Study on Tooth Bearing of Bevel Gears*
(1st Report, Theoretical Calculation of
Tooth Bearing and Experiment)

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To make clear the relation between the tooth bearing of bevel gear and assembly errors quantitatively and theoretically, a theoretical calculation of tooth bearing was carried out for Gleason and Reinecker type straight bevel gears and for Reinecker type skew bevel gear considering the geometric tooth profile of these gears. And the tooth bearing of each bevel gear was also measured using a tooth bearing tester which was produced for this study. Comparison of the calculated tooth bearing with the experimental result confirmed the propriety of the present calculation method, and revealed the characteristic of tooth bearing of each gear.

Key Words : Gear, Bevel Gear, Straight Bevel Gear, Skew Bevel Gear, Tooth Bearing, Assembly Error, Theoretical Study, Experimental Study

1. Introduction

For the bevel gear, to assemble gear pairs exactly is as important a problem as to produce gears with high accuracy. Since two shafts of the bevel gear are not parallel to each other unlike the spur and helical gears, the tooth bearing of bevel gear changes sensitively with assembly error and shaft angle error, and the strength and the performance of bevel gear also are influenced considerably by them. Therefore, the tooth bearing test which is done before assembly of gears is important for bevel gear as well as for cylindrical gear.

With the cylindrical gear, such individual errors as tooth profile, tooth trace and pitch errors can be easily measured by various testers available on the market, but it is difficult with the bevel gear to measure the tooth profile and tooth trace errors, because its tooth profile is complicated and is different depending on the gear cutting machines; further, the tooth depth and the tooth thickness change between the toe and the heel of gear tooth, and the tester for the bevel gear is not yet put into practice. Therefore, in the case of the bevel gear in place of the measurement of gear errors, the overall judgment of accuracies which include not only the accuracy of one gear but also the accu-

racy of gear box, the quality of assembly adjustment, the whole stiffness of the gear equipment and the quality of mounting is done by the tooth bearing test.

Some studies on the tooth bearing of bevel gear have already been reported (1), (2). However, all of them are nothing further than the qualitative results and considerations, and the quantitative and theoretical elucidation of the relation between the assembly error or the gear shaft angle error and the tooth bearing of bevel gear are not reported either.

In the present study, the tooth bearing tests on three kinds of Gleason and Reinecker type straight bevel gears and Reinecker type skew bevel gear were carried out using a tooth bearing tester built as a trial. The tooth bearing was measured changing the assembly error of bevel gear under a light load (almost no load), and the relation between the assembly error and the tooth bearing of bevel gear was investigated quantitatively. Furthermore, the tooth bearing was calculated theoretically considering the geometrical form of bevel gear, and the calculation results were compared with the tooth bearing obtained from experiment. The details are reported below.

2. Theoretical Calculation of Tooth Bearing

2.1 Calculation method of tooth bearing

The tooth bearing of bevel gear is calculated by a method as follows. First of all, one of a pair of the meshing tooth flanks (for example the driving tooth flank) is divided into many meshes as shown in Fig.1, and the coordinates of each intersection point m_i of these meshes on the tooth flank are calculated. The coordinate on the tooth flank can be calculated

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surface [Q] with the pitch surface [S], and the locus surface of the line of contact obtained with ψ changed variously is the generated flank of bevel gear. α_c is complementary angle to the angle between the pitch surface [S] and the cutter surface [Q], and it is the cutter pressure angle.

