Investigations of Mach Reflection of a Shock Wave*

(4th Report, The Transition between Regular and Mach Reflections)

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The transition between regular and Mach reflections of an oblique shock wave has been investigated by many researchers for steady, pseudo-steady and unsteady flows. However the problem for unsteady flows such as the reflection of a shock wave at cylindrical concave or convex walls has not been fully understood.

The problem, considered in this report, is that of transition from Mach to regular reflection of a shock wave over a concave wall. Experimental investigations have been performed using a model of concave wall. The wall is composed of two straight sections, and the downstream second section is more inclined than the upstream first one. The process of shock reflection over this wall has been observed in detail, and the transition criterion from Mach to regular reflection in this case has been presented experimentally.

Key Words: Compressible Flow, Unsteady Flow, Shock Wave, Mach Reflection, Regular Reflection, Transition

1. Introduction

In general, when an oblique shock wave is reflected from a rigid wall, two principal types of reflections are observed. They are regular reflection (RR) and Mach reflection (MR), and the latter is classified further into simple, complex and double Mach reflections. In the previous reports^{(1)~(3)}, the domains and the boundaries among these four types of reflection were presented, and the geometry and the strength of the reflected shock wave were discussed in detail.

When a plane moving shock wave encounters a curved wall, the configuration of the reflection of the shock wave changes as the shock propagates along the wall, and the transition from MR to RR and the reversed transition take place respectively in the case of a concave wall and a convex wall.^{(4)~(7)} The detachment criterion⁽⁸⁾ and the mechanical equilibrium criterion⁽⁹⁾ are usually applied as a theoretical criterion of the transition between MR and RR. According to the previous experiments, the former criterion shows good agreement with experimental

results in pseudo-steady flows^{(1),(11)}, while the latter agrees with the experimental results in steady flows where the incident shock Mach number is rather high.^{(9),(10)} However, the transition of non-stationary reflections such as a reflection of a moving shock wave on a curved wall has not yet been well understood, despite several theoretical and experimental investigations.

In the present paper, an experiment is performed using a model of concave wall in order to investigate the process of the transition from MR to RR at a concave wall. The wall is composed of two straight sections, as shown in Fig.1, and is called a double-staged inclined wall. The process of shock reflection from this wall is observed and the transition criterion from MR to RR in this case is discussed precisely.

2. Types of Shock Reflection at Double-staged Inclined Wall

As shown in Fig.1, when a moving plane shock wave perpendicular to the wall WO_1 encounters a double-staged inclined wall, the resulting process of reflection is classified into two types depending on the type of reflection, i.e. MR or RR, which takes place at the first inclined section O_1O_2 . In the present paper, the inclination of the first wall, θ_{w1} , is fixed at 20 degrees, and the case that MR takes place at the first inclined section is discussed. In this case, if the inclination of the second section, θ_{w2} , is larger than θ_{w1} , the process of shock reflection can be classified into three types; that is, cases of A, B and C, as shown respectively in Fig.1(a), (b) and (c). In these figures, the configuration of the shock reflection changes with time as (1)+(2)+(3)+(4). The stage (1) represents the

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shock system appearing immediately after the impingement on the first section. This reflected shock system is the same as the system appearing in a pseudo-steady flow. As the incident shock moves to right, the Mach shock T_1M_1 is reflected from the second wall, taking a configuration of either MR or RR. The configuration depends on the Mach number of the Mach shock, i.e. $M_{M1} = M_s \cos \chi_1 / \cos(\chi_1 + \theta_{w1})$, and the inclination of the second wall relative to the direction of the propagation of the Mach shock, $\theta_{w2} - \theta_{w1}$; where M_s and χ_1 denote respectively the incident shock Mach number and the angle between the trajectory of the triple point T and the first wall.

