Effects of Rim and Web Thicknesses on Bending Fatigue Strength of Internal Gear*

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This paper presents a study on the effects of rim and web thicknesses on bending fatigue strength of internal gears. A stress analysis by means of the 3-dimensional finite element method (FEM) and a static loading test for internal gears of different rim and web thicknesses were carried out and the effects of rim and web thicknesses on the root, rim and web stresses of internal gears were clarified. A bending fatigue test for thin-rimmed internal gears with a thin web was carried out and the effects of rim and web thicknesses on the bending fatigue strength were investigated.

Furthermore, the validity of the calculating formula of tooth bending strength of internal gear proposed by ISO was examined.

Key Word: Gear, Fatigue, Internal Gear, Bending Fatigue Strength, Rim Thickness, Web, Root Stress, Crack, FEM

1. Introduction

Recently, with increased demands for higher reduction ratio, size-down and lighter weight, planetary gear units have come increasingly into use and the load-carrying capacity of internal gear is now a matter of great concern. The rim and web thicknesses of an internal gear used in a planetary gear unit are often designed thin in order to distribute the transmitting power uniformly to every planetary gear by using elastic deformation of the rim. Although some studies on the root stress and bending fatigue strength of a thin-rimmed internal gear have been published(1)~(3), those of a thin-rimmed internal gear with a thin web are few. It would be necessary to clarify the root stress and bending fatigue strength of a thin-rimmed internal gear with a thin web for a more exact design of internal gears to be used in planetary gear unit.

In the present paper, the effects of rim and web thicknesses on the root stress and bending fatigue strength of internal gears were investigated. A stress analysis by FEM and a static loading test for internal gears of different rim and web thicknesses were carried out and the effects of rim and web thicknesses on the root, rim and web stresses of internal gears were clarified. The bending fatigue strength of internal gears of various rim and web thicknesses was investigated by making use of a bending fatigue testing machine, which had been developed by the authors.

Furthermore, the validity of the calculating formula of tooth bending strength of internal gear proposed by ISO was examined on the basis of the results of the static loading and bending fatigue tests.

2. Stress Analysis by 3-dimensional Finite Element Method

2.1 Computer program system for 3-dimensional analysis

The computer program used in this computation is SANAS-F(4) developed by

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Fig.1 Mesh pattern for gear model
2.2 Partition of gear model and constraint condition

A finite element analysis was made on an internal gear model with single web at the middle of face width. The main dimensions of the gear model are module \( m = 4 \), pressure angle \( \alpha_0 = 20^\circ \), number of teeth \( z = 45 \), face width \( b = 30 \text{ mm} \). The analysis was made on gear models with three teeth, the middle tooth of which was loaded. Figure 1 shows a mesh pattern of the gear model which is determined on the basis of the results in ref. (4). Each tooth of the gear model is approximated by four rectangles inscribed inside an internal gear tooth as shown in Fig. 2(a). Figure 2(b), (c) show the mesh patterns of rim and web parts respectively (web length in radial direction 30 mm). The numbers of elements and nodes for the gear models are 762 and 875 respectively. In the stress analysis by FEM, each node on outer circumference of web disk was constrained to have zero displacement. The FEM analysis was also made on the gear models with rack teeth as shown in Fig. 2(a) (two-dot chain line) and compared with the case of the gear models with internal gear teeth.

2.3 Computation of root stresses, rim (tooth bottom) and web stresses and tooth deflections

The root stresses, rim and web stresses and tooth deflections including deformation of rim and web parts were computed for a concentrated normal tooth load \( P = 10 \text{ N} \) (1.02 kgf) which was applied to 11 points along the tip of a tooth one after another as shown in Fig. 2(a). Application of this analysis was extended to a study on the stresses due to a distributed load. The root, rim and web stress distributions due to the distributed load were computed under the condition that the tooth deflection was equal along the tooth trace considering the mating solid gear a rigid one. The deflection in the direction of the line of action was taken as the tooth deflection. The root and rim stresses were obtained for the elements at E-zone 4 in Fig. 2(a) and at tooth bottom respectively. The web stresses \( \sigma_{w0} \) in circumferential direction and \( \sigma_{w} \) in radial direction were obtained for the elements shown in Fig. 2(c).

3. Experimental Method and Apparatus

3.1 Test gear

Test gears used in this experiment (Fig. 3) are standard internal spur ones.