Next, Eq.(1) of the line of contact is transformed into an orthogonal coordinate system $O-x_{\varphi}y_{\varphi}z_{\varphi}$ fixed to the bevel gear, where the coordinate system is such that the generation gear shaft is z_{φ} -axis and the coordinate plane which includes x_{φ} -axis and z_{φ} -axis of generation gear shaft as shown in Fig.2 passes through the pitch point P. P is the pitch point at the cone distance R. Transforming Eq.(1) into the orthogonal coordinate system $O-x_{\varphi}y_{\varphi}z_{\varphi}$, the following equation is obtained:

$$\frac{x_G - x_{G0}}{P_{xG}} = \frac{y_G - y_{G0}}{P_{yG}} = \frac{z_G - z_{G0}}{P_{zG}} \quad \dots\dots\dots (2)$$

In the above equation, x_{G0} , y_{G0} and z_{G0} are the coordinates of point O' in Fig.2, and defined as

$$\left. \begin{aligned} x_{G0} &= \frac{\rho}{\sin \psi} \cos \varphi \sin \delta_0 \\ y_{G0} &= -\frac{\rho}{\sin \psi} \sin \varphi \sin \delta_0 \\ z_{G0} &= \frac{\rho}{\sin \psi} \cos \delta_0 \end{aligned} \right\} \quad \dots\dots\dots (3)$$

Moreover, P_{xG} , P_{yG} and P_{zG} are obtained from the following equations:

$$\left. \begin{aligned} P_{xG} &= P_x \cos \varphi \cos \delta_0 + P_y \sin \varphi \\ &\quad + P_z \cos \varphi \sin \delta_0 \\ P_{yG} &= -P_x \sin \varphi \cos \delta_0 + P_y \cos \varphi \\ &\quad - P_z \sin \varphi \sin \delta_0 \\ P_{zG} &= -P_x \sin \delta_0 + P_z \cos \delta_0 \end{aligned} \right\} \quad \dots\dots\dots (4)$$

where P_x , P_y and P_z are equal to each denominator of Eq.(1), and are expressed as

$$\begin{aligned} P_x &= \sin \psi, \quad P_y = \sin \psi \cos \psi \cot \alpha_c, \\ P_z &= \cos^2 \psi \cot \alpha_c + \tan \alpha_c. \end{aligned}$$

δ_0 in Eqs.(3) and (4) is the pitch cone angle of generated gear, and φ is the rolling angle of gear at generation. As shown in Fig.2, the relation between φ and ψ is expressed by

$$\left. \begin{aligned} \varphi &= (\psi - \psi_0) / \sin \delta_0 \\ \psi_0 &= \sin^{-1}(\rho/R) = \beta \end{aligned} \right\} \quad \dots\dots\dots (5)$$

where β is helix angle on the pitch cone at cone distance R.

In the above, the formulas for the line of contact on the right flank of left hand tooth (driving gear) are expressed, and in the same way those for the mating gear (driven gear, right flank of right hand tooth) are obtained putting the sign of ρ in Eqs.(3) and (5) as minus. Moreover, the formulas for the line of contact of Reinecker type straight bevel gear are easily obtained putting the eccentricity ρ of cutter as zero. In this case, x_{G0} , y_{G0} and z_{G0} are each equal to zero as known from Eq.(3).

2.2.2 Tooth profile of Gleason type straight bevel gear

In Fig.3, the generation of Gleason type straight bevel gear (right flank) is explained. As shown in Fig.3(a), the gear material is set inclined to the machine plane [M] by the root cone angle δ_f , and both are in contact with each other on line OE. The plane [Q] is the cutter surface, and it is inclined at $90^\circ - \alpha_c$ (α_c is cutter pressure angle) to machine plane. The rotation ratio of the generation motion of gear material to cutter is obtained by $\sin \delta_0$, and if cutting pitch cone angle is expressed by $\delta_f + \gamma_F$, the following equation is obtained:

$$\tan \gamma_F = (\sin \delta_0 - \sin \delta_f) / \cos \delta_f \quad \dots\dots\dots (6)$$

