Scroll Compressor Dynamics*

(1st Report, The Model for the Fixed Radius Crank)

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The scroll compressor has been introduced for air conditioning application. This paper describes the dynamic behavior of the scroll machine with the fixed radius crank mechanism. The equations of motion are established for the orbiting scroll and the Oldham coupling, including the friction and the motion of the Oldham coupling, is shown. The bearing load, the torque and the power to drive the compressor are obtained. The overturning moment of the orbiting scroll is calculated and the stability condition is clarified.

Key Words : Fluid Machine, Dynamics of Machinery, Scroll Compressor, Refrigeration, Air Conditioning

1. Introduction

The scroll compressor is used for air conditioning application because of its high efficiency and low noise/vibration. The principles and the basic theory of the machine have been already reported.^{(1)~(3)} The dynamic behavior of the orbiting scroll and the Oldham coupling is to be taken into account, as well as the pressure load calculation. The force and the moment which act during the operation were obtained by Ikegawa⁽⁴⁾ and the authors.⁽⁵⁾ However the dynamic behavior of the orbiting scroll and Oldham coupling is only partly revealed by the authors.

The model for the fixed throw crank is established and the behavior during operation is obtained. The model for the compliant crank will be examined in the 2nd report, as well as the model for the compressor system.

Symbols

F:	force
g:	gravitational acceleration
h:	scroll height
M:	moment
m_o :	mass of Oldham coupling
m_s :	mass of orbiting scroll
N:	number of scrolls - 1/4
P:	pressure
p:	scroll pitch
r:	crank radius
t:	scroll wall thickness
V_s :	displacement volume

X,Y:	coordinate, orbiting scroll
x,y:	coordinate, Oldham coupling
$\bar{E}$:	thrust reaction point/thrust bearing radius
θ :	angle of rotation
κ :	specific heat ratio
μ :	friction coefficient
ρ :	density
ω :	angular velocity

Suffixes

B:	bearing
c:	centrifugal force
d:	discharge
m:	inertia force
o:	Oldham coupling
r:	radial
s:	suction
t:	thrust
θ :	tangential
1,2,3,4:	Oldham coupling surface

2. Dynamic Model for Scroll Compressor

2.1 Equations of motion for the orbiting scroll and the Oldham coupling

The model for the fixed throw crank is shown in Fig. 1. O_1 is the axis of the rotation and O_2 is the center of the orbiting scroll. The angular displacement of the orbiting scroll around O_2 is restricted by the Oldham coupling. The distance (r) between O_1 and O_2 is kept constant. Forces F_1 and F_2 act on the sliding surface between the orbiting scroll and the Oldham coupling.

The forces which act on the orbiting scroll are tangential acceleration $F_m (= m_s r (d^2\theta/dt^2))$, centrifugal force $F_c (= m_s r (d\theta/dt)^2)$, tangential gas force F_{θ} , radially inward gas force F_r , thrust F_t , friction of the thrust bearing $\mu_t F_t$, contact forces F_1 and F_2 , sliding friction $\mu_1 F_1$ and $\mu_2 F_2$ and bearing resistance moment M_B .

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Fig. 1 Orbiting Scroll Motion

(The force of the orbiting scroll due to gravity is neglected compared to thrust F_t .)

The model of the Oldham coupling is shown in Fig. 2. O_3 is the center of the Oldham coupling moves reciprocally along the y-axis guided by the channel of the compressor housing. The key of the Oldham coupling along the x-axis is coupled to the orbiting scroll and maintains an angular relationship between them.

Fig. 2 Oldham Coupling Motion

The forces which act on the Oldham coupling, i.e., the inertia force $m_o(d^2/dt^2)(r \sin \theta)$ contact force F_i ($i=1,2$), sliding friction $\mu_i F_i$ ($i=1,2$), contact force F_i ($i=3,4$), sliding friction $\mu_i F_i$ ($i=3,4$), gravitational force $m_o g$ and friction $\mu_o m_o g$.