Table 1 Rim and web thicknesses in static loading test

<table>
<thead>
<tr>
<th>Sector angle</th>
<th>( \psi )</th>
<th>( l_w ) mm</th>
<th>( b_w ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°</td>
<td>2m</td>
<td>30, 22, 14, 8, 4</td>
<td></td>
</tr>
<tr>
<td>72°</td>
<td>2m</td>
<td>30, 22, 14, 8, 4</td>
<td></td>
</tr>
<tr>
<td>120°</td>
<td>2m</td>
<td>30, 22, 14, 8, 4</td>
<td></td>
</tr>
</tbody>
</table>

* \( m \): module

Fig. 3 Internal spur gear with sector web
The main dimensions of test gears are module $m = 4$, pressure angle $\alpha_0 = 20^\circ$, number of teeth $z = 45$ and face width $b = 30$ mm. The test gears are made of S45C steel and generated with a pinion cutter after normalizing (heat treatment condition: $850^\circ$C x 1.5h water quenched and $620^\circ$C x 2h air cooled, Vickers hardness: $H_v = 210$). Dimensions of the external gears to apply a load to the internal gears are $m = 4$, $\alpha_0 = 20^\circ$, $z = 23$, $b = 40$ mm.

### 3.2 Static loading test

The root stresses were measured for various rim and web thicknesses shown in Table 1 by thinning the rim and web of the internal gear (Fig. 3) by an end mill. Figure 4 indicates the pasted positions of strain gages (gage length: 0.3 mm). Strain gages 1 - 6 were pasted on the root fillets on both the tensile and the compressive side at both ends (3 mm toward the middle) and the middle of face width in tooth height direction. The applied load (per unit face width) was chosen $P_0/b = 65.4$ N/mm.

Figure 5 shows a static loading testing machine used in this experiment. The test internal gear is fixed to the supporting frame at arbitrary meshing positions and loaded by driving the mating external gear with hydraulic pressure through a manual oil pump. The applied load value is detected from the readings of the strain gages pasted on the shaft. The measurement of root stresses was carried out under the single tooth pair contact realized by cutting off adjacent teeth of the loading tooth of the external gear.

![Fig. 4: Positions of strain gages](image)

1 N = 0.102 kgf

![Fig. 5: Static loading and bending fatigue testing machine for internal gear](image)

### 3.3 Bending fatigue test

Bending fatigue tests were carried out on the internal spur gears of sector angle of web $\psi = 60^\circ$, web thickness $b_w = 4$ mm, rim thickness $t_{rw} = 1.5$ mm, 2m (m: module). The bending fatigue testing machine used is of hydraulic type and consists of a fuel injection pump and its driving apparatus and an internal gear supporting frame as shown in Fig. 5. The frequency of load applications is about 600 c/min.

![Fig. 6: Effect of web thickness on root stress](image)

$1$ MPa = $0.102$ kgf/mm², $1$ N = $0.102$ kgf

![Fig. 7: Effect of rim thickness on root stress](image)

$1$ MPa = $0.102$ kgf/mm², $1$ N = $0.102$ kgf
4. Computed and Experimental Results and Discussions

4.1 Effects of rim and web thicknesses on root stresses

Figure 6(a), (b) show the measured root stresses on the tensile side of the internal gears of \( t_w = 2m, 5m, \gamma = 40^\circ \) with various web thicknesses. It is seen from this figure that the stress values at the middle of face width (just above web) are higher than those at the ends and that the maximum ones increase with a decreasing web thickness for \( t_w = 5m \). Figure 7 shows the measured root stresses for the internal gears of \( b_w = 4 \text{ mm}, \gamma = 40^\circ \) with various rim thicknesses. The stress values at the middle of face width increase with a decreasing rim thickness. Figure 8 shows the computed root stresses (2-dimensional FEM) for the internal gear with a fan-shaped hole \( (b_w = 0 \text{ mm}, \gamma = 40^\circ) \) and the measured root stresses for the internal gear with a thin web \( (b_w = 4 \text{ mm}, \gamma = 40^\circ) \).

\[ 1 \text{ MPa} = 0.102 \text{ kgf/mm}^2, \quad 1 \text{ N} = 0.102 \text{ kgf} \]