Hence, in Gleason type bevel gear, the reference pitch cone angle δ_0 and the cutting pitch cone angle $\delta_f + \gamma_F$ do not coincide with each other (4). Putting the cutting pitch point as C as shown in Fig.3, the plane [N] includes the cutting pitch point C, it is perpendicular to the cutter surface [Q] and intersects the machine plane [M] on line OB. Moreover, the intersection line OD of the plane [N] and the cutter surface [Q] is the line of contact of both flanks of the generated gear and the imaginary gear. In the same manner as in the case of Reinecker tooth profile, putting the apex of generated gear as the origin O of coordinate and the gear shaft as z_{φ} -axis, and expressing the formulas for the line of contact in the orthogonal coordinate $O-x_{\varphi}y_{\varphi}z_{\varphi}$ of which the coordinate plane including the x_{φ} - and z_{φ} -axes passes through the reference pitch point P of generated gear, the following equations are obtained:

$$\frac{x_G}{P_{xG}} = \frac{y_G}{P_{yG}} = \frac{z_G}{P_{zG}} \quad \dots\dots\dots (7)$$

where, P_{xG} , P_{yG} and P_{zG} are expressed as follows:

Fig.3 Generation of Gleason tooth profile (right flank)

$$\left. \begin{aligned}
 P_{xG} &= P_x \cos \varphi \cos \delta_f + P_y (\sin \phi \cos \varphi \\
 &\quad \times \sin \delta_f + \cos \phi \sin \varphi) \\
 &\quad + P_z (\cos \phi \cos \varphi \sin \delta_f - \sin \phi \sin \varphi) \\
 P_{yG} &= -P_x \sin \varphi \cos \delta_f - P_y (\sin \phi \sin \varphi \\
 &\quad \times \sin \delta_f - \cos \phi \cos \varphi) \\
 &\quad - P_z (\cos \phi \sin \varphi \sin \delta_f + \sin \phi \cos \varphi) \\
 P_{zG} &= -P_x \sin \delta_f + P_y \sin \phi \cos \delta_f \\
 &\quad + P_z \cos \phi \cos \delta_f \\
 \text{here, } P_x &= \sin w, P_y = \sin w \cos w \\
 &\quad \times \cot \alpha_c, P_z = \cos^2 w \cot \alpha_c + \tan \alpha_c \\
 w &= \phi + u.
 \end{aligned} \right\} \dots\dots\dots (8)$$

ϕ and u in Eq.(8) are angles shown in Fig. 3, and the locus surface of the line of contact obtained from Eqs.(7) and (8) with u changed variously is the generated flank of bevel gear. ϕ can be obtained as follows from Fig.3(b). Figure 3(b) shows the intersection line (large circle) of each plane explained in Fig.3(a) and the spherical surface of unit radius with center at the apex O of bevel gear, putting O_1 as the center of generated gear and the origin of coordinate, and putting a large circle by which O_1 and the center O_2 of imaginary gear are joined as the initial line⁽⁴⁾. By defining the sizes of each part and the angles as shown in the Fig.3(b), and using the spherical triangle theorem, ϕ is obtained from the following equations with the parameter u :

$$\left. \begin{aligned}
 \cos b &= \cos u \cos \gamma_F, \tan \lambda_A = \tan u / \sin \gamma_F, \\
 \tan \lambda_B &= \tan \gamma_F / \sin u, \\
 \tan \lambda_C &= \tan(\lambda_B + \alpha_c) / \cos b, \\
 \tan \phi &= -\sin \gamma_F \tan(\lambda_A + \lambda_C).
 \end{aligned} \right\} \dots\dots\dots (9)$$

Further, φ in Eq.(8) is the rotation angle of the gear material at generation and it is obtained as follows:

$$\varphi = \frac{u - u_P}{\sin \delta_0} + \mu_P \dots\dots\dots (10)$$

where u_P and μ_P are u and μ when generating the reference pitch point P on the tooth profile [see Fig.3(b)].

