Study on Mechanical Interface between Head and Media in Flexible Disk Drive
(1st Report, Static Characteristics)

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This paper presents an analytical method for mechanical interface between head and media in flexible disk drive. Static Green function in disk deflection is first derived by using Fourier series approximation in circumferential direction and finite element method in radial direction. Deflection distribution of a stationary disk including the base plate and the penetrating spherical head is next obtained by developing a novel numerical method named the virtual variable stiffness spring (VVSs) iteration method for solving the three dimensional contact problem. It is found from experiments that the analytical results of contact force, deflection distribution around the spherical head and contact position are in good agreement with experimental ones when disk deflection is as small as the disk thickness.

Key words: Business Equipment, Date Processor, Elasticity, Flexible Disk, Contact Problem, Head to Disk Interface

1. Introduction

Floppy disk storage is playing a more and more important role in data processing systems, particularly in small and personal computer systems. Advanced technology for higher density recording has been continuously utilized to provide a larger capacity disk without an increase of disk size or a smaller size disk and drive without a decrease of storage capacity. A new product with higher recording density has been realized by optimizing physical and dimensional parameters of head and medium for better contact condition as well as by developing higher density recording medium and head. There have been many papers on the mechanical interface between head and flexible disk (1)-(7). Some of them concern air bearing characteristics (2), (5)-(7), and others concern the fundamental dynamic behavior of a rotating floppy disk produced by a point force or mass spring system (3), (4). However, these studies scarcely give useful data for designing head contour and estimating contact pressure in the development of enhanced contact recording floppy disk drive.

The objective of this study is to develop an analytical method of contact pressure and disk deflection in an actual contact recording floppy disk drive. For this purpose, the static Green's function of a non-rotating flexible disk is first derived using Fourier series expression in the circumferential direction and finite element numerical method in the radial direction. Contact problem between flexible disk and base plate including a point contact head is next analyzed by a new analytical method which has a potential to solve the actual head and flexible medium contact problem. The validity and limitation of Green's function is examined in detail by experiment.

2. Nomenclature

\[ a = \text{inner radius of disk, mm} \]
\[ b = \text{penetrating depth of point contact head from clamping height, mm} \]
\[ D = \text{outer radius of disk, mm} \]
\[ E = \text{Young's modulus, Pa} \]
\[ F_t = \text{concentrated force, N} \]
\[ G = \text{disk deflection produced by a concentrated unit force, mm/N} \]
\[ g = \text{acceleration of gravity, mm/s}^2 \]
\[ H = \text{thickness of disk, mm} \]
\[ k = \text{length of finite element, mm} \]
\[ k_h = \text{clamping height at disk inner radius, mm} \]
\[ K_w = \text{stiffness of virtual spring, N/mm} \]
\[ \theta, \theta_0, \theta = \text{polar coordinates} \]
\[ w = \text{disk deflection, mm} \]
\[ w_t = \text{disk deflection produced by gravity force, mm} \]
\[ z = \text{deflection of virtual spring from rigid surface, mm} \]
\[ \rho = \text{density of disk, kg/mm}^3 \]
\[ v = \text{Poisson ratio} \]

3. Analysis
3.1 Green's function of non-rotating disk deflection

Figure 1 shows an analytical model of non-rotating flexible disk which is clamped at \( r = a \) and free at \( r = b \), where unit force is applied at the position where \( r = \xi \) and \( \delta = 0 \). Under the assumption of small linear deflection, disk deflection \( G \) is governed by the differential equation

\[
D \nabla^4 G = \frac{1}{2} \delta (r - \xi) \delta \theta
\]

where \( \delta (r) \) is Dirac delta function, and

\[
\nabla = \left( \frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \phi} \right)
\]

The boundary conditions of the disk are given by

\[
G_{r=0}, \frac{\partial G}{\partial r}_{r=a} = 0
\]

\[
\frac{\partial^2 G}{\partial r^2} + \frac{1}{r} \frac{\partial G}{\partial r} + \frac{1}{r} \frac{\partial^2 G}{\partial \theta^2} = 0
\]

\[
\frac{\partial^2 G}{\partial r^2} + \frac{1}{r} \frac{\partial G}{\partial r} = \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2}
\]

\[\frac{1}{r^2} \frac{\partial^2 G}{\partial \phi^2} - \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} = 0 \]

The solution of Eq. (1) can be expressed by Fourier cosine series which is symmetrical with respect to loading position, as follows:

\[ G = G_r + \sum G_c \cos \theta \]

Substituting Eq. (4) into Eq. (1) and using orthogonality of Fourier series, we obtain an ordinary differential equation for disk deflection function \( G \) with respect to the radial coordinate and as follows:

\[
D \frac{\partial^4 G}{\partial r^4} + \frac{\partial^2 G}{\partial r^2} + \frac{1}{r} \frac{\partial G}{\partial r} + \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} = 0
\]

At the same time, substitution of Eq. (4) into Eq. (3) gives a set of boundary conditions for \( G \) of the forms,

\[
G_{r=0}, \frac{\partial G}{\partial r}_{r=a} = 0
\]

The differential equation (5) can be solved in a closed form, but the finite element numerical analysis is employed here considering further extension of the analysis to the dynamic contact problem of a rotating flexible disk.

