The Behaviour of Fasteners under External Loading*

(Force Ratio of Threaded Connections)

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In the design of fasteners subjected to an external service load, it is very important to know the relationship between bolt tensile load and service load and to assure a good tightening. However, there are many difficult points in the analysis of fasteners, because the shape of threaded parts of bolts and nuts is very complex and a threaded fastener is, from a view point of the theory of elasticity, equivalent to a contact problem of many elastic bodies. This paper introduces a finite element technique to numerically analyze the threaded fastener subjected to external loads. As concrete examples of standard threaded fasteners, the results for the force ratios of the through bolt, the tap bolt, the stud and the T-type flange are numerically obtained by using an axi-symmetric and a three-dimensional model.

Key Words : Fixing Element, FEM, Force Ratio, Threaded Connection, Through Bolt, Tap Bolt, Stud Bolt.

1. Introduction

This paper is concerned with the analysis of threaded connections subjected to an external load, regarding them as a problem of contact by using the axi-symmetric and the three dimensional finite element method. Threaded connections are the most widely used machine elements to fasten various parts of an assembly together. Although the mechanism of threaded connection is simple and easy to understand, the connection is one of the weakest portions where a stress concentration occurs. Sometimes, the failure or the self-loosening of only one bolt becomes the cause of a serious accident.

Although there have been many studies on the threaded connection subjected to an external force, adequate design standards for severe operation of machine have not yet been established. Sometimes the designers must depend on the perceptions acquired from their experiences. These trends result from the fact that the analytical and the experimental study of threaded connections are very difficult because of complicated geometry of threaded parts.

It is very important to investigate the effects of various types of external load upon the strength of threaded connection. So far there have been many studies on threaded connections subjected to an external load by using the theory of elasticity[1],[2]. Although the fundamental method based on the theory of elasticity is

very easy to use, the range of its applications is very narrow, and it is very difficult to simulate the actual behaviour of a threaded connection. The method based on the theory of elasticity enables us, to some extent, to analyze the threaded connection in consideration of its shape and the boundary condition. However, the geometry of general type of threaded connection is so complicated that the use of a large scale computer becomes inevitable to develop a mathematical model, and this reduces the theory of elasticity to a simple numerical calculation technique. Contrarily, the advantage of the use of the finite element and the boundary element methods is that solutions of complicated structures such as bolt-nut joints can be easily obtained. The authors have been studying the application of the finite element method to the threaded connections subjected to an external load [4],[5],[6]. These methods permit a speedy and accurate analysis of the change in the axial force and the relationship between contact surface and external load.

In this paper, applying these methods to the concrete problem of a threaded connection subjected to an external service load, the force ratio, the distribution of contact pressures and the limiting load for opening of the joint are numerically analyzed.

2. Analytical Method

In this paper, by applying the unit partitioning method, the threaded connections as a contact problem were analyzed. It is unavoidable to use methods of iteration and convergence in order to investigate variations in the contact area which are associated with the external load variation. As the involved computational time in an iteration process becomes very long, it is inevitable to formulate the

* Received 19th March, 1984.

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system equations on the basis of a reduced matrix which are necessary for the calculation at least. The reduced matrix which is at least necessary for the calculation is constructed of the components of the nodes on the contact surfaces and the point at which is applied an external service load.

An axial tension is caused by applying either a forced displacement to the cross section of the bolt or setting an interference between the interfaces of the fastened plates. The reduced matrix must also contain the components of the nodes at these places. The process to introduce a reduced matrix has been already discussed in the previous paper[4]. In this paper, this method is improved so that more than two substructures can hold some nodes in common and threaded connections can be adequately modeled.

A discretization is carried out by selecting any favourite type of element from among linear, quadratic and cubic ones. The nodal forces in the finite element method do not represent the actual values of the force acting on the node because they are discrete equivalent forces. So it is necessary to calculate the correct value of the contact pressure before finding the correct contact area. In the case of higher order elements, the direction of the nodal force does not always coincide with that of the pressure. It is preferable to transform the nodal forces acting on the contact area into tractions.

