Minimum-energy Control of a Manipulator with Two-degree of Freedom*

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A performance criterion for minimum-energy control calculated by dynamic programming is defined considering the characteristics of driving source. Such minimum-energy control is compared with near-minimum-energy control whose input signal is rectangular, and the dynamic characteristics of a manipulator controlled by these methods are analyzed theoretically and investigated experimentally.

Key Words: Control Theory, Control Devices, Minimum Energy Control, Manipulator

1. Introduction

In general, optimum control of a manipulator is employed for minimizing the working time or minimizing the consumed energy. Minimum-time control of the system is made in consideration of dynamic characteristics of the driving source⁽²⁾⁽³⁾. And, minimum-energy control of the system is made by approximating the nonlinear system to a linear system and approximating a response of angular velocity to a trapezoid wave⁽⁴⁾, but consideration of dynamic characteristics of the driving source is not enough.

In this paper, a performance criterion for a minimum-energy control calculated by dynamic programming is defined considering the characteristics of a DC servomotor. Such minimum-energy control is compared with near-minimum-energy control under the condition that the input signal is rectangular, and the dynamic characteristics of the manipulator controlled by these methods are analyzed theoretically and investigated experimentally.

2. Minimum-energy control in consideration of characteristics of a driving source

2.1 Characteristics of a driving source

In general, the DC servomotor is employed to drive a system. In the armature of the servomotor, the output torque M_t is directly proportional to the electric current i_a , and their relation is expressed as

$$M_t = k_t i_a \quad (1)$$

And the applied voltage of the servomotor is

$$e = k_v \dot{\theta} + R_a i_a + L_a \frac{di_a}{dt} \quad (2)$$

where

e : applied voltage of servomotor
 $\dot{\theta}$: angular velocity of servomotor
 R_a : resistance of armature
 L_a : inductance of armature

The relation between the output torque of the armature and the output torque of the servomotor is

$$M_t = M + J_m \ddot{\theta} + D_m \dot{\theta} \quad (3)$$

where

M : output torque of servomotor
 J_m : moment of inertia of armature
 D_m : coefficient of viscous damping

Then, the electric current is

$$i_a = \frac{M_t}{k_t} = \frac{1}{k_t} (M + J_m \ddot{\theta} + D_m \dot{\theta}) \quad (4)$$

When the inductance is small, the applied voltage is

$$e = b_1 \dot{\theta} + b_2 \ddot{\theta} + b_3 M \quad (5)$$

where

$$b_1 = k_v + \frac{R_a}{k_t} D_m, \quad b_2 = \frac{R_a}{k_t} J_m, \quad b_3 = \frac{R_a}{k_t} \quad (6)$$

The performance criterion of the minimum-energy control is defined by the consumed energy of the motor, and the energy is derived in consideration of dissipation of energy which is generated by the motor. The consumed energy is

$$E = \int_0^T e i_a dt \quad (7)$$

where

$$1(x) = \begin{cases} 1 & (x \geq 0) \\ 0 & (x < 0) \end{cases} \quad (8)$$

2.2 Dynamical equation of the manipulator

When the mechanism of the manipulator is open-loop, the consumed energy increases with the weight of a motor which is on the moving link. Then, we shall take a manipulator whose mechanism is close-loop.

The parameters of the manipulator with two degrees of freedom as shown in Fig.1 which is able to move in a vertical plane are as follows.

l_j : length of link j
 m_j : mass of link j

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m_L : mass of the load
 l_{oj} : length between the end of a link and its center of gravity
 I_j : moment of inertia about the center of gravity
 M_j : torque which acts on link
 θ_j : angular displacement of link

The condition to maximize the workspace of the manipulator is

$$l_1 = l_2 = l_3 = l_4 = l, l_5 = 0$$

Then, dynamical equations of the system are

$$M_1 = a_{11}\ddot{\theta}_1 + a_{21}\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + a_{21}\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + a_{31} \cos \theta_1 \quad (9)$$

$$M_2 = a_{12}\ddot{\theta}_2 + a_{21}\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + a_{21}\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + a_{32} \cos \theta_2 \quad (10)$$

where

$$\begin{aligned}
 a_{11} &= I_1 + m_1 l_{o1}^2 + m_2 l^2 + I_3 + m_3 l_{o3}^2 + m_L l^2 \\
 a_{21} &= (m_2 l_{o2} + m_3 l_{o3} + m_L l) l \\
 a_{31} &= (m_1 l_{o1} + m_2 l + m_3 l_{o3} + m_L l) g \\
 a_{12} &= I_4 + m_4 l_{o4}^2 + m_3 l^2 + I_2 + m_2 l_{o2}^2 + m_L l^2 \\
 a_{32} &= (m_4 l_{o4} + m_3 l + m_2 l_{o2} + m_L l) g
 \end{aligned} \quad (11)$$