Referring to a finite element model of a one dimensional thin beam as shown in Fig. 2, beam deflection \( G \) specified by local radial coordinate \( r' \) can be approximated by the linear combination of the boundary deflections \( G_0 \), \( G_1 \) and inclinations \( \phi_0 \), \( \phi_1 \), in the form

\[
G(r') = G_0 + N_1 G_1 + N_2 \phi_0 + N_3 G_1 + N_4 \phi_1 \]

where the trial interpolation functions with respect to \( r' \), \( N_r \) to \( N_r \) are given by

\[
N_1 = \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_2 = \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_3 = \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_4 = \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

Fig. 1 Analytical model of flexible disk

Fig. 2 Finite element model in radial direction

\[
N_1 = \left( r' \right) \left( r' \right) \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_2 = \left( r' \right) \left( r' \right) \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_3 = \left( r' \right) \left( r' \right) \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

\[
N_4 = \left( r' \right) \left( r' \right) \frac{1}{2} \left( r' \right) \frac{1}{2} \left( r' \right)
\]

Putting

\[
N_1 = D_{12} \phi_0 + D_{13} \phi_1 \]

\[
N_2 = D_{22} \phi_0 + D_{23} \phi_1 \]

\[
N_3 = D_{32} \phi_0 + D_{33} \phi_1 \]

in Eq. (5) and using the trial function (8), Gelerkin's formulation of the system gives a set of equations, which determine the status variables \( G_r \) and \( \phi_k \), as follows:

\[
\frac{\partial^2 G}{\partial r^2} + \frac{\partial G}{\partial r} = \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2}
\]

\[
\frac{1}{r^2} \frac{\partial^2 G}{\partial \phi^2} - \frac{1}{r^2} \frac{\partial^2 G}{\partial \theta^2} = 0 \]

where the superscript (e) means the local coordinate expression in the element of index number e and

\[
[N^{eq}] = [N_1, N_2, N_3, N_4] \]

Applying the integration partially to Eq. (11), we can get the following set of equations for each element.

\[
\frac{\partial G}{\partial r} \left. \bigg|_{r=a} \right. = \frac{\partial G}{\partial r} \left. \bigg|_{r=b} \right.
\]

\[
\int_{N^{eq}} \frac{\partial G}{\partial r} \frac{\partial G}{\partial r} d \theta d \phi
\]

\[
\int_{N^{eq}} \frac{\partial G}{\partial r} \frac{\partial G}{\partial r} d \theta d \phi
\]

\[
\int_{N^{eq}} \frac{\partial G}{\partial r} \frac{\partial G}{\partial r} d \theta d \phi
\]

\[
\int_{N^{eq}} \frac{\partial G}{\partial r} \frac{\partial G}{\partial r} d \theta d \phi
\]
\[ + \int \left[ N^{(m)} \right] R^{(m)} \delta^{(m)} \cdot \delta^{(m)} - \int \left[ N^{(m)} \right] S^{(m)} \delta^{(m)} \cdot \delta^{(m)} + \int \left[ N^{(m)} \right] T^{(m)} \delta^{(m)} \cdot \delta^{(m)} \]  
\[ \delta(x+(e-1)h+r) \delta(x+(e-1)h+r) \]  
\[ \int \left[ N^{(m)} \right] \delta^{(m)} \cdot \delta^{(m)} \]  
\[ \int \left[ N^{(m)} \right] S^{(m)} \delta^{(m)} \cdot \delta^{(m)} \]  
\[ \int \left[ N^{(m)} \right] T^{(m)} \delta^{(m)} \cdot \delta^{(m)} \]  
\[ (e=1) \]  

