Characteristics of the Flow around Four Circular Cylinders

Arranged in Line*

By Tamotsu IGARASHI**

An experimental investigation was carried out on the characteristics of the flow around four circular cylinders arranged in line. Concerning the behavior of the shear layers separated from the first cylinder to the downstream ones, the flow patterns were classified according to the longitudinal spacing between the axes of the cylinders and Reynolds numbers. Flow characteristics of these patterns were elucidated. Especially, the Reynolds numbers corresponding to the reattachment of the shear layers onto the second cylinder, were obtained. The flow characteristics around the downstream cylinders changed drastically in this transition region. Thereby, a bistable flow and a hysteresis phenomenon emerged.

Key words : Unsteady Flow, Four Circular Cylinders, Strouhal Number, Pressure Distributions, Flow Patterns, Reattachment, Hysteresis

1. Introduction

A flow around two circular cylinders arranged in tandem which is the most simple arrangement of tube banks in line, has been investigated by many workers for a long time. Zdravkovich[1] has reviewed a number of investigations in this field. Recently, the author carried out detailed experimental investigations on a flow around two circular cylinders with equal diameters[2, 3] and with different diameters[4] arranged in tandem. The unclear flow in the previous reports was clarified and the flow patterns were classified according to the spacing and Reynolds numbers. In a close arrangement, a unique flow pattern which was not reported at that time has been revealed.

The fluid flow and heat transfer around two circular cylinders with equal diameters and with different diameters have been published by Kostic and Oka [5] and Hiwada et al.[6,7]. The heat transfer measurements around three and four circular cylinders closely spaced have been carried out by Aiba and Yamazaki[8] and Aiba et al. [9-11], and the flow measurements around five circular cylinders by Ishigai and Nishikawa[12]. However, as to the case of more than three cylinders, the measurements of the pressure distribution around them and the Strouhal numbers behind them were carried out for a specified spacing only. In the case of more than three cylinders, the flow characteristics remain unclarified until now, hence the author investigated a flow around three cylinders arranged in line [13]. The flow patterns are divided by the behavior of a shear layer separated from the first cylinder, and the flow characteristics are clarified. Moreover the

reattachment Reynolds number, at which the shear layer reattached onto the second cylinder, was obtained and a hysteresis and a bistable flow were clarified.

The purpose of the present work is to elucidate the basic characteristics of the flow around tube banks arranged in line and to obtain the basic data for comparison with the data about two or three cylinders. The longitudinal spacing between the axes of the cylinders was restricted to within narrow limits.

Nomenclature

C_D : drag coefficient = $\int_0^\pi C_p \cos\phi d\phi$
 C_p : pressure coefficient = $(p - p_o)/0.5\rho u_o^2$
 C_p' : fluctuating pressure coefficient
 $= \Delta p / 0.5\rho u_o^2$
 d : diameter of circular cylinder
 f : vortex shedding frequency, frequency
 L : longitudinal spacing between the axes of cylinders
 p_o : static pressure at free stream
 $\bar{p}$: time-averaged pressure
 p' : fluctuating pressures
 Δp : R.M.S. of fluctuating pressure
 $= (\overline{p'^2})^{0.5}$
 Re : Reynolds number = $u_o d / \nu$
 Re_c : reattachment Reynolds number
 $\bar{S}$: Strouhal number = fd / u_o
 u_o : free stream velocity
 ν : kinematic viscosity of fluid
 ρ : density of fluid
 ϕ : circumferential angle on cylinders
 Subscripts
 1, 2, 3, 4 : first, second, third and fourth cylinders

2. Experimental Apparatus and Procedure

The configurations of four cylinders are shown in Fig. 1. The diameter of every cylinder, d , was 34 mm and the free stream velocity, u_o , ranged from 4 to 18 m/s. The Reynolds numbers were in the range $8.72 \times 10^3 \leq Re \leq 3.92 \times 10^4$. The longitudinal distance

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** Associate Professor,
The National Defense Academy, (1-10-20
Hashirimizu, Yokosuka 239, Japan).

between the centers of the four cylinders, L/d , varied from 1.0 to 2.65. Experimental apparatus and procedure were identical to those described in the previous papers [2, 4, 13] in detail. And the turbulence intensities of the free stream in a low speed wind tunnel and in a smoke tunnel were 0.55 and 0.60 % in this range of Reynolds numbers, respectively. Both the time-averaged and the R.M.S. of fluctuating pressure distributions around the cylinders were measured by a manometer and by a semi-conductor pressure transducer, respectively. The vortex shedding frequencies from the cylinders were obtained by the frequency analysis of the velocity fluctuations measured using a hot-wire anemometer. The flow around the cylinders was visualized using a smoke tunnel and the flow on the surface of the cylinders was by an oil-film method.