The equations of motion are obtained as follows:

X-wise balance for the Oldham coupling

$$\mu_1 F_1 + \mu_2 F_2 - F_3 + F_4 = 0 \quad (1)$$

Y-wise momentum for the Oldham coupling

$$m_o \frac{d^2}{dt^2} (r \sin \theta) = -F_1 + F_2 - \mu_3 F_3 - \mu_4 F_4 - \mu_o m_o g \quad (2)$$

moment balance for the Oldham coupling

$$F_1 \cdot n + F_2 \cdot n - F_3 \cdot n - F_4 \cdot n - \mu_1 F_1 e + \mu_2 F_2 e + \mu_3 F_3 e - \mu_4 F_4 e - \mu_o m_o g x' = 0 \quad (3)$$

moment balance for the orbiting scroll

$$F_\theta \cdot \frac{r}{2} + (\mu_t F_t \sin \theta \cdot y - \mu_t F_t \cos \theta \cdot x) - F_1 \cdot n - F_2 \cdot n + \mu_1 F_1 - \mu_2 F_2 e + M_B = 0 \quad (4)$$

where x and y are obtained from the moment balance around the thrust bearing in Fig. 1 as follows.

$$x = \left\{ \frac{F_c}{F_t} \left(\frac{l}{2} + c \right) - \frac{F_r}{F_t} \left(\frac{h}{2} + \delta + \frac{l}{2} \right) - \frac{r}{2} \right\} \cos \theta + \left\{ \frac{F_\theta}{F_t} \left(\frac{h}{2} + \delta + \frac{l}{2} \right) + \frac{l}{2} \cdot \mu_t + \frac{F_m}{F_t} \left(\frac{l}{2} + c \right) \right\} \sin \theta + \frac{\mu_1 F_1 + \mu_2 F_2}{F_t} \left(\frac{l}{2} - \frac{b}{2} \right) \quad (5)$$

$$y = \left\{ \frac{F_\theta}{F_t} \left(\frac{h}{2} + \delta + \frac{l}{2} \right) + \frac{l}{2} \mu_t + \frac{F_m}{F_t} \left(\frac{l}{2} + c \right) \right\} \cos \theta + \left\{ -\frac{F_c}{F_t} \left(\frac{l}{2} + c \right) + \frac{F_r}{F_t} \left(\frac{h}{2} + \delta + \frac{l}{2} \right) + \frac{r}{2} \right\} \sin \theta + \frac{F_1 - F_2}{F_t} \left(\frac{l}{2} - \frac{b}{2} \right) \quad (6)$$

From the moment balance around the c.g. of the Oldham coupling, x' and y' are obtained as follows.

$$x' = \frac{(\mu_1 F_1 + 2 F_2)}{m_o g} \cdot \frac{(b + w)}{2}$$

$$+ \frac{F_3 - F_4}{m_0 g} \cdot \frac{(b + w)}{2} \quad (7)$$

$$y' = \frac{-F_1 + F_2}{m_0 g} \cdot \frac{(b + w)}{2} + \frac{\mu_3 F_3 + \mu_4 F_4}{m_0 g} \cdot \frac{(b + w)}{2} + \mu_0 \cdot \frac{w}{2} \quad (8)$$

Eqs. (1) - (4) are solved simultaneously and F_i ($i = 1 \sim 4$) are obtained as follows.

$$F_1 = \frac{1}{E} \{D \cdot F - B \cdot G\} \quad (9)$$

$$F_2 = \frac{1}{E} \{-C \cdot F + A \cdot G\} \quad (10)$$

$$F_3 = \frac{1}{E \cdot E'} \{(C'D - CD') \cdot F + (AD' - BC') \cdot G\} \quad (11)$$

$$F_4 = \frac{1}{E \cdot E'} \{(C''D - CD'') \cdot F + (AD'' - BC'') \cdot G\} \quad (12)$$

where

$$A = (n - e\mu_1) + \mu_1\mu_2\left(\frac{\ell}{2} - \frac{b}{2}\right) \cdot \cos \theta$$

$$+ \mu_t\left(\frac{\ell}{2} - \frac{b}{2}\right) \sin \theta$$

$$B = (n + e\mu_2) + \mu_2\mu_t\left(\frac{\ell}{2} - \frac{b}{2}\right) \cdot \cos \theta$$

