curve of the reflected shock $S(\xi)$ and an extension of the perpendicular from the point O' (which is identical with the point O' in Fig.1.) to the wall OW be the point N. It was observed that the shock angle at the point N was nearly 90° for any case of M_s and θ_w in the present experiment. The point N determined by the present shadowgraphs is indicated by open circles in Figs.4(a) and (b). The dash-dotted line which connects these open circles agree roughly with the broken line which represents the minimum point of the

Fig.3 Typical shadowgraphs

(a) $\theta_w = 20^\circ$

(b) $\theta_w = 40^\circ$

Fig.4 Distribution of strengths of reflected shock wave along its wave front

theory. The experimental values of CMR (half-closed symbols) and DMR (closed symbols) show good agreement with the three-shock theory, whereas those of SMR (open symbols) are smaller than the theoretical values in any case of θ_w . The discrepancy increases when M_s becomes smaller. This fact means that the experimental value of the angle χ is smaller than that predicted by the three-shock theory. Next, the component of M_{2t} normal to the reflected shock front, $(M_{2t})_n$, is compared with the theoretical value in Fig.8. In this case, the experimental value agrees with the theoretical one only at a certain value of M_s (near the boundary between SMR and CMR), and the experimental values are lower than the theoretical ones in either case of a larger or a smaller value of M_s than that. The disagreement in the case of a smaller M_s is due to the fact that the experimental value of M_{2t} is smaller than the theoretical one as shown in Fig.7. The discrepancy in the case of a larger M_s is considered to be due to the fact that the angle of reflection α_r measured in the experiment is smaller than the theoretical one, because $(M_{2t})_n$ depends on not only χ but also α_r . From the facts mentioned above, it may be concluded that the strength of the reflected shock near the triple point is weaker than that predicted by the three-shock theory, especially when M_s is different from the value where a transition between SMR and CMR takes place.

Next, the ratio of the pressures on both sides of the slip surface TC in Fig.1, p_{4t}/p_{3t} , is shown against M_s in Fig.9. The symbols in this figure are the same as those in Fig.7. According to the three-shock theory, the pressure p_{4t} must be equal to the pressure p_{3t} . Figure 9 shows that this condition is well satisfied when $M_s=3.0$ in the cases of $\theta_w=20$ and 30° , and when $M_s=2.0$ in the case of $\theta_w=40^\circ$, but in other cases p_{4t} is larger than p_{3t} . This unsatisfactory pressure condition arises from the facts that the reflected shock is not straight but curved, and even the Mach shock is slightly curved⁽¹⁵⁾ toward the region 4 in Fig.1, and that the flow field in the region 4 is not uniform due to viscous effect of gas⁽¹⁵⁾.

Let δ_i , δ_r and δ_m be the deflection

angles of flow by the incident, reflected and Mach shocks, respectively, then the flow which passes through both the incident and reflected shocks is deflected by $\delta_i - \delta_r$, and the flow which passes through the Mach shock is deflected by δ_m . According to the three-shock theory, the difference between those angles, $\Delta\delta = (\delta_i - \delta_r) - \delta_m$, must be equal to zero, because both sides of flow of the slip surface should be parallel. The angle $\Delta\delta$ is plotted against M_s in Fig.10, which shows that the experimentally determined values of $\Delta\delta$ are not zero but positive and amount to about 10° when M_s increases. This means that the flow of the region 3 in Fig.1 collides with the flow of the region 4. The behavior and the characteristic of the slip surface seem to be a problem not fully solved. It should be investigated from a microscopic point of view.

As was described above, the experimental values of the flow properties near the triple point agree well with the value predicted by the three-shock theory when M_s is close to the one where a transition between SMR and CMR takes place, although the difference between the experimental and theoretical values increases in either case of a larger or a smaller value of M_s than that. For the case of a smaller M_s , the discrepancy, that is von Neumann paradox, becomes considerable in the case of SMR. In this case, as was shown in Fig.7, M_{2t} decreases with an increasing M_s in the theory and the angle of reflection of the shock near the triple point, β_r , is greater than 90° according to the three-shock theory. However, Semennov et al.⁽¹⁶⁾ and Walenta⁽¹⁷⁾ showed that the value of β_r is nearly equal to or smaller than 90° in their experiments. This fact may be one of the reasons for the discrepancy in the case of a weak incident shock.

5. Conclusions

Experimental studies were performed to investigate the reflection of a shock wave from a wedge and the strength of the reflected shock was discussed in detail. The conclusions are summarized as follows:

(1) The distribution of the strengths of the reflected shock wave along its wave front has been calculated from its experi-

Fig.9 Ratio of pressures on both sides of slip surface

Fig.10 Difference of angles between both sides of slip surface

mentally determined shape. In the case of a regular reflection and a simple Mach reflection, the strength varies smoothly from the head to tail of it and there exists a point where its strength becomes minimum between them. In the case of the complex and double Mach reflections, the strength changes discontinuously at the kink point and there also exists a minimum point between the kink point and the tail of it.

(2) The minimum point described above is nearly equal to the point where the flow downstream of the incident shock wave is normal to the reflected shock wave in a ξ - η plane, where ξ and η denote x/t and y/t in terms of a Cartesian coordinate (x , y) and time t , respectively.

(3) The reason why the strength of the reflected shock becomes minimum between the head or the kink point and the tail of it was explained qualitatively in connection with the shape of the reflected shock.

(4) The flow properties near the triple point in the cases of simple, complex and double Mach reflections were obtained from the experimentally determined triple point trajectory and the angle of reflection, and they were compared with the values by the three-shock theory. The experimental values agree well with the theoretical ones when the incident shock Mach number M_s is close to the one where a transition between the simple and complex Mach reflections takes place. In other cases, the experimental results do not agree with the theoretical values, and the assumption in the three-shock theory that the pressure and the flow direction on one side of the slip surface must be equal to those on the other side does not hold. The discrepancy

between experiment and theory at a lower M_s arises in the case of the simple Mach reflection, and the degree of discrepancy increases as M_s tends to unity.

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