3. Experimental Results and Discussions

3.1 Pressure distribution and drag coefficient

The pressure distributions around the downstream cylinders at $Re=2.18 \times 10^4$ are shown in Fig. 2. The profiles of the third and the fourth cylinders are almost the same. As to the maximum value of the pressure coefficient and its position, the profiles are classified into three types according to the spacing ratios, L/d . The situation without reattachment of the shear layer, in which the C_p value has no maximum for $L/d=1.18$, belongs to the first type. The situation with reattachment of the shear layer, in which the C_p value has maximum at $\phi=40^\circ$ for $L/d=1.47$, belongs to the second type. The situation of rolling up of the shear layer in front region of the cylinder, in which the C_p value has maximum at $\phi=0^\circ$ for $L/d=2.50$, belongs to the third type. The three situations are designated as W, R and J hereinafter, respectively. While the second cylinder differs somewhat in profile of C_p from the downstream ones, the above three types roughly apply here too. The type J seems to occur at $L/d \geq 3.82$ [2], but it is beyond the scope of the present experiment ($L/d \leq 2.65$).

The variations of drag coefficients obtained from the pressure distributions of the cylinders with L/d are shown in Fig. 3. The value of the first cylinder, C_{D1} , is roughly from 0.9 to 1.0, it has a maximum for $L/d=1.32$ and then decreases gradually with an increase in L/d . The value of the second cylinder, C_{D2} , is minus and it acts as a thrust. For $L/d=1.40$ the shear layer reattaches onto the second cylinder, the value of C_{D2} increases stepwise from 0.6 to

Fig. 1 Coordinate system and symbols

-0.3. As L/d increases the value of C_{D2} decreases and has a minimum -0.4 at $L/d=1.76$, and then increases gradually. The value of the third cylinder, C_{D3} , increases

(a)

(b)

Fig. 2 Pressure distributions around the downstream cylinders

Fig. 3 Drag coefficients of four cylinders

monotonously from 0 to 0.6 with an increase in L/d . The value of the fourth cylinder, C_{D4} , is slightly larger than that of the third one and its profile is analogous to that of the second one. It makes a discontinuous change for the same spacing of $L/d=1.40$. In Fig. 3, comparison of the results on the present four cylinders and those of three ones[13] indicated with a dotted line shows their excellent agreement except the maximum values of C_{D2} for about $L/d=1.5$ and 2.5.

3.2 Reattachment Reynolds number

When the shear layer separated from the first cylinder reattaches onto the second one, the flow characteristics around

Fig. 4 Pressure distribution on the second cylinder around reattachment Reynolds numbers

Fig. 5 Reattachment Reynolds numbers

the downstream ones change drastically. As shown in Fig. 4, the reattachment appears beyond a specific Reynolds number. Such a Reynolds number is hereinafter called a reattachment Reynolds number, Re_c . In Fig. 5, the symbols $\circ$ and $\bullet$ indicate the cases accompanied and not accompanied with a reattachment of the shear layers, respectively. The reattachment Reynolds number decreases with an increase in L/d , and is given by

$$Re_c = 7.1 \times 10^4 (L/d)^{-4.0} \dots\dots\dots(1)$$

Furthermore, a broken line in Fig. 5 is the critical Reynolds number obtained by Aiba et al.[10], at which the heat transfer around the four cylinders changes abruptly. Their results are expressed by

$$Re_c = 1.14 \times 10^5 (L/d)^{-5.84} \dots\dots\dots(2)$$

Although they state that the drastic change in the flow pattern accounts for the drastic change in heat transfer at Re_c , this change can be ascribed to the reattachment of the shear layer onto the second cylinder. As can be seen from the figure, the reattachment for the four cylinders involves a slight delay than that for the three ones[13].

3.3 Flow patterns

The classification of the flow patterns around the four cylinders was accomplished by a combination of three types, W, R and J of the pressure distribution on individual cylinders. As exhibited in Fig. 6, they can be divided into the following five patterns: A(W,W,W), B(W,R,R), C(R,R,R), D(R,R,J) and E(R,J,J). Besides, an

Fig. 6 Classification of flow patterns according to pressure distribution

Fig. 7 Flow visualization by smoke tunnel and sketches of the flow patterns

intermediate pattern (W,W,R) between the patterns A and B, was found to be absent. It can be considered that immediately after the shear layer separated from the first cylinder reattaches to the fourth cylinder, the other shear layer reattaches to the third one. After all, the flow eventually turns from patterns A(W,W,W) to B(W,R,R). Furthermore, leaving a wide space between the cylinders, the pattern F(J,J,J) comes to be followed by the pattern E.

