Steady Mass Transfer at the Wall of a Narrow Rectangular Stenosed Channel*

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In the present study abnormalities in the exchange of substance from fluid to wall of a narrow rectangular stenosed channel have been studied. The transfer of substance such as oxygen is considered, and is numerically estimated by calculating the nonlinear diffusion equation. Furthermore, both the local and the mean Sherwood numbers on the wall downstream of the stenosis are measured by the electrochemical technique, in which a redox system comprises the electrolyte and the test nickel electrode. Measurements are carried out in the Reynolds number range of 100-300 and a Schmidt number of about 1000. Consequently, the local Sherwood number is maximum near a reattaching point, and it asymptotically decreases at the wall downstream of this point and steeply decreases at the upstream wall. Above $Re=130$, the mean Sherwood number on the wall just behind the stenosis is lower than that away from the stenosis.

Key Words: Fluid Mechanics, Mass Transfer, Physiology, Electrochemical Method, Biological Engineering, Vascular Model, Atherosclerosis

1. Introduction

Flow through a channel with a locally constricted section is fundamentally important in engineering. The orifice flow meter is typical example of this application. The obstacle such as a constricted section in a channel affects the flow pattern. Recently, it has been well known that various abnormalities generated at vascular wall are associated with blood flow in artery. Especially, the stenosis generated at vascular wall is closely related to both the generation and the promotion of atheroma, a degeneration of the arterial wall which leads to the so-called hardening of the artery⁽¹⁾.

From a viewpoint⁽²⁾ that arterial wall is degenerated by an excessively high rate of shear, Lee & Fung⁽³⁾, Young and Tsai⁽⁴⁾, Cheng et al.⁽⁵⁾, Daly⁽⁶⁾ and Tanahashi et al.⁽⁷⁾ analysed the flow through a locally constricted channel. Back et al.⁽⁸⁾, Schneiderman et al.⁽⁹⁾ analysed the mass transfer from blood flow to boundary wall in a locally constricted channel, and indicated that the mass transfer decreases toward the downstream side of constriction⁽¹⁰⁾.

It is difficult to obtain an accurate solution of the nonlinear diffusion equation in the numerical analysis, because the Schmidt number of a liquid such as blood is very large. Furthermore, there are only a few reports⁽¹¹⁾⁽¹²⁾ about the

mass transfer that was measured by the electrochemical technique⁽¹²⁾⁽¹³⁾.

In a steady flow passing through the stenosis of the two-dimensional narrow rectangular channel as shown in Fig.1, the mass transfer on the wall downstream of the stenosis has been considered in the present study. Furthermore, the effect of the decrease in the mass transfer on the degeneration of artery has been suggested. Considering the fact that when the Schmidt number is very large the concentration boundary layer becomes very thin, the mass transfer is numerically calculated by a finite difference method. In the experiment, an electrolytic solution that is composed of potassium ferri-cyanide and potassium ferrocyanide is employed as the working fluid. The Schmidt number of this solution is very large. The mass transfer is measured by using an electrochemical technique.

2. Numerical analysis

The flow field is assumed to be a two-dimensional stenosed channel schematically shown in Fig.1. The following reference quantities are introduced: the channel width H , the mean velocity U , the difference $(C_\infty - C_w)$ between the bulk concentration C_∞ at the infinite upstream section and the concentration C_w on the wall FG downstream of the stenosis. In the calculation, a Newtonian fluid is assumed and the difference of density due to the concentration distribution is neglected. The stenosed ratio L/H is 0.475 and the stenosed thickness δ/H is 0.12. The governing equations expressed by the stream function ψ , the vorticity ω and the concentration θ are as follows:

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$$\omega = -(\partial^2 \psi / \partial x^2 + \partial^2 \psi / \partial y^2) \dots (1)$$

$$Re(\partial \psi / \partial y \cdot \partial \omega / \partial x - \partial \psi / \partial x \cdot \partial \omega / \partial y) \\ = (\partial^2 \omega / \partial x^2 + \partial^2 \omega / \partial y^2) \dots (2)$$

$$ReSc(\partial \psi / \partial y \cdot \partial \theta / \partial x - \partial \psi / \partial x \cdot \partial \theta / \partial y) \\ = (\partial^2 \theta / \partial x^2 + \partial^2 \theta / \partial y^2) \dots (3)$$

where $Re = UH/\nu$ is the Reynolds number, $Sc = \nu/D$ is the Schmidt number, ν is the kinematic viscosity and D is the diffusion coefficient. In the calculation, at first estimating both the stream function and the vorticity by Eqs. (1) and (2), subsequently the concentration can be estimated by Eq. (3).