2.3 Coordinate transformation for calculation of tooth bearing

Since the formulas for the lines of contact obtained in the section 2.2 are those for the rotating coordinate system

Fig.4 Coordinate system for calculation of tooth bearing

located on each gear, they must be transformed into an orthogonal coordinate system common to both gears (the coordinate system for calculation of tooth bearing, the stationary coordinate system) in the calculation of tooth bearing of a pair of meshing gears. Furthermore, to introduce the assembly condition of bevel gear, the assembly error in the three directions (the deviation from regular position) and the shaft angle error are defined, and the formulas for the line of contact which consider them are obtained as follows.

2.3.1 Coordinate system for calculation of tooth bearing

Figure 4 shows the coordinate system $O-XYZ$ for calculation of tooth bearing. We put the apex of driving gear at designing as the origin O of coordinate system, the driving gear shaft as X -axis, and assume Y -axis to be perpendicular to X -axis. If the shaft angle of bevel gear is 90° and the assembly and shaft angle errors are zero, the driven gear shaft coincides with Y -axis.

Expressing the formulas for the line of contact of driving gear in the coordinate system $O-XYZ$, in the case of Reinecker tooth profile the following equations are obtained from Eq.(2):

$$\left. \begin{aligned}
 \frac{X_1 - X_{10}}{P_{x1}} &= \frac{Y_1 - Y_{10}}{P_{y1}} = \frac{Z_1 - Z_{10}}{P_{z1}} \\
 X_{10} &= z_{G10} - f_{a1} \\
 Y_{10} &= x_{G10} \cos \theta_1 - y_{G10} \sin \theta_1 \\
 Z_{10} &= -x_{G10} \sin \theta_1 - y_{G10} \cos \theta_1 \\
 P_{x1} &= P_{xG1} \\
 P_{y1} &= P_{xG1} \cos \theta_1 - P_{yG1} \sin \theta_1 \\
 P_{z1} &= -P_{xG1} \sin \theta_1 - P_{yG1} \cos \theta_1
 \end{aligned} \right\} \dots\dots\dots (11)$$

where, suffix 1 signifies the driving gear, and as shown in Fig.5 f_{a1} in these equations is the deviation of the apex O_c at cutting from the apex O_0 at designing of bevel gear (the apex error). Here, it is defined as plus with O_c outside O_0 . x_{G10} , y_{G10} and z_{G10} are given by Eq.(3), and P_{xG1} , P_{yG1} and P_{zG1} are given by Eq.(4). In the case of Gleason tooth profile, as x_{G10} , y_{G10} and z_{G10} are equal to zero, $X_{10} = -f_{a1}$ and $Y_{10} = Z_{10} = 0$ are applicable. And if P_{xG1} ,

Fig.5 Deviation of apex at cutting from apex at designing

P_{yG1} and P_{zG1} are calculated from Eq.(8), the same equations as Eq.(11) can be used. The rotation angle θ_1 of driving gear in Eq.(11) is the angle between the coordinate plane XOY and the perpendicular line to the X-axis through the pitch point on tooth flank, and has the minus sign in approach contact and the plus sign in recess contact, respectively.

2.3.2 Introduction of assembly condition

For calculating the tooth bearing considering the assembly condition, the followings are defined. Namely, if the designing apexes of the driving and driven bevel gears coincide with each other and the shaft angle of bevel gear is equal to the design value, this position of bevel gear is named the regular one. If the driving gear is fixed, the assembly errors f_X , f_Y and f_Z are defined as the deviations of the designing apex O_2 of driven gear in X, Y and Z directions from the regular position, and also the shaft angle error f_Σ is defined as the difference from the regular shaft angle. In Fig.6, the assembly and shaft angle errors (in this figure they are plus) are shown. The driven gear shaft is defined as the Y_2' -axis in this figure, and the coordinate plane $X_2'O_2Y_2'$ of the system $O_2-X_2'Y_2'Z_2'$ is parallel to the plane XOY of the coordinate system for calculation of tooth bearing. Using Fig.6, the formulas for the line of contact of driven gear for the coordinate system O-XYZ are obtained as follows:

$$\left. \begin{aligned} \frac{X_2 - X_{20}}{P_{X2}} &= \frac{Y_2 - Y_{20}}{P_{Y2}} = \frac{Z_2 - Z_{20}}{P_{Z2}} \\ X_{20} &= (x_{G20} \cos \theta_2 + y_{G20} \sin \theta_2) \cos \gamma \\ &\quad - (z_{G20} - f_{a2}) \sin \gamma + f_X \\ Y_{20} &= (x_{G20} \cos \theta_2 + y_{G20} \sin \theta_2) \sin \gamma \\ &\quad + (z_{G20} - f_{a2}) \cos \gamma + f_Y \\ Z_{20} &= -x_{G20} \sin \theta_2 + y_{G20} \cos \theta_2 + f_Z \\ P_{X2} &= (P_{xG2} \cos \theta_2 + P_{yG2} \sin \theta_2) \cos \gamma \\ &\quad - P_{zG2} \sin \gamma \\ P_{Y2} &= (P_{xG2} \cos \theta_2 + P_{yG2} \sin \theta_2) \sin \gamma \\ &\quad + P_{zG2} \cos \gamma \\ P_{Z2} &= -P_{xG2} \sin \theta_2 + P_{yG2} \cos \theta_2 \end{aligned} \right\} \dots (12)$$

here, $\gamma = \Sigma - 90^\circ + f_\Sigma$

Fig.6 Assembly error and gear shaft angle error

where suffix 2 signifies the driven gear, and Σ is the shaft angle at designing of bevel gear. In the same manner as in the case of driving gear, θ_2 is defined as the rotation angle of driven gear and f_{a2} is defined as the deviation (apex error) of the apex at cutting from that at designing. As Gleason tooth profile has $x_{G20} = y_{G20} = z_{G20} = 0$ in Eq.(12), $X_{20} = f_{a2} \sin \gamma + f_X$, $Y_{20} = -f_{a2} \cos \gamma + f_Y$ and $Z_{20} = f_Z$ are applicable.

2.4 Division of tooth flank and calculation of tooth bearing

Figure 7 shows a method of dividing the tooth flank of driving gear. In the tooth trace direction, the tooth flank is divided by the intersection lines of the spherical surfaces with various radii whose center is the apex of bevel gear and the tooth flank, and in the tooth profile direction, it is divided by the intersection lines of the cone surfaces with various cone angles whose apex is the one of bevel gear and the tooth flank. In this calculation, the tooth flank is divided into 60 equal parts in the tooth trace direction and into 50 equal parts between the tip and root cones in the tooth profile direction. Thus, the tooth flank is composed of 3000 meshes in total. By calculating the clearance between each intersection point m_i of these meshes and the driven gear flank, the tooth bearing is obtained. The coordinate (X_{1i}, Y_{1i}, Z_{1i}) of the intersection point m_i can be obtained by solving the simultaneous equations which are composed of Eq.(11) for the line of contact and the equations for sphere and cone.

The calculation method for the clearance between each intersection point m_i of meshes and the tooth flank of driven gear is described below. Figure 8 shows the meshing tooth profiles of the driving and driven gears, which are cut by the plane $X = X_{1i}$ (the plane $m_i O_i n_i$ in Fig.1 or $m_i O_i$ in Fig.7) perpendicular to the driving gear shaft through the intersection point m_i . In this figure, O_i is the axial center of driving gear, and n_i is the intersection point of the tooth flank of driven gear and the circular arc with the radius of $\overline{m_i O_i} = r_i$ and with the center O_i . θ_i is the angle of the points m_i and n_i against O_i . Radius