Fig.8 Effect of rim thickness on root stresses of internal gear with a thin web \( (b_w = 4 \text{ mm}) \) and those of internal gear with a fan-shaped hole \( (b_w = 0 \text{ mm}) \)

\[ P_y = \frac{P}{b} \]

(a) Tensile side

(b) Compressive side

Fig.9 Load components on tooth

The abscissa denotes the ratio of rim thickness to module \( (l_w/m) \) and the ordinate the root stresses (compressive stress : absolute value). The root stresses \( (l_w \geq 3.375m) \) calculated by ISO formula are also indicated. In this figure \( \text{FEM}_\theta = 45^\circ \) and \( \text{FEM}_{\text{max}} \) denote the root stresses at the positions of tangential angle \( \theta = 45^\circ \) (\( \theta \) : angle between the center line of the tooth and the tangent to the fillet curve) and the maximum root stresses respectively. It is seen from Fig.8 that the root stresses on both the tensile and the compressive side of the internal gears with a thin web and with a fan-shaped hole increase with a decreasing rim thickness, and that the root stresses of the internal gear with a thin web are

\[ \sigma_\alpha, \sigma_\beta, \sigma_\gamma \]
almost equal to the case with a fan-shaped hole on both the tensile side and the compressive side for \( t_w \geq 4\text{mm} \) and larger on tensile side and smaller on compressive side for \( t_w < 4\text{mm} \).

The normal tooth load \( P_n \) is resolved into components \( P_1 \) and \( P_2 \) normal and parallel to the center line of the tooth respectively as shown in Fig.9. The root stress due to \( P_n \) is obtained by composing the bending stress \( \sigma_{b_1} \) due to \( P_1 \) and compressive stress \( \sigma_{cr} \) due to \( P_2 \) and bending stress \( \sigma_{b_2} \) due to \( P_2 \). Figure 10 shows each root stress component (at the position of \( \theta = 45^\circ \)) computed by 2-dimensional FEM for the internal spur gear with a fan-shaped hole (\( b_w = 0\text{mm}, \gamma = 40^\circ \)) with \( t_w/\alpha \) on the abscissa. \( \sigma_{b_1} \) increases sharply with a decreasing rim thickness on both the tensile and the compressive side. \( \sigma_{b_2} \) is almost constant irrespective of rim thickness on both the tensile and the compressive side. \( \sigma_{cr} \) decreases on tensile side and increases on compressive side with a decreasing rim thickness. These phenomena might be due to the larger deformation of the rim part with a decreasing rim thickness. Since the deformation of rim part of the internal gear with a thin web is considered smaller due to the web than that with a fan-shaped hole, the variation of \( \sigma_{cr} \) due to the rim thickness becomes smaller compared with the case of the internal gear with a fan-shaped hole. Hence in Fig.8

the root stresses of the internal gear with a thin web are considered larger on tensile side and smaller on compressive side than those with a fan-shaped hole for \( t_w < 4\text{mm} \). Since the root stress of the internal gear (\( t_w \geq 3.375\text{mm} \)) calculated by ISO formula is smaller than the measured ones as shown in Fig.8, the estimation of ISO formula is considered to be on critical side as in the case of the internal gear with a fan-shaped hole.

### 4.2 Effect of sector angle of web on root stresses

Figure 11(a), (b) show the measured root stresses of the internal gears of \( \gamma = 72^\circ \), \( t_w = 2\text{mm} \) with various web thicknesses. It is seen from this figure that the stress values at the middle of face width (just above web) are higher than those at the ends and tend to increase for \( b_w \geq 8\text{mm} \) and decrease for \( b_w < 8\text{mm} \) with a decreasing web thickness. These tendencies differ from the case of \( \gamma = 40^\circ \) [Fig.6(a)]. Figure 12(a), (b) show the measured and computed root stresses for the case of \( t_w = 2\text{mm}, b_w = 4\text{mm} \). The root stresses at the middle of face width decrease with an increasing sector angle of web on both the tensile and the compressive side. This might be because the rigidity of the rim becomes smaller with an increasing sector angle of web and the range of the deformation of the rim part produced by the application of load becomes larger. The differences between the measured root stresses at the middle of face width for \( \gamma = 120^\circ \) and the computed ones are 7% on tensile side and 9% on compressive side.