The basic equations (1), (2) and (3) are subject to specific boundary conditions. Therefore, the velocity profile at the infinite upstream section is determined by a two-dimensional Poiseuille flow. In the present problem, attention is paid to the mass transfer on the wall downstream of the stenosis. Thus, the concentration θ_w at this wall FG is zero and the bulk concentration θ_∞ at the infinite upstream fluid is unity. The other boundary wall except the wall FG are impermeable.

In the numerical analysis, the coordinate x along the flow is transformed into ξ except the region DE as follows:

$$\left. \begin{aligned} \xi &= 1 - e^{\alpha_u x} \quad (x < 0) \\ &= 1 - e^{-\alpha_d x} \quad (x > 0.12) \end{aligned} \right\} \dots (4)$$

where α_u, α_d are 5.06, 0.280, respectively. The grid sizes at the upstream region and the downstream region of the stenosis are $\Delta x = 0.005$ and 0.06 , respectively. The divisions of the upstream region and the downstream region of the stenosis are 40 and 60, respectively.

The thickness δ_c of the laminar concentration boundary layer along a flat plate is related to $\delta_c = \delta_u / \sqrt[3]{Sc}$, where δ_u is the thickness of the laminar boundary layer⁽¹⁴⁾. Considering the diffusion of oxygen in blood, the Schmidt number 1000 is employed in the calculation⁽¹⁵⁾. Therefore, the thickness δ_c is about 0.1 times as large as the thickness δ_u . This suggests that the grid spacing adjacent to the mass transfer surface must be a finer grid size in estimating the concentration field. So, the coordinate y is transformed into ζ as follows⁽¹⁶⁾:

$$y = \zeta(\zeta + 2\sigma)/(2\sigma + 1) \dots (5)$$

Fig.1 Sketch of flow field

where $\sigma = 0.2$ and the total division number with respect to y is 50. Eventually, the size Δy just adjacent to the mass transfer surface is 0.006 .

The equations (1), (2) and (3) transformed by Eqs. (4) and (5) are employed in the calculation. The numerical calculation is carried out by a finite difference approximation of the governing equations. The Reynolds number is a few hundred and the Schmidt number is about 1000. This implies that the magnitude of the convective term in both the vorticity transported equation and the diffusion equation is considerably larger than that of the diffusion term, therefore the numerical solutions are apt to diverge. Therefore, the technique of Dennis & Cheng⁽¹⁷⁾ can be employed in the present calculation; the unknown functions ω, θ are represented using the function f ⁽¹⁸⁾ as follows:

$$C_u(f^{n+1}) - D_c(f^{n+1}) \\ = a\{C_u(f^n) - C_c(f^n)\} \dots (6)$$

where the coefficient a implies that the convective terms are approximated by the upstream finite difference and the central finite difference in the cases of $a=0$ and $a=1$, respectively. In the calculation, the coefficient a gradually increases from zero, and finally the flow field is estimated at $a=1$. Therefore, the flow pattern is estimated by the central finite difference. Next, although the coefficient a gradually increases from zero for the concentration, the converged solution can not be obtained above $a=0.7$. Thus, the concentration field is estimated at $a=0.7$.

3. Experimental

3.1 Experimental apparatus and procedure

The flow system is shown in Fig.2. The electrolytic solution which is composed of 0.002M of potassium ferrocyanide $K_4Fe(CN)_6$, 0.002M of potassium ferricyanide $K_3Fe(CN)_6$ and 0.2M of potassium hydroxide KOH as an unreactive electrolyte is employed as the working fluid. The electrolytic solution is introduced through a head tank into a test channel. The reservoir, the test channel and the head tank are immersed into the reservoirs which are controlled at a constant temperature by a heater and a thermostat. Consequently, the temperature is maintained at the set temperature with an accuracy of $\pm 0.2K$. The temperature is measured by the CC thermocouple installed on the wall of the test channel. In order to prevent the resolution of the fluid due to light, the head tank, the test channel and the reservoir are made of black acrylic resin. Furthermore, the electrolytic solution is inactivated by continuously injecting the nitrogen gas N_2 . The viscosity is measured by a capillary viscometer.

The test channel is shown in Fig.3 in detail. Its cross section is $4 \times 50 \text{ mm}^2$ and the inlet section has a semi-bellmouthed shape. The stenosis is placed at 360 mm

downstream of the inlet section. The experiment is carried out in laminar flow condition, and therefore the flow rate is low. The quantity of the electrolytic solution can not be measured in the atmospheric condition because of oxidization of the solution. So, a bucket flow meter made of acrylic resin, as shown in Fig. 4, is devised and the flow rate is calibrated.