When the output torque of the motor is sufficiently large as compared with the torque of the load, the applied voltages are given as

$$e_1 = b_{11}\dot{\theta}_1 + b_{21}\dot{\theta}_1 + b_{31}[a_{11}\ddot{\theta}_1 + a_{21}\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + a_{21}\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + a_{31} \cos \theta_1] \quad (12)$$

$$e_2 = b_{12}\dot{\theta}_2 + b_{22}\dot{\theta}_2 + b_{32}[a_{12}\ddot{\theta}_2 + a_{21}\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + a_{21}\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + a_{32} \cos \theta_2] \quad (13)$$

3. Control method

3.1 Dynamic programming

It is difficult to calculate the manipulated variables for the minimum-energy control by the maximum principle because the dynamical equations of the system are highly nonlinear. Then, a numerical approach is necessary.

When the initial position is θ_{ij} , the desired position is θ_{fj} , the maximum applied voltage is $e_{j \max}$, and the reduction ratio of the driving source is large, the maximum velocity is approximated as follows⁽²⁾.

$$\dot{\theta}_{j \max} = [e_{j \max} - b_{3j} a_{3j} \cos((\theta_{ij} + \theta_{fj})/2)] / b_{1j} \quad (14)$$

Fig.1 Mechanism of manipulator

And, the range of angular displacements, θ_j , which is variable during the working time, T , is

$$\theta_{ij} - \Delta\theta_j \leq \theta_j \leq \theta_{fj} + \Delta\theta_j \quad (15)$$

where

$$\Delta\theta_j = (\dot{\theta}_{j \max} T - \theta_{fj} + \theta_{ij}) / 2$$

Then, we shall search for the path of minimum-energy control in the parallelogrammatic region as shown in Fig.2(a) which is enclosed with the lines whose gradients are $\pm \dot{\theta}_{j \max}$. The working time is divided into $K+1$ elements, and the angular displacement is divided into N elements, and the lattice points are defined as shown in Fig.2(b). The mean values of the angular displacement, the angular velocity and of the angular acceleration are as follows.

When $0 < t < \Delta t / 2$,

$$\begin{aligned}
 \bar{\theta}_{0nj} &= \theta_{ij} + (\theta_{1nj} - \theta_{ij}) / 4 \\
 \bar{\dot{\theta}}_{0nj} &= (\theta_{1nj} - \theta_{ij}) / (2\Delta t) \\
 \bar{\ddot{\theta}}_{0nj} &= 4\bar{\dot{\theta}}_{0nj} / \Delta t
 \end{aligned} \quad (n=1, 2, \dots, N) \quad (16)$$

When $(k-1/2)\Delta t < t < (k+1/2)\Delta t$,

$$\begin{aligned}
 \bar{\theta}_{knj} &= \theta_{knj} \\
 \bar{\dot{\theta}}_{knj} &= \frac{1}{2} \left(\frac{\theta_{(k+1)qj} - \theta_{knj}}{\Delta t} + \frac{\theta_{knj} - \theta_{(k-1)pj}}{\Delta t} \right) \\
 \bar{\ddot{\theta}}_{knj} &= \frac{1}{\Delta t} \left(\frac{\theta_{(k+1)qj} - \theta_{knj}}{\Delta t} - \frac{\theta_{knj} - \theta_{(k-1)pj}}{\Delta t} \right)
 \end{aligned} \quad (p=1, 2, \dots, N, \quad q=1, 2, \dots, N) \quad (17)$$

Fig.2 Region for searching for paths

When $(K+1/2)\Delta t < t < (K+1)\Delta t$,

$$\left. \begin{aligned} \bar{\theta}_{(K+1)n_j} &= \theta_{Kn_j} + 3(\theta_{fj} - \theta_{Kn_j})/4 \\ \bar{\dot{\theta}}_{(K+1)n_j} &= (\theta_{fj} - \theta_{Kn_j})/(2\Delta t) \\ \bar{\ddot{\theta}}_{(K+1)n_j} &= -4\bar{\dot{\theta}}_{(K+1)n_j}/\Delta t \end{aligned} \right\} \dots\dots\dots (18)$$

Then, the consumed energy during the time, Δt , is calculated from Eqs.(4) through (13). The number of paths from the lattice points, $\theta_{(k-1)Pj}$, to a lattice point, $\theta_{(k+1)Qj}$, is N^2 . And, the number of paths from the initial position to the desired position is $N^3 \times K$.