Transformation of traction into nodal force can be performed in quite the same manner as the evaluation of the mass matrix. The values of displacement u and traction p on the boundary are expressed in terms of nodal values.

$$\left. \begin{aligned} u &= \sum_{k=1}^n \phi_k u_k = [\phi] \{u\} \\ p &= \sum_{k=1}^n \phi_k p_k = [\phi] \{p\} \end{aligned} \right\} \dots\dots\dots (1)$$

where n is the number of nodes on one side of element, ϕ_k is an interpolation function. Figure 2 shows the interpolation functions in the case of quadratic element.

Putting the nodal forces as $\{f\}$ and the virtual displacement as u^* , the work done by the nodal force $\{u^*\} \{f\}$ becomes equal to the one done by the traction $\int u^* p ds = \{u^*\} \left(\int [\phi]^T [\phi] ds \right) \{p\}$. From this figure we get the relationship between the nodal force and

Fig.1 Interpolation functions
(Quadratic element)

traction as

$$\{f\} = [\Phi] \{p\} = \left(\int [\phi]^T [\phi] ds \right) \{p\} \dots\dots\dots (2)$$

Applying Eq(2) to the contact problem the nodal forces can be transformed into tractions, and the calculation can be performed by a procedure similar to that used in the boundary element method. The search for contact area is made by cancelling the contact when a tensile pressure emerges between mating surfaces and by coupling the overlapping node pairs whose relative displacements are less than the interference setting. For the calculation of the change in the axial tension and the contact pressure in the axi-symmetric model of threaded connection subjected to an external load, we can rely on a large scale computer and employ not only the iterative procedure to find the contact area in a single step but also the incremental procedure to follow the loading history until the limiting load for opening of the joint is reached. In the previous paper, the threaded connection under an axi-symmetric loading was analyzed with the model fixing one side of virtual cross section of the bolt axis and causing a forced displacement of another that is equal to the axial movement of the nut following its rotation. This method is applicable only to the model in which the axial tension is uniformly distributed over the cross section of the bolt. However, in the case where the bolt is very short such as stud and tap bolt, this method is not so convenient to use. So, in this paper the authors have developed a general purpose program applicable to various types of threaded connections.

Figure 2 shows a schematical relation between the external load acting on the connection and the axial tension. The loads on points C and D act simultaneously in opposite directions with same magnitude. Point D is axially fixed to prevent the

Fig.2 Threaded connection subjected
to an external load.

rigid body displacement, so the load on point D appears in a reactive form.

The reduced matrix used in this study consists of components of the nodal point subjected to an external load and of the contact surfaces. The threaded connection clamped with a nut can be easily simulated by introducing an interference setting between the mating surfaces that is equivalent to the axial movement of the nut following its rotation. At first, analysis is made under the assumed amount of interference setting in the case with no external load. The axial tension obtained is compared with the input data on the initial tension, and the amount of the interference setting is so scaled as to give a correct value of initial tension. The axial tension is calculated as the sum of axial components of the nodal forces on the interfaces. The size of the load increment is determined after the calculation of limiting load for opening of the joint.

The nodal forces at the innermost points on the mating surfaces become zero when all the interfaces of the fastened plate completely separate. The displacement and the force components at points A, B and C are denoted as δ_A , δ_B and δ_C , and F_A , F_B and F_C respectively. When the interfaces of a fastened plate completely separate, the condition of $F_A=F_B$ gives the reasonable relationship between nodal forces and displacements at points A, B and C.

$$\begin{Bmatrix} 0 \\ 0 \\ F_C \end{Bmatrix} = \begin{bmatrix} K_{AA} & K_{AB} & K_{AC} \\ K_{BA} & K_{BB} & K_{BC} \\ K_{CA} & K_{CB} & K_{CC} \end{bmatrix} \begin{Bmatrix} \delta_A \\ \delta_B \\ \delta_C \end{Bmatrix} \quad \dots\dots\dots(3)$$