In the cases of A and B in Fig.1, the Mach shock T_1M_1 is reflected from the second wall as Mach reflection, and the triple point T_2 approaches the point T_1 forming a new triple point T_3 . According to the present experiment, as mentioned later, the trajectory of T_3 can be assumed as an almost straight line. In the case of A in Fig.1(a), the angle between the trajectory T_3 and the second wall, i.e. χ_3 , is positive. Then the transition from MR to RR does not occur because the triple point T_3 never impinges on the wall. On the other hand, in the case of B in Fig.1(b), since the angle χ_3 is negative, and since the triple point T_3 impinges on the wall, the transition from MR to RR takes place. In the case of C in Fig.1(c), the Mach shock T_1M_1 is reflected from the second wall as regular reflection because of the large value of the angle θ_{w2} . In this case, the reflection point P_1 collides with the triple point T_1 at the point F on the second wall as the incident shock moves to right. The shock configuration after the collision remains as regular reflection, because that the incident shock with reference to the point P_1 changes from T_1P_1 to IP_2 and the angle

of incidence of the shock decreases by the collision. The cases of A, B and C described above are summarized in Table 1.

As for the transition between MR and RR, the detachment and mechanical equilibrium criterions^{(8),(9)} are calculated for carbon dioxide, assuming a case of the reflection of a shock wave at a single wedge which makes an angle θ_{w2} against the direction of the shock propagation. The results are shown as the curves 1 and 2 in Fig.2, respectively. The curve 3 is obtained by applying the detachment criterion for the reflection of the Mach shock T_1M_1 from the second wall in Fig.1. Three types of shock reflections described above will each take place in the shaded domains in Fig.2. The boundary between the case of B and the case of C may be represented by the curve 3 and it is expected that this boundary agrees with the experimental results, because it is the same as to the case of a pseudo-steady flow⁽¹¹⁾. However, it is not clear at this stage whether the boundary between the case of A and the case of B agrees with curve 1 or 2, or not. From this view point, the problem of this transition criterion is discussed experimentally in this paper.

3. Experimental Procedures

The shock tube and the instruments used in the present study are the same as those in the previous reports⁽¹⁾⁻⁽³⁾. The double-

Table 1 Cases of shock reflection over double-staged inclined wall

case	Reflection of incident shock from 1st. wall	Reflection of Mach shock from 2nd. wall	Final configuration of shock reflection
A	MR	MR	MR
B	MR	MR	RR
C	MR	RR	RR

Fig.2 Domains of shock reflection over double-staged inclined wall ($\theta_{w1}=20^\circ$)

Fig.1 Process of shock reflection over double-staged inclined wall

staged inclined wall shown in Fig.1 is installed in the test section. The first wall has a length of 29.2mm and an angle θ_{w1} of 20° , while the second wall has a length of 50mm and its angle θ_{w2} can be varied in the range of 20° to 60° . As in the previous paper⁽³⁾, carbon dioxide was used as a test gas.

4. Experimental Results and Discussions

4.1 Optical observations

Typical shadowgraphs showing shock reflections from the double-staged inclined wall are represented in Fig.3. Figure 3(a) shows the case of A, and (a-1), (a-2) and (a-3) correspond to the stages (1), (2) and (3) in Fig.1(a), respectively. In Fig.3 (a-3), the slip stream issuing from the triple point T₁ (see Fig.1(a)) interacts with the slip streams from T₁ and T₂, and the flow field is rather complicated. Figure 3(b) shows the case of B, and (b-1), (b-2) and (b-3) are the stages (1), (3) and (4) in Fig.1(b), respectively. In Fig.3 (b-3), a shock wave is observed immediately downstream of the reflection point P₁ (see Fig.1(b)). This shock is formed as a result of the transition from MR to RR; the reason will be discussed in the following section.