$$- \mu_t\left(\frac{\ell}{2} - \frac{b}{2}\right) \sin \theta$$

$$C = (\mu_1\mu_4 - 1) \cdot \{(n - e\mu_3)$$

$$+ (n + e\mu_4)\} - (\mu_3 + \mu_4)$$

$$\{\mu_1(n + e\mu_4) + (n - e\mu_1)$$

$$+ \mu_0\mu_1(b + w)\}$$

$$D = (\mu_2\mu_4 + 1) \cdot \{(n - e\mu_3)$$

$$+ (n + e\mu_4) - (\mu_3 + \mu_4)$$

$$\{\mu_2(n + e\mu_4) + (n + e\mu_2)$$

$$+ \mu_0\mu_2(b + w)\}$$

$$C' = \mu_1(n + e\mu_4) + (n - e\mu_1)$$

$$+ \mu_0\mu_1(b + w)$$

$$D' = \mu_2(n + e\mu_4) + (n + e\mu_2)$$

$$+ \mu_0\mu_2(b + w)$$

$$C'' = -\mu_1 \cdot (n - e\mu_3) + (n - e\mu_1)$$

$$+ \mu_0\mu_1(b + w)$$

$$D'' = -\mu_2 \cdot (n - e\mu_3) + (n + e\mu_2)$$

$$+ \mu_0\mu_2(b + w)$$

$$E = A \cdot D - B \cdot C$$

$$E' = (n - e\mu_3) + (n + e\mu_4) \quad (13)$$

$$F = F_\theta \cdot \frac{r}{2} + M_B + \mu_t \{F_r \left(\frac{h}{2} + \delta + \frac{\ell}{2}\right)$$

$$- F_c \left(c + \frac{\ell}{2}\right) + F_t \frac{r}{2}\}$$

$$G = E' \{ \mu_0 m_0 g + m_0 r \cos \theta \frac{d^2 \theta}{dt^2}$$

$$- m_0 r \sin \theta \left(\frac{d\theta}{dt}\right)^2 \} \quad (14)$$

The friction coefficients are given as follows.

$$\mu_1 = |\mu_1| \operatorname{sgn}(\sin \theta)$$

$$\mu_2 = |\mu_2| \operatorname{sgn}(\sin \theta)$$

$$\mu_3 = |\mu_3| \operatorname{sgn}(\cos \theta)$$

$$\mu_4 = |\mu_4| \operatorname{sgn}(\cos \theta)$$

$$\mu_0 = |\mu_0| \operatorname{sgn}(\cos \theta) \quad (15)$$

The friction changes its direction depending on the relative movement between the contact surfaces. The friction moment M_B is obtained as follows.

$$M_B = \mu_B r_B F_B \quad (16)$$

where μ_B is the friction coefficient and r_B is the bearing radius. F_B is unknown when the friction exists. The iteration procedure may be necessary to obtain an accurate F_B . However F_B without the friction can be employed for approximation.

F_i ($i = 1 \sim 4$) can be obtained from Eqs. (9) $\sim$ (12).

Without the friction, F_i ($i = 1 \sim 4$) are obtained, respectively, as follows.

$$F_1 = \frac{F_\theta \cdot \frac{r}{2}}{2n} - \frac{m_0 \frac{d^2}{dt^2} (r \sin \theta)}{2} \quad (17)$$

$$F_2 = \frac{F_\theta \cdot \frac{r}{2}}{2n} - \frac{m_0 \frac{d^2}{dt^2} (r \sin \theta)}{2} \quad (18)$$

$$F_3 = \frac{F_\theta \cdot \frac{r}{2}}{2n} \quad (19)$$

$$F_4 = \frac{F_\theta \cdot \frac{r}{2}}{2n} \quad (20)$$

The inertia force of the Oldham coupling is transmitted to the sliding surface between the Oldham coupling and the orbiting scroll.

Eqs. (17) $\sim$ (20) (no friction) may be employed for the ordinary design.

2.2 Forces which act on the orbiting scroll bearing

The x and y components of the bearing load F_B are obtained, respectively, from the force balance in Fig. 1.

$$F_{Bx} = F_\theta \cdot \sin \theta - F_r \cdot \cos \theta + F_c \cdot \cos \theta + F_m \cdot \sin \theta + \mu_t F_t \cdot \sin \theta + \mu_1 F_1 + \mu_2 F_2 \quad (21)$$

$$F_{By} = F_\theta \cdot \cos \theta + F_r \cdot \sin \theta - F_c \cdot \cos \theta + F_m \cdot \cos \theta + \mu_t F_t \cdot \cos \theta + F_2 - F_1 \quad (22)$$

The tangential and the radial components of F_B are given as follows:

$$F_{B\theta} = F_\theta + F_m + \mu_t F_t + (\mu_1 F_1 + \mu_2 F_2) \sin \theta + (-F_1 + F_2) \cos \theta \quad (23)$$

$$F_{Br} = F_c - F_r + (\mu_1 F_1 + \mu_2 F_2) \cos \theta + (F_1 - F_2) \sin \theta \quad (24)$$