Substituting Eqs. (7) and (14) into Eq. (13) and adding each two adjacent equations, we have, including the boundary conditions (6),

\[ \begin{bmatrix} 1 & 0 & B_{12} & 0 & \ldots & B_{12} & \ldots & 0 \\ 0 & 1 & B_{13} & 0 & \ldots & B_{13} & \ldots & 0 \\ 0 & 0 & B_{14} & 0 & \ldots & B_{14} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & B_{12} & 0 & \ldots & B_{12} & \ldots & 0 \\ \end{bmatrix} \begin{bmatrix} B_{12} \\ B_{13} \\ B_{14} \\ \vdots \\ B_{12} \end{bmatrix} + \frac{D}{h} \begin{bmatrix} \frac{D}{h} (G_{12} + \ldots + G_{12}) \\ \frac{D}{h} (G_{13} + \ldots + G_{13}) \\ \frac{D}{h} (G_{14} + \ldots + G_{14}) \\ \vdots \\ \frac{D}{h} (G_{12} + \ldots + G_{12}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]  

where \( m \) is the index number of the element where unit force is applied. The values of \( G \) at all nodal points can be obtained solving Eq. (15). G1 at any radial position can be obtained from Eq. (7) and disk deflection \( C(r, \alpha) \) can be further calculated from Eq. (4).

Figure 3 shows a typical calculated result of Green's function where a concentrated force is applied at \( \xi=70 \) mm.

3.2 Deflection of non-rotating disk produced by a point contact head including base plate

Using Green's function derived above, deflection of non-rotating disk produced by point contact spherical head is analyzed including base plate and gravity force. Basic equation for the deflection due to gravitational force is given by

\[ DV^{(m)} w = -\rho g H \]  

(a) Radial distribution

\[ G m/N \]

(b) Circumferential distribution

\[ G m/N \]

Fig. 3 Calculated result of Green's function

The boundary conditions for \( W \) are obtained from Eq. (3) by neglecting the terms differentiated by \( \bar{e} \). Therefore, \( u_0 \) can be calculated in the same manner as described above.

Next consider the analytical method of disk deflection produced by a point contact spherical head including contact condition between disk and base plate. For this purpose, a novel numerical method named here the virtual variable stiffness spring (VWSS) iteration method is proposed.

VWSS iteration method is characterized by assuming a set of virtual springs which are individually connected to the disk at each mesh point and whose natural length positions are just on the base plate surface, as shown in Fig 4(a) and (b). If we denote the stiffness,

Fig. 4 Analytical model of disk deflection including contact to base plate
deflection and restoring force of the spring by $K_d$, $x_d$ and $f_d$, respectively, the disk deflection $w_{kl}$ at the point $(k,l)$ is given by

$$w_{kl} = \frac{K_d}{a^2} G_{10} f_{kl} + G_{kl} F_k + x_{kl}$$

where $G_{kl}$ is a discrete Green's function, i.e., an influence coefficient from the point $(i,j)$ to the point $(k,l)$. Spring force $f_d$ is given by

$$f_d = -K_d x_d$$

The relationship between clamping height $h_c$ and $x_d$ is

$$w_0 + h_c = x_d$$

Substituting Eqs. (18) and (19) into Eq. (7), we have

$$w_{kl} = \frac{K_d}{a^2} \left[ G_{kl} + \delta_{kl} \delta_{ij} \right] x_d$$

Considering that Green's function and disk deflection are symmetrical in the circumferential direction with respect to head loading point and that Green's function depends only on the distance in the circumferential direction, we can put

$$G_{kl} = G_{kl0}$$

where the head loading point is identified by subscript number $j=l$. From the symmetrical property and Eq. (21), we can have a set of linear algebraic equations of the form

$$\sum_{kl} \left\{ \left[ G_{kl} + \delta_{kl} \delta_{ij} \right] F_k + x_{kl} \right\}$$

If we want to calculate the head load $F_k$ for a given penetrating depth $R$, we have only to interchange these two terms after putting $R$ into $x_{kl}$.

In reality, however, the stiffness of the individual spring is infinite in the contact region and zero in the non-contact region. Then, after solving first Eq. (22) assuming an appropriate initial homogeneous stiffness, we carry out an iterative

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Fig. 5 Calculated results of contact region between disk and base plate

(a) $\xi=90\text{ mm}$
(b) $\xi=70\text{ mm}$
(c) $\xi=50\text{ mm}$

Fig. 6 Experimental apparatus
calculation, by putting, in each step,

\[ K_u = 0.1K_u \left( x_u > 0 \right) \]
\[ K_u = 10K_u \left( x_u < 0 \right) \]  

the convergent solution is defined to be obtainable when the successive deflection solutions satisfy the following criteria

\[ \frac{\Delta \left( w_i - w_{i+1} \right) + 2 \Delta w_{i+1} - w_{i+1}}{2 \Delta w_i + \Delta w_{i+1}} < \varepsilon \]  

where the superscript \((n)\) means the iteration number and \( \varepsilon \) is chosen as a small value.