4.2 Transition process from MR to RR in case of B

A schematic sketch showing the transition process from MR to RR on the second wall, which is shown from the state (3) to the state (4) in Fig.1(b), is shown in detail in Fig.4. The incident shock propagates from left to right, and the triple point T₁ moves along the dash-dotted line and collides against the wall at the point E, where the transition from MR to RR

takes place. And then, a normal shock KS is formed behind the reflection point P₁. The shock polar diagram representing this transition process is shown for the case of $M_s=1.40$, $\theta_{w1}=20^\circ$ and $\theta_{w2}=50^\circ$ in Fig.5, where δ denotes the angle of deflection of flow and P_1 is the pressure ahead of the incident shock. Figure 5(a) and (b) indicate the states before and after the transition, respectively. Points (1), (2), (3) and (4) in Fig.5 represent the states of regions 1, 2, 3 and 4 in Fig.4. In Fig.4, since the strength of the reflected shock KR in the state (4) may be equal to that of the reflected shock T₁R in the state (3), the pressure in the region 6 is equal to that in the region 4. Therefore, the pressure in the region 5, which is predicted by the two-shock theory, differs from that in the region 6, and the shock KS must be formed to adjust the flow field. Since the pressure in the region 5 is lower than the pressure in the region 6, the shock KS is represented by the polar R' in Fig.5 (b), where the points (5) and (6) on the polar R' indicate the states of the regions 5 and 6 in Fig.4. Consequently, it is evident that the shock KS propagates toward the region 5, and this behavior is confirmed by the optical observation.

Fig.4 Schematic sketch of the transition process in case B

(a) case A, $\theta_{w1}=20^\circ$, $\theta_{w2}=48^\circ$, $M_s=2.20$ (b) case B, $\theta_{w1}=20^\circ$, $\theta_{w2}=54^\circ$, $M_s=1.80$

Fig.3 Typical shadowgraphs

Fig.5 Shock polar diagrams of the transition process

Fig.6 Angle of triple point trajectory

4.3 Triple point trajectory

It is found by the optical observation that the trajectory of the triple point T_3 is almost a straight line. The angle between the trajectory of T_3 and the second wall, i.e. χ_3 , is shown against θ_{w2} for the case of $M_s=3.0$ in Fig.6. Solid line represents the theoretical angle predicted by the three-shock theory⁽⁶⁾. The point a on this line, where χ_3 is equal to zero, shows the transition point by the mechanical equilibrium criterion. The left side of the point a where χ_3 is positive corresponds to the domain of the direct Mach reflection. On the other hand, the right side of it where χ_3 is negative corresponds to that of the inverted Mach reflection⁽¹²⁾. Assuming that the strength of the Mach shock after the collision of the triple points T_1 and T_2 in Fig.1(a) remains unchanged and that the strength is equal to that of the Mach shock T_2M_2 , the angle χ_3 can be calculated by the following equation;

$$\chi_3 = \tan^{-1} \left(\frac{\cos \theta_{w2} - M_s/M_{m2}}{\sin \theta_{w2}} \right)$$

,where M_{m2} is the Mach number of the Mach shock T_2M_2 . The dashed line represents the results obtained from the above equation. Open circles representing the experimental points are located between the solid and the dashed lines. Therefore, the strength of Mach shock increases by the collision of the triple points T_1 and T_2 , but it is weaker than that predicted by the three-shock theory. This fact means that the strength of Mach shock after the collision depends not only on the value of θ_{w2} but also on θ_{w1} .

Fig.7 Angle of reflection

Fig.8 Typical shadowgraph showing reflection in case A' ($\theta_{w1}=20^\circ$, $\theta_{w2}=56^\circ$, $M_s=3.02$)

4.4 Angle of reflection

The angle of reflection, α_r , after the collision of the triple points T_1 and T_2 is shown against θ_{w2} in the case of $M_s=1.8$ in Fig.7. The solid and dashed lines represent the theoretical values respectively in the case of MR in the three-shock theory⁽⁸⁾ and in the case of RR in the two-shock theory⁽¹⁰⁾. According to the detachment criterion and to the mechanical equilibrium criterion, the transition between MR and RR takes place at the points a and b, respectively. Open and closed circles represent the experimental points of MR and RR. In the region of case of A since the transition from MR to RR never arise, only one value of α_r is obtained in experiments for any value of θ_{w2} , and it is larger than the theoretical value obtained by the three-shock theory. The reason why the experimental value deviates from the theoretical estimation may be attributed to the fact that the width of slip stream becomes wide as shown in Fig.3. In the region of case B, since the transition from MR to RR takes place, two values of α_r are obtained for any value of θ_{w2} . These are denoted by open and closed circles, which respectively represent the value of MR before the transition and the values of RR after the transition. The measured values of MR are located above the solid line as in the case of A, and the values of RR show good agreement with the values of the two-shock theory. The decrease in α_r by the transition from MR to RR in the case of B corresponds to the fact that the shock KS (see Fig.4) is formed by the transition.