3.2 Near-minimum-energy control

A long calculating time is necessary to search for a path of the minimum-energy control by means of the dynamic programming. Then, we propose the near-minimum-energy control as follows.

When the applied voltage for accelerations is e_j and the applied voltage for retardation is e_{fj} , the switching time t_{qj} , of the applied voltage and the time, t_{fj} , when the link arrives at the desired position are easily calculated using an approximated expression for the velocity of the DC servomotor⁽²⁾. The equations are

$$t_{qj} \approx \frac{b_{1j}}{e_j - A_j} \left[\theta_{fj} - \theta_{ij} + \frac{e_{fj} - A_{fj}}{b_{1j}} T_{mj} \right] \times \ln \left(\frac{-e_{fj} + A_{fj}}{e_j - e_{fj} - A_j + A_{fj}} \right) \dots\dots\dots (19)$$

Fig.3 Flow chart of simulation for near-minimum-energy control

$$t_{fj} \approx t_{qj} - T_{mj} \ln \left[\frac{-e_{fj} + A_{fj}}{e_j - e_{fj} - A_j + A_{fj}} \right] \dots\dots\dots (20)$$

where

$$\left. \begin{aligned} A_j &= b_{3j} a_{3j} \cos[(\theta_{ij} + \theta_{fj})/2] \\ A_{fj} &= b_{3j} a_{3j} \cos \theta_{fj} \\ T_{mj} &= (b_{2j} + b_{3j} a_{1j})/b_{1j} \end{aligned} \right\} \dots\dots\dots (21)$$

Then, the consumed energy is approximated as follows.

$$\begin{aligned} E_j &\approx \frac{e_j}{k_{tj}} \left[\frac{e_j - A_j}{b_{1j}} \{a_{1j} + J_{mj} + D_{mj}(t_{qj} - T_{mj})\} \right. \\ &\quad \left. + t_{qj} a_{3j} \cos \theta_{ij} \right] + \frac{e_{fj}}{k_{tj}} \frac{e_j - e_{fj}}{b_{1j}} (D_{mj} T_{mj} \\ &\quad - a_{1j} - J_{mj}) \left[1 - \exp \left(-\frac{t_{fj} - t_{qj}}{T_{mj}} \right) \right] \\ &\quad + \frac{e_{fj}}{k_{tj}} \left(D_{mj} \frac{e_{fj} - A_{fj}}{b_{1j}} + a_{3j} \cos \theta_{fj} \right) (t_{fj} - t_{qj}) \end{aligned} \dots\dots\dots (22)$$

A flow chart of the simulation for the near-minimum-energy control is shown in Fig.3.

4. Simulations

We shall take the parameters of the system as shown in Table 1. The simulations of the system are done as follows.

A response of the manipulator for the minimum-time control from the initial position ($\theta_{i1}=1.2217$ rad, $\theta_{i2}=-1.2217$ rad), to the desired position ($\theta_{f1}=2.6180$ rad, $\theta_{f2}=2.0944$ rad) is shown in Fig.4. The applied voltage to the motor, e_j , is not expressed as a step function because of the internal resistance of the power source. When the manipulator is accelerated or decelerated, the electric current increases rapidly. And, the consumed energy is about 95.6 joule.

The energy, which is necessary to move the load from the initial position to the desired position, is expressed as follows.

$$E_w = \int_{\theta_{i1}}^{\theta_{f1}} a_{31} \cos \theta_1 d\theta_1 + \int_{\theta_{i2}}^{\theta_{f2}} a_{32} \cos \theta_2 d\theta_2$$

Table 1 Parameters of manipulator

Parameter	Value	Parameter	Value
l	m	I_1	kgm ² 1.22×10^{-2}
l_{q1}	m	I_2	kgm ² 7.09×10^{-3}
l_{q2}	m	I_3	kgm ² 7.45×10^{-3}
l_{q3}	m	I_4	kgm ² 1.22×10^{-2}
l_{q4}	m	J_{m1}	kgm ² 0.280
m_1	kg	J_{m2}	kgm ² 0.240
m_2	kg	D_{m1}	Nsm/rad 0.200
m_3	kg	D_{m2}	Nsm/rad 0.230
m_4	kg	b_{11}	Vs/rad 3.395
e_{1max}	V	b_{12}	Vs/rad 3.352
e_{2max}	V	b_{21}	Vs ² /rad 0.085
k_{t1}	Nm/A	b_{22}	Vs ² /rad 0.083
k_{t2}	Nm/A	b_{31}	V/Nm 0.397
		b_{32}	V/Nm 0.344

And, this is about 5.6 percent of the consumed energy for the minimum-time control.