If x is the amount of interference setting, δ_B is expressed as $\delta_A - x$. Eq(3)

becomes

$$\begin{Bmatrix} 0 \\ 0 \\ F_C \end{Bmatrix} + \begin{Bmatrix} K_{AB} \\ K_{BB} \\ K_{CB} \end{Bmatrix} X = \begin{bmatrix} K_{AA} & K_{AB} & K_{AC} \\ K_{BA} & K_{BB} & K_{BC} \\ K_{CA} & K_{CB} & K_{CC} \end{bmatrix} \begin{Bmatrix} \delta_A \\ \delta_A \\ \delta_C \end{Bmatrix} \quad \dots\dots\dots(4)$$

We may determine the unknown variables by solving Eq(4) as follows.

$$\begin{Bmatrix} \delta_A \\ \delta_C \end{Bmatrix} = [K_{AA} + K_{BB} \quad K_{AC}]^{-1} \begin{Bmatrix} K_{AB} \\ K_{CB} \end{Bmatrix} X \quad \dots\dots\dots(5)$$

$$F_C = [K_{CA} + K_{CB} \quad K_{CC}] \begin{Bmatrix} \delta_A \\ \delta_C \end{Bmatrix} - K_{CB} X \quad \dots\dots\dots(6)$$

F_C is the limiting load for opening of the fastened plates. The increment external load is determined from F_C , and in each incremental step the relation between the axial tension and the external load is calculated by using an iteration technique. All these procedures are executed automatically in sequence by use of a computer.

3. Symmetric Analysis of Threaded Connection

One of the simplest way to analyze a threaded connection is to use an axisymmetric model. The threaded connections treated in this paper are those clamped with through bolt, tap bolt, stud bolt and hexagon socket head cap bolt. Figure 3 shows the finite element models divided into quadratic elements. Threaded size of these models is M12, and the size of fastened plates and the locations of external loads are selected the same if possible to compare the force ratios under same circumstances.

Figure 4 shows the relation between the axial tension and the external load in

Fig.3 Division of the threaded connection into finite element meshes.

the case without friction. In these analyses, the load acts on point C_1 and the amount of load increment is selected as one-seventh the limiting load. Generally, when the external load remains small, the axial tension slowly increases. When the external load exceeds the value of limiting load and the joint completely separates, the axial tension shows a very rapid increase for the same increment of the external load. In the case of through bolt, when the

Fig.4 Relation between axial tension and external load.

Fig.5 Force ratio

Fig.6 Relation between axial tension and external load (Through bolt)

external load remains small, the axial tension slightly decreases because the inner side of nut bearing surface moves downward in the direction of the axis as the result of leverage. This phenomenon is a typical example obtained by finite element analyses that has never occurred in the traditional techniques.

Comparing the limiting loads obtained from the analyses, it is found that the threaded connection clamped with tap bolt is less liable to open (8976 N) than others, followed by through bolt (8551 N), stud (8367 N) and hexagon socket head cap bolt (8075 N) in this order. This comes from the fact that the hexagon socket head cap bolt has smaller area of bearing surface than others, and the stud has two threaded parts.

In the design of threaded connection, the value of the force ratio - or the loading factor - is controversial. The force ratio is the ratio of the axial tension increased to the external load increase. If the force ratio is small, the effect of the external load upon the axial tension is small and this gives a substantial margin of safety against external loading. It is evident from Fig.4 that the force ratio is not constant, but variable in the history of loading according to the

Fig.7 Force ratio (Through bolt)

Fig.8 Contact pressure distribution (Through bolt)

magnitude of external load. In this study the force ratio is defined as the ratio of the total amount of increase in axial tension to the external load.