2.3 Torque to drive the compressor

The torque to drive the scroll compressor is the sum of the torque to counter the bearing tangential load and the friction moment of the bearings, which is given as follows:

$$T = F_{B\theta} \cdot r + \Sigma M_B = \{F_\theta + F_m + \mu_t F_t + (\mu_1 F_1 + \mu_2 F_2) \sin \theta + (-F_1 + F_2) \cos \theta\} r + \mu_B F_{Br} r_B + \mu_{B1} F_{B1} r_{B1} + \mu_{B2} F_{B2} r_{B2} \quad (25)$$

where $F_{B\theta}$ is given by Eq.(23). The last three terms of the right hand side of Eq. (25) represent the friction moments of the orbiting scroll bearing, the main bearing and the sub-bearing, respectively. The first term in { } of Eq. (25) represents the gas compression force, the second term the acceleration, and the third term the thrust bearing friction. The fourth and fifth term in { } of Eq.(25) represent the frictional and the inertia forces of the Oldham coupling, respectively. See Eq. (2).

2.4 Power to drive the compressor

Now that the torque T to drive the compressor is obtained, the power L to drive the compressor can be calculated by using Eqs.(2) and (25) when angular velocity ω is constant. ($\theta = \omega t$, $F_m = 0$) The power L is given as follows.

$$L = T \cdot \omega$$

$$\begin{aligned} &= F_\theta \cdot r \omega - m_0 r^2 \omega^3 \sin \theta \cos \theta \\ &+ \mu_t F_t \cdot \omega r \\ &+ (\mu_1 F_1 + \mu_2 F_2) r \omega \sin \theta \\ &+ (\mu_3 F_3 + \mu_4 F_4 + \mu_0 m_0 g) r \omega \cos \theta \\ &+ \mu_B F_{Br} r_B \omega + \mu_{B1} F_{B1} r_{B1} \omega \\ &+ \mu_{B2} F_{B2} r_{B2} \omega \end{aligned} \quad (26)$$

where the first term of the right hand side represents the gas compression energy and the second term represents the Oldham coupling reversible energy consumption which changes twice as fast as the rotation speed. The other terms of the right hand side of Eq.(26) show the frictional energy losses.

2.5 The overturning moment of the orbiting scroll

The radial force as well as the thrust acts on the orbiting scroll as shown in Fig. 1. Therefore the overturning moment always exerts a force on the orbiting scroll.

The thrust reaction F_t at (x, y) in Fig. 1 counters the overturning moment caused by the radial force. The thrust reaction moment around the x-axis (which passes the bearing center) is given as follows.

$$M_x = F_t \cdot (y - r \sin \theta) \quad (27)$$

The thrust reaction around the y-axis is given as follows.

$$M_y = F_t \cdot (x + r \cos \theta) \quad (28)$$

where x and y are given by Eq.(5) and Eq. (6), respectively. The overturning moment is given by

$$M = \sqrt{M_x^2 + M_y^2} \quad (29)$$

The direction of the moment is

$$\theta_M = \tan^{-1} \frac{M_y}{M_x} \quad (30)$$

The thrust reaction point r_t measured from the thrust bearing center is

$$r_t = \sqrt{(x + r \cos \theta)^2 + (y - r \sin \theta)^2} \quad (31)$$

The ratio $\tilde{\epsilon}$ of r_t against the thrust substantial outer radius $d_t/2$ is

$$\tilde{\epsilon} = \frac{2M}{F_t \cdot d_t} \quad (32)$$

where $M = r_t \cdot F_t$. The stability condition for the orbiting scroll is $\tilde{\epsilon} < 1$.⁽⁵⁾ The moment together with the thrust acts

on the thrust bearing and the thrust reaction is eccentric.⁽⁶⁾

3. Example

The calculated results in the air conditioning operation ($P_d/P_s = 2.06/0.59$ MPa) are shown in this chapter for the scroll compressor, with the displacement volume of 90.5 cc/rev and the rotation speed of 3600 rpm. The number of scrolls is 3.25 and the built-in volume ratio is around 3. The friction coefficients are all assumed to be 0.01 and the other dimensions are shown in the figures and in Table 1.