Fig.7 Division of tooth flank of driving gear

r_i of circular arc is obtained using the coordinates (X_{1i}, Y_{1i}, Z_{1i}) of the intersection point m_i as follows:

$$r_i = \sqrt{Y_{1i}^2 + Z_{1i}^2} \quad (13)$$

The point n_i on the tooth flank of driven gear can be determined as follows. As the point n_i is that on the tooth profile cut by the plane $X=X_{1i}$, by substituting $X_2=X_{1i}$ into Eq.(12), the following equations are obtained:

$$\left. \begin{aligned} Y_2 &= P_{r2}(X_{1i} - X_{20})/P_{x2} + Y_{20} \\ &\equiv F(\psi_2 \text{ or } u_2) \\ Z_2 &= P_{z2}(X_{1i} - X_{20})/P_{x2} + Z_{20} \\ &\equiv G(\psi_2 \text{ or } u_2) \end{aligned} \right\} \quad (14)$$

Y_2 and Z_2 in Eq.(14) express the tooth profile cut by the plane $X=X_{1i}$ (they are not the coordinates of point n_i), and are functions of ψ for Reinecker tooth profile or u for Gleason tooth profile as known in Eqs.(2)~(5) or Eqs.(7)~(10). Next, the following equation must be satisfied in the relation between Y_2 and Z_2 in order for the point n_i to be on the circular arc of radius r_i :

$$\sqrt{Y_2^2 + Z_2^2} = r_i \quad (15)$$

Thus, the coordinate of point n_i (X_{2i}, Y_{2i}, Z_{2i}) (X_{2i} is known and equal to X_{1i}) can be obtained by calculating Eqs.(14) and (15) with ψ_2 or u_2 assumed repeatedly and by satisfying Eq.(15) finally. The angle θ_i between the points m_i and n_i can be calculated by the following equations using the coordinates of these points:

$$\theta_i = 2 \sin^{-1} \frac{\sqrt{(Y_{2i} - Y_{1i})^2 + (Z_{2i} - Z_{1i})^2}}{2r_i} \quad (16)$$

Figure 9 shows two tooth pairs in the meshing position, and the symbols P and G express the teeth of the driving and driven gears respectively. (P1) (G1) is the tooth pair for the calculation of tooth bearing, and in the same manner as Fig.8 it expresses the tooth profile cut by the plane $X=X_{1i}$ which includes the intersection point m_{1i} (suffix 1 is number of tooth pair, and i shows the position of the intersection point of meshes) and which is perpendicular to the driving gear shaft. The following tooth pair (P2) (G2) shows the tooth profile cut by the plane $X=X_{1a}$ which includes the intersection point m_{2a} of meshes (the mesh pattern on the flank of tooth (P2) is the same as that of (P1)). Therefore, the tip

Fig.8 Tooth profile cut by plane $X=X_{1i}$

and root circles of teeth (P2) and (G2) are different from those of teeth (P1) and (G1) respectively. In the figure, a state where neither of two tooth pairs mesh is shown, but in practice it is assumed that only tooth pair (P2) (G2) is in contact with each other (here (P1) and (P2) are in the position shown with the dotted line). The points m_{1i} , n_{1i} , m_{2a} and n_{2a} in the figure are all on the tooth profiles shown with the solid lines. Putting the angle between the points m_{1i} and n_{1i} made with the axial center O_i of the driving gear and the angle between the points m_{2a} and n_{2a} made with O_a as θ_{1i} and θ_{2a} respectively (they are called the provisional clearance angles), they can be calculated from Eqs.(13)~(16). Since both gears are in contact with each other at only the point n_{2a} as shown in Fig.9, θ_{1i} is larger than θ_{2a} , and θ_{2a} becomes the minimum value θ_{min} . When (P2) and (G2) are in contact with each other at the point n_{2a} , the actual clearance angle θ_{1i} between the points m_{1i} and n_{1i} on the tooth pair (P1) (G1) for calculating the tooth bearing is obtained from $\theta_{1i} = \theta_{1i} - \theta_{min}$, and $\theta_{2a} = \theta_{2a} - \theta_{min} = 0$ is at the contact point n_{2a} . Thus, by repeating the same procedure as the above for each intersection point of meshes on the flanks of all tooth pairs in meshing position and by searching for the position having a clearance angle $\theta=0$ on the flank of tooth pair for calculating the tooth bearing, we can obtain the tooth bearing in which the adjacent tooth pairs in simultaneous meshing are considered.