3.2 Principle of measurements

The process chosen for the measurement is reduction of ferricyanide ions at a nickel cathode in the presence of a large volume of excess potassium hydroxide. The electric circuit is shown in Fig. 5. According to Faraday's law, the relation between mass flux N and electric current i generated by the reduction expressed in Eq. (7)(11)(12) can be expressed by Eq. (8) when a constant electric potential is provided between anode and cathode.

$$i = A_c F N \quad (8)$$

where A_c is the cathode area and F is Faraday's constant. The relation between mass transfer rate α and mass flux N is as follows:

$$N = \alpha(C_\infty - C_w) \quad (9)$$

where C_∞ is the bulk concentration, C_w is the concentration at the cathode surface. Under limiting current, the concentration C_w is zero. The Sherwood number (non-

dimensional mass transfer rate) is defined as follows:

$$Sh = \alpha H / D = (\partial \theta / \partial y)_{\text{wall}} \quad (10)$$

From Eqs. (8), (9) and (10), the following equation is obtained.

$$Sh = iH / (A_c F C_\infty D) \quad (11)$$

Therefore, the Sherwood number is calculated from the electric current i measured under the limiting current.

3.3 Method of measurements

The arrangements of cathode for measuring both the mean and the local Sherwood numbers are shown in Fig. 6. The width of the test nickel cathode in the direction perpendicular to this paper is 10mm. The cathode is enclosed with araldite resin and its surface is polished smoothly.

In the measurement of the mean Sherwood number (Fig. 6(a)), the current i_1 through the cathode 1 is obtained from the potential drop V across the resistance R . Furthermore, the total current i_2 through both the cathode 1 and the cathode 2 is measured by the potentiostat.

$$Sh_{m1} = i_1 H / (A_{c1} F C_\infty D) \quad (12)$$

$$Sh_{m2} = i_2 H / ((A_{c1} + A_{c2}) F C_\infty D) \quad (13)$$

$$Sh_r = Sh_{m1} / Sh_{m2} \quad (14)$$

where A_{c1} and A_{c2} are the area of the cathode 1 and the cathode 2, respectively. The mean Sherwood number Sh_{m1} is the mean mass transfer rate on the wall (cathode 1: $0.63 \leq x \leq 2.13$) just behind the stenosis. The mean Sherwood number Sh_{m2} is the mean mass transfer rate on the wall (cathode

Fig. 2 Experimental apparatus for flow system

Fig. 3 Test channel

Fig. 4 Flowmeter

Fig. 5 Experimental apparatus for electric circuit

downstream of this point.

The development of the concentration field is shown in Fig. 11. The thickness δ_c of the concentration boundary layer increases very steeply as the flow approaches the stenosis from the reattaching point. This suggests that the Sherwood number becomes lower in the separated region, particularly in the region just behind the stenosis.

4.3 Experimental results

An example of the flow visualization around the stenosis is shown in Fig. 12. Dye is injected into the fluid from a small aperture ($d=0.5\text{mm}$) on the bottom wall just behind the stenosis. This reattaching point agrees well with the numerically obtained value of $x_r=2.98$ for $Re=100$.

The local Sherwood number Sh_x in the region downstream of the stenosis is shown in Fig. 13. The experimental and the numerical results are shown in Figs. 13(a) and (b), respectively. For each Reynolds number, the local Sherwood number Sh_x is maximum near the reattaching point, asymptotically decreases in the region downstream of this point and steeply decreases in the upstream region. Particularly, this decrease in Sh_x is remarkable in the region $1 < x < 2$. It is found that the mass transfer becomes low in the region just behind the stenosis because the flow stagnates there.

The deterioration of the mass transfer rate is expressed by the relative Sherwood number Sh_r which is the ratio of Sh_{m1} to Sh_{m2} . The relation between the Reynolds number and the mean Sherwood number Sh_{m1} , Sh_{m2} and the relative Sherwood number Sh_r is shown in Fig. 14. The symbols $\circ$ and $\square$ denote the experimental and the numerical results, respectively. It is found that the numerical results agree with the experimental ones. The local Sherwood number Sh_x is maximum near the reattaching

point excluding the region of the mean Sherwood number Sh_{m1} . As the Reynolds number increases, the reattaching point moves downstream. In consequence the mean Sherwood number Sh_{m1} decreases and the mean Sherwood number Sh_{m2} increases as the Reynolds number increases. At $Re=130$, Sh_{m1} is approximately equal to Sh_{m2} , and therefore $Sh_r=1$. Above $Re=130$, the relative Sherwood number decreases steeply. In general, the mean Reynolds number in the animal artery is a few hundred⁽²⁰⁾. This implies that one of the reasons for the degeneration of the arterial wall is the deterioration of the mass transfer rate near the wall just behind the stenosis.

At each Reynolds number, both the general tendency of the local Sherwood number Sh_x and the location x of its maximum value obtained by numerical calculation

Fig. 12 Flow visualization
($Re=107$, $H=10\text{mm}$, $L/H=0.5$, $\delta/H=0.1$)

Fig. 10 Velocity profiles at various sections

Fig. 11 Concentration profiles at various sections

a. Experiment

b. Numerical calculation

Fig. 13 Local Sherwood number