Table 1. Dimensions for Calculation

Scroll pitch	p	20 mm
Scroll thickness	t	4 mm
Scroll height	h	24 mm
Number of scrolls - 1/4	N	3
Orbiting scroll shaft length	l	40 mm
Orbiting scroll diameter	d_o	135 mm
Orbiting scroll shaft diameter	d_s	28 mm
Orbiting scroll base plate thickness	δ	11 mm
Orbiting scroll density	ρ_s	7.8 g/cm ³
Oldham coupling mean radius	n	60 mm
Oldham coupling key half width	e	3 mm
Oldham coupling key height	b	8 mm
Oldham coupling thickness	w	4.2 mm
Oldham coupling outer diameter	d_1	134 mm
Oldham coupling inner diameter	d_2	106 mm
Oldham coupling density	ρ_0	7.8 g/cm ³
Friction coefficient	μ_{suffix}	0.01
Specific heat ratio	κ	1.17

The force acting on the sliding surface of the Oldham coupling is shown in Fig. 3. F_1 and F_2 change periodically during the rotation. This is the effect of the reciprocal movement of the Oldham coupling. See Eqs.(17) and (18). These forces appear to counter the torque which is half of the compression torque.

The calculated results for x' and y' of Eq.(8) (Oldham coupling reaction point) are shown in Fig. 4. The average diameter of the Oldham coupling is assumed to be 120 mm ($n = 60$ mm in Fig. 2) and condition $y' > n$ appears. The substantial reaction point of the Oldham coupling is the outer radius when $y' > n$, and the coupling tends to float around it. This is because the point where the force acts is different from that where the force is supported. To avoid the floating of the Oldham coupling, the axial clearance between the orbiting scroll and the coupling may be reduced. The sign change of x' at $\theta = \pi$ occurs because the sliding direction becomes opposite. The other discontinuity of x' is caused by the commencement of the gas discharge.

The scroll bearing load F_B together with the tangential component $F_{B\theta}$ and the radial component F_{Br} is shown in Fig. 5.

Fig. 3 Oldham Coupling Reaction

Fig. 4 Reaction Position of Oldham Coupling

See Eqs.(23), (24) etc. The bearing load is fairly smooth in the air conditioning condition because the scroll compressor operates near the built-in pressure ratio. F_{Br} is primarily the centrifugal force, and $F_{B\theta}$ is caused by the gas pressure (in Fig. 5). The frequency, which is twice as fast as the rotation speed, is found in F_B , $F_{B\theta}$ and F_{Br} , and this is the effect of the Oldham coupling. For example, this effect on F_{Br} , is shown by the fourth term of the right hand side of Eq.(24). (Substitute F_1 and F_2 of Eq.(17) and (18) into the fourth term of the right hand side of Eq.(24).)

Fig. 5 Bearing Load of Orbiting Scroll

The torque of Eq.(25) is shown in Fig. 6. The result is fairly smooth in the air conditioning condition and is similar to the bearing load. The torque includes the friction and the inertia force of the Oldham coupling and is therefore slightly different from the theoretical one.

The power to drive the scroll compressor of Eq.(26) is shown in Fig. 7. It is shown in Fig. 7 that the effect of the second term of the right hand of Eq. (26) is noticeable. The energy of the

Fig. 6 Torque

Fig. 7 Compression Energy

Oldham coupling is proportional to the cube of the rotation speed at the constant speed ($\omega = \text{const}$). The magnitude of the energy changes twice as fast as the rotation speed. The theoretical energy, $L_g (= F_{\theta} r \cdot \omega)$, to compress the gas is also shown in Fig. 7. The difference between these two is partly due to the Oldham coupling.

The overturning moment M of Eq.(29) is shown in Fig. 8. $\tilde{\epsilon}$ of Eq.(32) is shown in Fig. 1, and the stability condition ($\tilde{\epsilon} < 1$) is satisfied.

4. Conclusions

An analytical model for the scroll compressor was established and the dynamic behavior of the Oldham coupling and the orbiting scroll was clarified.

(1) The equations of motion, including the friction, are established for the Oldham coupling and the orbiting scroll with the fixed throw crank. The forces which act on the coupling are obtained analytically.

(2) There appeared the effect of the reciprocal movement of the Oldham coupling where the coupling and the orbiting scroll make contact. The contact force changes periodically and the amplitude of the force is proportional to the mass of the Oldham coupling and the square of the rotation speed.

(3) The load of the scroll bearing, the torque and the power can be calculated, including the friction.

(4) The tangential and the radial forces of the overturning moment are supported by different planes from which they act, and the overturning moment appears accordingly. The overturning moment was estimated and the stability condition for the orbiting scroll was clarified.

Fig. 8 Overturning Moment of Orbiting Scroll

Fig. 9 Orbiting Scroll Stability

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