The above calculation method in which the clearance angle θ is calculated for the tooth bearing is used for the reason as follows. Namely, first of calculation of the tooth bearing the rotation angles θ_1 and θ_2 of the driving and driven gears in Eqs.(11) and (12) must be decided for the meshing position. However, they can not be decided unconditionally, because the values of θ_1 and θ_2 have relation to the contacting position of two teeth. Specially, when the assembly and shaft angle errors of gears are considered, it is very difficult to determine the values of θ_1 and θ_2 . Therefore, the meshing position is determined from only the rotation angle θ_2 of driven gear, and the rotation angle θ_1 of driving gear is assumed as an arbitrary

Fig.9 Two tooth pairs in meshing

value. And the provisional clearance angles θ_{1i} and θ_{2a} are calculated under the state of the teeth separated as shown in Fig.9; furthermore, the actual clearance angles θ_{1i} and θ_{2a} are calculated using the method described in the above. Thus the tooth bearing can be obtained easily. This is the reason why the clearance angle θ is adopted.

In this experiment, in the measurement of tooth bearing the red lead is used as described later. Therefore, the tooth bearing is calculated under the condition that two flanks are in contact with each other when the clearance between two flanks is less than the thickness of red lead. In this case, the condition of $\theta \leq t_p / (r_f \cos \alpha_c \times \cos \beta)$ is used approximately for the tooth bearing calculation in place of the above-mentioned condition of the clearance angle $\theta=0$, where t_p is the thickness of red lead, and β is equal to zero for the straight bevel gear.

Fig.10 Flow chart of calculation of tooth bearing

Figure 10 shows a rough flow chart of the computer program for the calculation of tooth bearing.

3. Experimental Apparatus and Experimental Procedure

The dimensions of Gleason and Reinecker type straight bevel gears and Reinecker type skew bevel gear used in the tooth bearing test are shown in Table 1. All of these bevel gear pairs are miter bevel gears having the gear ratio 1, and their material is SCM420. The tooth flanks of Gleason type bevel gear are finished by only cutting with a Gleason gear cutting machine, and both Reinecker type straight and skew bevel gears are ground by using a Reinecker grinding machine after they are cut and carburized. Helix angle β_m of skew bevel gear in Table 1 is one at mid-face-width. All of the bevel gears used in this experiment have no crowning on the tooth flank.

The tooth bearing test was carried out using a tooth bearing tester designed and produced for this experiment, and the changes of the tooth bearing pattern with the changing of the assembly errors were measured. This tester is composed of two bearing stands and two gear shafts fixed to them, and the test gears rotate slipping around the gear shaft. The assembly errors of f_x and f_y (see Fig.6) are given to the test gear pair by fixing one bearing stand and moving the other in two directions of the horizontal. Furthermore, by inserting shims having various thicknesses under each bearing stand, error f_z is given to the test gear pair. The movement of the bearing stand in the horizontal direction is measured by dial gauges attached on two sides of one bearing stand which make an angle of 90° with each other. The tooth bearing is measured by the following method using red lead: The red lead applied on the right