The flow visualizations using smoke tunnel, which correspond to the pressure distributions in Fig. 6 and the sketches of them are illustrated in Fig. 7. For the pattern A, the shear layers are not accompanied with any reattachments onto the downstream cylinders, and a pressure difference between the individual cylinders is not produced. For the pattern B, the two shear layers reattach periodically onto the third and the fourth cylinder alternately. An important feature of pressure distributions is that the value of C_p at the front stagnation point on the third cylinder is lower than that at the rear stagnation point on the second one. For the pattern C, the shear layers reattach periodically onto the second and third cylinder alternately, while the pressure difference between the cylinders is nearly equal. For the pattern D, the undisturbed flow on the outside of the shear layer rolls up into the front region of the fourth cylinder. The value of C_p near the front stagnation point on the fourth cylinder increases and it is larger than that of C_p at the rear stagnation point of the third one. For the pattern E, the shear layer on one side reattaches onto the second cylinder and the opposite one rolls up into the front region of the third one and forms a vortex. The pressure difference between the second and the third cylinder becomes discontinuous.

Fig. 8 Fluctuating pressure distributions

3.4 Fluctuating pressure distribution

Figure 8 shows typical distributions of the R.M.S. of fluctuating pressures, C_p' , around the cylinders. For the pattern A without reattachment, the values of C_p' on the four cylinders are very small and the distribution is flat along the circumference. For the pattern B appearing only at $L/d=1.18$ and 1.32 , the value of C_p' on the first cylinder increases and has a maximum at the separation point, the profile

being analogous to that of a circular cylinder. The value of Cp'_2 increases slightly over that for the pattern A. While the value of Cp'_3 increases remarkably and has a peak at about $\phi=40^\circ$, the value of Cp'_4 is considerably smaller than that of Cp'_3 . On the basis of the fact mentioned above, it can be considered that the pattern B is regarded as the flow around two cylinder-pairs: the first and second cylinders make one pair and the third and fourth ones make the other pair.

For the pattern C, the value of Cp'_2 increases considerably and has two peaks at the points of reattachment and separation. The values of Cp'_3 and Cp'_4 increase remarkably and have a maximum at about $\phi=40^\circ$ and 100° . The profiles are analogous to those of the jump flow for two circular cylinders[2]. For the pattern D, the values of Cp'_1 and Cp'_2 decrease, and the profiles of the downstream cylinders are almost the same as those of the pattern C, respectively. As compared with those for the pattern C, the values of Cp'_3 around the rear stagnation point and those of Cp'_4 around the front stagnation point decrease and the difference between the two values increases. For the pattern E, the values of Cp'_1 and Cp'_2 on the front faces of the cylinders increase. Moreover, the profiles are discontinuous between the rear stagnation point on the upstream cylinder and the front stagnation point on the downstream one. And the profiles of the third and the fourth cylinder are identical in the same manner as those of the pressure distributions. It is clear from the facts that the flow around the downstream cylinders is dominated by the behavior of the shear layers separated from the first cylinder.

3.5 Strouhal number

As pointed out by Ishigai et al.[12], for the pattern A, no cylinder except the last downstream cylinder has a vortex shedding. On the other hand, for the patterns B

Fig. 9 Strouhal number behind fourth cylinder

to E, the flow around all cylinders behind the reattachment one fluctuates and sheds vortices at the same frequencies. Figure 9 shows the Strouhal numbers behind the fourth cylinder. The value of the Strouhal number decreases abruptly at Re_c (patterns A to B and C) and increases gradually with an increase in L/d (patterns D and E). No particular change in the Strouhal number was found in the transition region of the reattachment patterns from B to E. Furthermore, a bistable flow was observed in the transition region of flow pattern associated with the reattachment of shear layers. An example of power spectrum of fluctuating velocity behind the fourth cylinder is shown in Fig. 10.

For $L/d=1.18$, the spectrum is characterized by two specified frequencies which occur at $Re=9.2 \times 10^3$. On the contrary, behind the third cylinder no specified frequency was found. As seen from Fig. 10, the flow is a bistable one in which two patterns occur alternately. Although each pattern is momentarily altered, its duration is relatively long. The larger specified frequency ($S=0.19$) is attributed to the pattern A, while the smaller specified frequency ($S=0.09$) observed at about $Re=9 \times 10^3$ is still unknown in itself. At $Re=2.18 \times 10^4$, the flow pattern A changes into the pattern B, and the value of the Strouhal number reduces abruptly to half. A bistable flow was observed in this region. Beyond $Re=3.7 \times 10^4$, the shear layer reattaches on the second cylinder (pattern C), while no discontinuous change in the Strouhal number occurs.