Next, in the range of θ_{w2} of 46° to 51° in Fig.7, it has been found in the present shadowgraph pictures that the value of χ_3 is negative immediately after the formation of the triple point T_3 , but the absolute value of it is so small that the transition from MR to RR does not take

Fig.9 Domains and boundaries of shock reflection over double-stage inclined wall

place. For this reason, this case is defined as the case A' and only the experimental value of MR is shown in Fig.7. Figure 8 shows a typical shadowgraph in the case A'. In this photograph, a very short Mach shock is observed at the reflection point. In this case where χ_s takes a small negative value, it may be considered that the triple point T₃ does not impinge on the wall, because the trajectory of T₃ is bent due to the effect of the boundary layer.

4.5 Domains and boundaries of shock reflection

In consequence of the above consideration, the domains and boundaries of the shock reflection on the double-stage inclined wall are represented in the M_s - θ_{w2} plane of Fig.9. The curves 1 and 2 are the same as those in Fig.2, and the curve 3 represents an experimental curve along which $\chi_s=0$ immediately after the formation of the triple point T₃. Open, closed and half-closed circles represent the experimental point of the cases of A, B and A', respectively. The curve 4 gives the boundary between the case of B and the case of A'. The experimental criterion for the transition from MR to RR can be regarded as the curve 4, and thus it agrees with the curve 1 for weaker incident shocks, while it agrees with the curve 2 for stronger shocks. This result shows a tendency closer to that in the case of steady flows⁽⁹⁾⁽¹⁰⁾ rather than that in the case of pseudo-steady flows⁽¹⁾⁽¹¹⁾. However, as shown in Fig.9, since the region of the case of A' is broadened as the incident shock becomes stronger, there arises a considerable limitation in determining the transition point precisely by means of optical observations alone.

5. Conclusions

To investigate the transition problem from Mach to regular reflection over a concave corner, an experimental study was performed using a model of concave wall composed of two straight walls (a double-staged inclined wall). The conclusions are summarized as follows:

(1) When an incident shock is reflected as a Mach reflection at the first wall, and further the Mach shock is reflected as a Mach reflection at the second wall, two triple points created by these two Mach reflections collide each other and a new (the third) triple point is formed. If the inclination angle of the second wall θ_{w2} is comparatively small, the third triple point is directed away from the wall surface and Mach reflection remains. On the other hand, if the angle θ_{w2} is large enough, the triple point impinges on the wall and the transition from Mach to regular reflection takes place.

(2) In the case that the trajectory of the third triple point makes an extremely small angle with the second section of the wall, the trajectory is bent and the transition from Mach to regular reflection does not occur, though the third triple point approaches the second wall. The range of θ_{w2} where this process takes place increases with the incident shock Mach number M_s .

(3) The criterion of the transition from Mach to regular reflection over the double-staged inclined wall is determined by experiments, and the result agrees well with the detachment criterion in case of weak incident shock wave, whereas for a strong incident shock it agrees with the mechanical equilibrium criterion.

(4) The result described in the above conclusion shows a tendency closer to that in the case of steady flows, rather than that in the case of pseudo-steady flows.

(5) The trajectory angle of the third triple point immediately after the collision of the first and the second triple point is smaller than the angle predicted by the three-shock theory, but it is larger than the angle calculated by assuming that the strength of the Mach shock wave is kept constant by the collision.

(6) The angle of reflection of the shock after the formation of the third triple point is larger than that predicted by the three-shock theory. And, the angle of reflection of the shock wave after the transition from Mach to regular reflection is in good agreement with the angle predicted by the two-shock theory.

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