When $T=0.62\text{ s}$, $K=20$, $N=1000$, a response of the minimum-energy control is shown in Fig.5. The calculating time, which is necessary to search for a path of the minimum-energy control by means of the dynamic programming, is 12 times as large as the time which is necessary for the minimum-time control. When the manipulator starts or stops, the electric current is small because the applied voltage changes slowly. And, the consumed energy for this control is about 25.8 percent of the consumed energy for the minimum-time control.

When $T=0.62\text{ s}$, a response of the near-minimum-energy control is shown in Fig.6. The calculating time for this control is the same as that for the minimum-time control. When the manipulator stops, the applied voltage and the electric current are small. And, the consumed energy for this control is about 31.4 percent of the consumed energy for the minimum-time control.

Figure 7 shows the relation between the working time and the consumed energy. When the working time increases from 0.5 seconds to 0.6 seconds, the consumed energy decreases rapidly. The consumed energy when the manipulator remains at rest is expressed as follows.

$$E_t = \int_0^T \frac{b_{31}}{k_{t1}} a_{31}^2 \cos^2 \theta_1 dt + \int_0^T \frac{b_{32}}{k_{t2}} a_{32}^2 \cos^2 \theta_2 dt$$

Fig.4 Minimum-time control

Fig.5 Minimum-energy control

Fig.6 Near-minimum-energy control

This energy increases as the working time increases. Then, the consumed energy, which is necessary to move the load from the initial position to the desired position under the condition that the angular velocities of the motors are constant, becomes minimum when the working time is about 3 times as large as the time which is necessary for the minimum-time control.

Figure 8 shows the paths of the minimum-energy control. The path in Fig.8(b) changes as the load changes. Then, the consumed energy is shown in Fig.9. When the manipulator starts, the angular velocity of the link 1 increases as the load increases. And, the response of the link 4 becomes smooth due to the inertial force of the link 1, and the consumed energy changes slowly. Thus, when the angular displacement of the link 1 from the initial position to the desired position is not the same as that of the link 4, the path changes as the load changes.

5. Experiment

5.1 Experimental apparatus

Experimental apparatus is shown in Fig.10. The parameters of this system are the same as given in Table 1. The motors 1 and 2 (rated 24 V, 60 W) are on the frame. And, the reduction ratio of the motor is 1/50. We shall measure the angular displacement of the links 1 and 2 by potentiometers. And, we shall measure the electric current as a potential difference of the resistor (rated 0.1 Ω , 40 W).

Fig.7 Relation between working time and consumed energy

Fig.8 Paths for minimum-energy control

Fig.9 Relation between mass of load and consumed energy

Fig.10 Experimental apparatus

Fig.11 Experimental result of minimum-energy control

5.2 Experimental result

Figure 11 shows a response of the minimum-energy control. The applied voltage, which is approximated to the polygonal lines, is controlled by the analog computer. When the manipulator stops, the electric current and the applied voltage of the motor 1 fluctuate due to the inertial force of the link 4. And, the experimental response (solid line) is the same as the theoretical result (dashed line).

Figure 12 shows a response of the near-minimum-energy control. When the electric current of the motor 2 increases rapidly, the applied voltage of the motor 2 fluctuates. And, the experimental result (solid line) is the same as the theoretical result (dashed line).

Here, we shall define the positioning error as follows.

$$\delta_j = \frac{\theta_{fj} - \theta_j}{|\theta_{fj} - \theta_{ij}|} \quad (j=1, 2)$$

When the initial conditions and the final conditions are the same as shown in Fig.11 and Fig.12, the relation between the mass of load and the positioning error is as shown in Fig.13. Figure 13(a) shows a positioning error of the minimum-energy

Fig.12 Experimental result of near-minimum-energy control

control. When the applied voltage is approximated by polygonal lines, the deviation between the desired voltage and the actual voltage increases. And, the positioning error is large. Figure 13(b) shows a positioning error of the near-minimum-energy control. When the applied voltage of the motor 1 is small, the positioning error of the link 1 is large because of the inertial force of the link 4.

From these results, it is seen that the experimental responses of the system are nearly equal to the theoretical responses.

Fig.13 Relation between mass of load and positioning error

6. Conclusions

The results obtained in this paper are summarized as follows.

(1) When the working time for the minimum-energy control is 1.2 ~ 1.3 times as long as the time for the minimum-time control, the consumed energy for the minimum-energy control is 1/3~1/4 times as much as the energy for the minimum-time control.

(2) Accurate switching times of the applied voltage for the near-minimum-energy control, whose input signal is rectangular, can be easily calculated using an approximated expression of the velocity of the DC servomotor.

(3) It has been revealed that the experimental responses of this control method are effective.

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