For $L/d=1.32$, at $Re=1.6 \times 10^4$ the flow becomes, bistable one in the transition region between the patterns A and B. For the pattern B, the value of Strouhal number reduces to half, and the most periodical shedding of the vortex proceeds, then a specified frequency can be observed also

Fig. 10 Power spectrum of velocity fluctuation behind the fourth cylinder for a bistable flow in transition region

behind the second cylinder. This fact corresponds to the fact that the fluctuating pressure coefficient of the first cylinder, $C_p'1$, has a maximum, as described in the preceding paragraph. Beyond $Re=2.4 \times 10^4$, the flow changes to the pattern C. For $L/d=1.47$, two Strouhal numbers are plotted in the range $1.31 \times 10^4 \leq Re \leq 1.53 \times 10^4$. This is not a bistable flow but a hysteresis as described later.

3.6 Hysteresis

A hysteresis due to an increase or a decrease in Reynolds number was observed only in transition region between the patterns A and C at $L/d=1.47$. The power spectrum of fluctuating velocity behind the fourth cylinder was examined in two cases; one in which the free stream velocity, u_0 , increases gradually and the other in which it decreases gradually. Figure 11 illustrates the variations of the power spectrum obtained. With an increase in Reynolds number, the flow has the pattern A(W, W, W) up to $Re=1.53 \times 10^4$, and changes into the pattern C(R, R, R) beyond $Re=1.60 \times 10^4$. On the other hand, with a decrease in Reynolds number, the pattern C is limited at $Re=1.31 \times 10^4$ and then changes into the pattern A at $Re=1.16 \times 10^4$. Figures 12 and 13 show the variations of the distributions of time-mean and fluctuating pressures on the downstream cylinders in the hysteresis phenomenon, respectively. These two distributions change into either the pattern A or the pattern C at the critical Reynolds number corresponding to an abrupt change in Strouhal number respectively.

No hysteresis phenomenon was observed with two cylinders arranged in tandem [2]. Besides, with three cylinders it was observed at $L/d=1.32$ and in a narrow range of the Reynolds numbers $Re=1.90 \times 10^4 \sim 1.94 \times 10^4$. With four cylinders the range of Reynolds numbers is from 1.31×10^4 to 1.53×10^4 , and its value is lower and the range expands wider than with three cylinders. Hereupon, the hysteresis for five cylinders was examined, and the result observed at $L/d=1.47$ agrees very closely with that for four ones. Namely, beyond $Re=Re_C$, the Strouhal number decreases remarkably from 0.19 to

Fig. 11 Power spectrum of velocity fluctuation behind the fourth cylinder at critical spacing

0.11. The conversion of the flow pattern A into C or the pattern C into A occurs at $Re=1.6 \times 10^4$ and 1.2×10^4 , respectively.

3.7 Classification of flow patterns

On the basis of the results mentioned above, the flow patterns classified according to L/d and Re are illustrated in Fig. 14. The fourth cylinder has exerted little effect on the flow around the upstream cylinders except in the pattern B. The reattachment Reynolds numbers, and the ranges of a bistable flow and a hysteresis phenomenon approximately agree with those for the three cylinders [13]. In the patterns A, B and C, the flow characteristic

Fig. 12 Hysteresis of pressure distributions around downstream cylinders between patterns A and C.

Fig. 13 Fluctuating pressure distributions around four cylinders at reattachment Reynolds numbers

around the fourth cylinder are almost the same as those around the third one. Meanwhile, those for the patterns D and E present the jump phenomenon.

4. Conclusions

Experimental investigations on the flow around four circular cylinders closely arranged in line were carried out in the range of subcritical Reynolds numbers. The following conclusions can be drawn from the results obtained:

(1) The behaviors of the shear layers separated from the first cylinder to the downstream ones are classified into three groups; the case accompanied with no reattachment (W), the case accompanied with a reattachment (R) and the case accompanied with a rolling up in front region of the downstream cylinders (J). The flow patterns are divided into five types according to W, R and J as follows; the patterns A(W,W,W), B(W,R,R), C(R,R,R), D(R,R,J) and E(R,J,J).

(2) These patterns are divisible by the longitudinal spacing and Reynolds numbers. The flows around individual cylinders are analogous to those around three cylinders arranged in line.

(3) Reattachment Reynolds number, at which the shear layers separated from the first cylinder reattaches onto the second one, is given by

$$Re_C = 7.1 \times 10^4 (L/d)^{-4.0}.$$

Beyond the critical Reynolds numbers, the flow characteristics around the downstream cylinders change abruptly. For example, the fluctuating pressure increases remarkably and the Strouhal numbers reduce to half. The value of Re_C is slightly larger than that for three cylinders.

(4) A bistable flow occurs in the transition region between $L/d=1.18$ and 1.47 .

(5) In the transition region between the patterns A and C for $L/d=1.47$, a hysteresis accompanied with an increase and a decrease in the Reynolds numbers, is observed in the range $1.3 \times 10^4 \leq Re \leq 1.6 \times 10^4$.

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Fig. 14 Classification of flow patterns

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