Figure 5 shows the ratios obtained from these analyses. It is found from this figure that the axial tension does not show a remarkable increase with the external load. The change in force ratio obtained in the analysis of hexagon socket head cap bolt is smaller than that in the other three cases. This comes from the fact that the

Total No. of elements : 46
Total No. of nodes : 343
o : Service load point

Fig.9 Division of T-flange into iso-parametric elements.

(a) Effect of bolt hole

(b) Effect of flange thickness

Fig.10 Relation between axial tension and external service load.

(a) Effect of bolt hole

(b) Effect of flange thickness

Fig.11 Force ratio (T-flange).

bearing surface of hexagon socket head cap bolt is small and the behaviour of connection is well simulated by the simple model based on the strength of materials.

The effects of the position of load point and the size of the fastened plate upon the change in the axial tension were investigated in the case of through bolt. The calculations were performed in the cases where the load points are point C₁, C₂ and C₃, and the radii of the fastened plates were 13.0 and 18.0 mm. Figure 6 shows the relation between the external load and the axial tension. Figure 7 shows the relation between the load and the force ratio. From Figs.6 and 7, it is found that the axial tensions under loading at point C₁ and C₂ whose distances from the bolt axis are equal show quite the same behaviour. The magnitude of the force produced by leverage is governed by the distance of the load point from the perimeter of the bearing surface that acts as a supporting point.

The force ratio for point C₂ is greater than ones for points C₁ and C₃ until the load increases to 5 kN. When the load exceeds this range, the force ratios for points C₁ and C₃ become greater than that for C₂. If the locations of load are same, the change in the radius of the fastened plate does not show any great effect upon the axial tension. Figure 8 shows the relationship between contact surfaces and external load. Even if no loads act on the

Fig.12 Contact pressure distribution.

fastened plate, all the interfaces of the fastened plate do not keep in contact. When load acts, high contact pressure takes place at the inner side of the interfaces.

4. T-flange subjected to Axial Load.

In the preceding paragraph the force ratio is analyzed by the finite element method assuming the geometry and the loading are axi-symmetrical. However, the geometry of general type of threaded connection and the loading are non-symmetrical and a three dimensional analysis is required. In this chapter, T-flange coupling clamped with two bolts is analyzed to find out the relation between axial tension and external service load that acts uniformly on the top.

Figure 9 shows one eighth of T-flange coupling modeled by iso-parametric element considering its symmetry. The boundary conditions used in this analysis are such that the cross section of the bolt axis is given a forced displacement and the external load acts on the top of the leg of flange. The load increment is chosen as 0.5 kN, and the total load is 12.0 kN. Six types of flanges with thicknesses 12.0, 21.0 and 30.0 mm and the distance of bolt hole $2C$ of 70.0 and 55.0 mm are analyzed to compare the force ratios.

Figure 10 shows the relation between the axial tension and the external service load. Consider here the effect of the distance between two holes shown in Fig. 10-(b). The axial tension linearly increases in the case of $C = 27.5$ mm, but in the case of $C = 35$ mm the axial tension shows an increasing rate with an increased external load. Figure 10-(b) show the effect of the flange thickness. The thinner the flange, the greater the increasing rate of the axial tension. The effect of h upon the axial tension is greater than that of C . Figure 11 shows the relation between the force ratio and the external load. It is clear from this figure that the greater the distance between two holes is, or the thinner the flange is, the greater the force ratio is. In the case where h is small, the axial tension slightly decreases and the force ratio becomes negative when the external load remains small.

When the external load acts in the axial direction on the top of the leg of the T-flange, the joint begins to separate on each side of center line. Figure 12 shows the successive stages of separation taking an example of $h = 30$ and $c = 27.5$ mm. Contact limit is the line of contact stress

Fig.13 Division of T-flange into iso-parametric elements.

Tab.1 Opening load.

C	Theory of elasticity	FEM	
		Start of separation	Separation along symmetric line
27.5	6.0 kN	1.5 kN	6.0 kN
35.0	1.8 kN	0.0 kN	2.5 kN

of 0 MPa. When the external load does not act, the contact occurs in whole interface. If the external load acts, the contact surface moves farther from the location of the load with an increase in the load.