Figure 5 shows some calculated results of the disk contact region using the VVSS iteration method. In this case a point contact head penetrates into the disk by \( B_t = 100 \mu m \) from the clamping height. When the head position is near the outer edge of the disk, there appears a contact region between head position and inner clamped position as seen in Fig. 5(a). Calculated results of disk deflection around head position and head load or disk reacting force are shown later in comparison with the experimental data.

4. Experimental Study

4.1 Experimental apparatus and methods

In order to examine the validity and limitation of the calculated results under the assumption of linear disk deflection, an experimental apparatus for measurement of disk reacting force and disk deflection

**Fig.7** Comparison between experimental and theoretical disk reacting forces

\( a = 25 \text{ mm}, \ b = 100 \text{ mm}, \ H = 78 \mu m, \ E = 4.9 \text{ GPa}, \ \rho = 1.3 \times 10^4 \text{ kg/mm}^3 \)
in the vicinity of point contact spherical head was made, as shown schematically in Fig. 6 (a) and (b). Figure 6 (a) shows the disk reacting force measurement system. Penetrating height $B_z$ and disk reacting force can be measured by the inductive gap detector and micrometer head by using calibration curve between load applied to the pin and the cantilever deflection. Figure 6 (b) shows the spacing measuring system between the spherical head and disk by interference fringe pattern. The radius of curvature of a glass spherical head is 20 mm and thus, the head contacts the disk at one point. Spherical head is supported by fine positioning stage in X, Y, Z directions. A commercially available 8 inch floppy disk was used for measurement. Young’s modulus, which is calculated from natural vibration test for several rectangular test specimens considering an additional inertia effect of the surrounding air, is 4.9 GPa.

4.2 Comparison between experiment and theory

Figures 7 (a) to (i) show the theoretical and experimental disk reacting forces versus head loading position, taking the clamping height and penetrating depth as the parameters. Experimental data measured at every 45 degrees circumferential point of the disk are plotted for each head position. It is seen from these figures that the theoretical curves agree well with the average values of the disk reacting force, if the amount of the disk deflection is within the same order of the disk thickness. However, the disk reacting force becomes larger than the calculated one, as the disk deflection increases over the disk thickness. Large variation of disk reacting force in the circumferential direction is considered to be due to the anisotropic deflection characteristics of the floppy disk.

Figures 8 (a) and (b) compare the experimental and theoretical spacing between disk and spherical head of 20 mm radius, i.e., the relative disk deflection from the head surface. The dotted line shows an experimental interference fringe pattern of 0.24 $\mu$m spacing difference. It is seen that the theoretical disk deflection around the point contact head agrees well with the experimental one, even if the clamping height and penetrating depth are several times the disk thickness.

Figures 9 (a) and (b) show the shift of disk contact point from the head apex versus head protruding position. Although the variation of measured data at every 45 degrees circumferential disk position is large, its average values agree well with the theoretical results.

5. Conclusions

The results of analytical and experimental investigations on the deflection characteristics of a non-rotating flexible disk are summarized as follows: (1) An analytic-numerical method, which combines Fourier series approximation and the finite element method, was proposed to obtain economically an accurate Green's function of a flexible disk deflection, which is useful for the analysis of mechanical interface problems between a

![Diagram](image-url)

Fig. 8 Comparison between experimental and theoretical spacing

(a) $h_e=140\mu m$, $B_z=0\mu m$

(b) $h_e=140\mu m$, $B_z=100\mu m$

![Diagram](image-url)

Fig. 9 Shift of contact point from apex

(a) $a=25\,\text{mm}, b=100\,\text{mm}, H=78\,\text{um}, E=4.9\,\text{GPa}$

(b) $a=25\,\text{mm}, b=100\,\text{mm}, H=78\,\text{um}, E=4.9\,\text{GPa}$
flexible disk and head.  

(2) A new numerical method named virtual variable stiffness spring (VVSs) iteration method was proposed to solve the three dimensional contact problem between a flexible disk and base plate.  

(3) Analytical results of contact force, disk deflection and contact position between the flexible disk and the spherical head are found in good agreement with the average experimental values, if the disk deflection is within about the same order of the disk thickness. Disk deflection around point contact head agrees well with the analytical one, even if the disk deflection is several times the disk thickness.  

The analytical method developed here will be extended to the analysis of the dynamic behavior of a rotating flexible disk under a point contact head or actual read/write head loading.  

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