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**Fig.11** Effect of web thickness on root stress (\( \gamma = 72^\circ \))

**Fig.12** Relation between sector angle of web and root stress
side. This might be because the computed root stresses are those for $\gamma = 360^\circ$ and the measured root stresses are considered to approach the computed ones with an increasing sector angle of web, since the measured root stresses decrease with an increasing sector angle of web. Hence the 3-dimensional finite element analysis method used in this investigation is considered valid for the thin-rimmed internal gear with a thin web as in the case of the thin-rimmed external gear with web arrangement.

4.3 Computed results of root, rim and web stresses

Figure 13(a), (b) show the computed root and rim stress distributions due to the distributed load ($P = 110$ N). It is seen from this figure that the root stress shows the highest value at the middle of face width (just above web) and a tendency to decrease monotonously toward the end of face width. The root stress distributions are almost equal irrespective of rim and web thicknesses on both the tensile and compressive sides for $l_w = 2m$, $b_w = 4$ mm. The rim stress increases with decreasing rim and web thicknesses on both the tensile and the compressive side. The maximum rim stress occurs at the middle of face width on tensile side and at the position between the middle and the end of face width on compressive side. The maximum rim stress for $l_w = 2m$, $b_w = 4$ mm is fairly large, which corresponds to 80% of the maximum root stress. Figure 14 shows the computed web stresses due to the distributed load ($P = 110$ N) for the case of $l_w = 2m$, $b_w = 4$ mm. In this figure $\sigma_{wT}$, $\sigma_{wR}$ indicate the web stresses in radial and circumferential directions respectively. The maximum web stress occurs at the inner circumference of web on tensile side and is fairly smaller, which corresponds to 23% of the maximum root stress. The maximum web stress for the cases of $l_w = 2m$, $b_w = 14$ mm and $l_w = 5m$, $b_w = 4$ mm correspond to 12% and 8% of the maximum root stress respectively. Thus the root, rim and web stresses of the internal gears of $l_w = 2m$, $b_w = 4$ mm, $P = 110$ N, $l_w = 2m$, $b_w = 4$ mm are shown in Figure 14.

1 MPa = 0.102 k gf/mm², 1 N = 0.102 k gf

Fig. 13 Computed root and rim stress distributions

Fig. 14 Computed web stress distribution (constraint condition: outer circumference of web is fixed)

Fig. 15 Bending fatigue test results
the center of gears. Hence it might be considered reasonable to take \( t_w = 2m \) as a minimum rim thickness for such internal gears of \( b_w \geq 4 \text{ mm} \) as used in this experiment.

5. Conclusions

The main results obtained from this investigation are summarized as follows.

1. In the internal gears of larger rim thickness \( t_w \), the maximum root stress is almost equal irrespective of web thickness \( b_w \) and sector angle of web \( \gamma \). In the internal gears of smaller \( t_w \), the maximum root stress increases monotonously with a decreasing \( t_w \) for smaller \( \gamma \) but has maximum at a certain \( b_w \) for larger \( \gamma \).

2. The maximum root stress of the thin-rimmed internal gear with a thin web shows a tendency to decrease with an increasing sector angle of web on both tensile and compressive sides.

3. The 3-dimensional finite element analysis method used in this investigation is considered valid for the thin-rimmed internal gear with a thin web as in the case of the thin-rimmed external gear with web arrangement.

4. The root, rim and web stresses of the internal gears of \( t_w \geq 2m \), web thickness \( b_w \geq 0.13b \) become smaller in the order mentioned.

5. The bending fatigue-limit-loads of the internal gears of \( t_w = 2m \) and \( 1.5m \), \( b_w = 0.13b \) are about 5%, 19% lower respectively than in the case of \( t_w = 8m \), \( b_w = 0m \).

6. In the thin-rimmed internal gears of \( t_w = 1.5m \), \( 2m \), \( b_w = 0m \), a tooth does not break, but a rim part breaks. In the case of \( t_w = 1.5m \), \( 2m \), \( b_w = 0.13b \), a tooth breaks.

7. It might be considered reasonable to take \( t_w = 2m \) as a minimum rim thickness for such internal gears of \( b_w \geq 0.13b \) as used in this experiment.

8. The estimation by the calculating formula of total bending strength of internal gear proposed by ISO might be on the critical side.

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References

5. ISO/DP6336/III.