Table 1 shows the opening load — a load at which separation occurs — obtained by the finite element analysis by comparing them with ones obtained by the theory of elasticity. It is found from these results that a three dimensional analysis is required for an investigation of the separating process. There are two types of opening loads. One is the load at which separation occurs at one point on the symmetric line $\bar{o}\bar{o}'$. The other is the load at which separation occurs at all points lying on the symmetric line $\bar{o}\bar{o}'$. The opening load obtained by the theory of elasticity corresponds to the load obtained by the finite element method at which the whole of $\bar{o}\bar{o}'$ separates. Although the axial tension is assumed to increase linearly in the analysis using the theory of elasticity, the actual behaviour of the axial tension

Fig.14 Relation between axial tension and bending moment.

slightly differs from a simple linear relationship because of the separation.

5. T-flange subjected to Bending Moment.

It is clear case that the threaded connection is subjected to an axi-symmetric load. Actually more various types of loads like a transverse load or a bending moment act on a threaded connection. This chapter shows the analysis of a flange coupling subjected to a bending moment. The symmetry can be used and one fourth of T-flange is modeled by iso-parametric elements as shown in Fig.13. Even if the initial tensions in both bolts are equal, effects of bending moment on bolts A and B are different.

The bending moment is assumed to produce the axial stress in the leg varying linearly from the neutral axis, i.e. satisfying the basic moment-stress relation $\sigma = My/I$. Positive (tensile) and negative stresses are produced in bolts A and B respectively. I and y are the second moment of the cross sectional area and the distance from the neutral axis of the leg respectively.

Figures 14-(a) and (b) show the relations between the bending moment and the axial tension in the cases of $h=21$ and $h=30$ mm respectively. The force ratios expressed in the form proposed by Sawa[7] are shown in Fig.15. The decrement of the axial tension in bolt B is less than the increment of that

Fig.15 Force ratio (bending).

Fig.16 Contact pressure distribution (bending)

in bolt A, and is linearly related to the bending moment. In the case with $h=21$ mm it is found that if the distance between the two bolts is short, the change in the axial tension becomes large. The difference between the analyses obtained by the finite element method and the theory of elasticity[7] seems to arise from the models assumed in analyses. Figure 16 shows the contact area caused by the bending moment in the case with $c=35$ mm and $h=21$ mm. High pressure and large contact area occur in the vicinity of bolt B in comparison with bolt A.

6. Conclusions

The results obtained may be summarized as follows:

1. This study presents a new simple method for the analysis of the force ratio by using the finite element technique. The force ratio obtained by the finite element method differs from that by the traditional method.

2. This method gives the finite element analyses of threaded connections clamped with through bolt, tap bolt, stud and hexagon socket head cap bolt using more exact models of real objects.
3. A speedy and accurate analysis of the threaded connections is possible by judging the contact condition by tractions. This technique was extended to the three dimensional analysis of threaded connections by making the calculation procedure of contact area in two dimensions.
4. In the case with same thickness of fastened plates, it is found that hexagon socket head cap bolt is the hardest to separate, followed by stud, through bolt and tap bolt in this order.
5. The axial tension is not linearly related to the external load. The force ratio is not constant, and its value changes according to the amount of external load. In some cases, the value of the force ratio becomes negative.
6. In the case of T-flange, the effect of flange thickness upon the force ratio is greater than that of the location of bolt hole.
7. It is found from the analysis of T-flange that when the external load remains small, the smaller the force ratio, the smaller is the opening load. The flange coupling whose force ratio is small is safe from a view point of strength. However, such flange is inadequate to seal the mating surfaces perfectly because of a lowering of opening load.

8. In the case of T-flange subjected to bending moment the force ratio of the bolt on tensile side is so great that the design of threaded connection considering the effects of bending moment becomes very important.

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