Table 1 Dimensions of test bevel gears

		Straight bevel gear		Skew bevel gear
		Gleason	Reinecker	Reinecker
Tooth profile				
Module	m	6		6
Cutter pressure angle	α_c	20°		20°
Number of teeth	z	32		32
Shaft angle	Σ	90°		90°
Reference diameter	d_g	192 mm		192 mm
Face width	b	30 mm		30 mm
Helix angle	β_m			20° (R.L.)
Offset of tooth trace	ρ			41.31 mm
Tooth depth	h	13.5 mm		13.5 mm
Addendum	h_k	6 mm		6 mm
Dedendum	h_f	7.5 mm		7.5 mm
Cone distance (Heel)	R_a	135.77 mm		135.77 mm
Pitch angle	δ_0	45°		45°
Dedendum angle	δ_f	3°10'		3°10'
Tip angle	δ_k	47°32'		47°32'
Root angle	δ_f	41°50'		41°50'
Material		SCM420		SCM420
Heat treatment		Normalized	Carburized	Carburized
Method of finishing teeth		Cut	Ground	Ground

tooth flank of driven gear is transferred to the tooth flank of driving gear by meshing both tooth flanks with each other under a light load (nearly no load), and the transferred red lead is further transferred to a transparent adhesive tape. The red lead is applied uniformly with utmost care to about $20\mu\text{m}$ in thickness in this experiment.

4. Calculated and Experimental Results and Consideration

Figures 11~13 show the change of the tooth bearing patterns of Gleason and Reinecker type straight bevel gears and Reinecker type skew bevel gear with the changing of the assembly errors. The measured tooth bearing pattern is that on the right flank of driving gear. The upper and lower figures at every assembly error show the experimental result and the theoretical calculation result (calculated using the gear dimensions in Table 1) respectively. The values of f_X and f_Y in these figures express the assembly errors defined in Fig. 6. $f_X=f_Y=0$ mm means that both of the driving and driven gears lie in the regular assembly position. And the decreasing of f_X and the increasing of f_Y mean that the driven gear moves away from the driving gear in the axis direction (X direction) of the driving gear and in the direction (Y direction) at right angles to the driving gear shaft, respectively. Therefore, the definition of plus and minus for f_X differs from that for f_Y . In this experiment, the tooth bearing is measured variously changing the assembly error f_Z (offset between two gear shafts), too. However, in Figs. 11~13, as an example, the tooth bearing for $f_Z=0$ mm and the shaft angle error $f_\Sigma=0^\circ$ is shown. A nearly complete tooth bearing pattern of each bevel gear is shown in each lowest figure in Figs. 11~13. From these figures, it is evident that the nearly complete tooth bearing occurs at one position different from the regular assembly position of $f_X=f_Y=0$ mm for all of bevel gears. The reason can be explained in the fact that there is the apex error f_a (the deviation of the apex at cutting or grinding from that at designing, see Fig. 5) on each bevel gear, which was found from the tooth profile of bevel gear measured using a base cone type bevel gear flank tester of our own production. Table 2 shows the apex errors of each bevel gear used in the tooth bearing calculation.

It is evident from Figs. 11~13 that the measured result and the calculated result of tooth bearing are in good agreement with each other in its shape and position for all of bevel gears. From this fact, it can be concluded that the theoretical calculation for the tooth bearing is valid. The apex error of each driven bevel gear in Table 2 agrees with the values of f_Y for the nearly complete tooth bearings of each gear shown in Figs. 11~13, and the apex error of each driving gear agrees with the values of f_X for the nearly complete tooth bearings (the sign of f_X is changed to minus because the definition of plus and

minus of f_X is different from that of f_Y). For the bevel gear having an apex error, it is found from the above fact that a satisfactory tooth bearing can be obtained by moving the bevel gear in its axial direction from the regular position by a dis-

Fig.11 Change of tooth bearing of Gleason type straight bevel gear ($f_Z=0$ mm, $f_\Sigma=0^\circ$)

Fig.12 Change of tooth bearing of Reinecker type straight bevel gear ($f_Z=0$ mm, $f_\Sigma=0^\circ$)

Fig.13 Change of tooth bearing of Reinecker type skew bevel gear ($f_Z=0$ mm, $f_\Sigma=0^\circ$)

Table 2 Apex error f_a of bevel gear used in calculation

	Straight bevel gear		Skew bevel gear
	Gleason	Reinecker	Reinecker
Driving gear	$f_a = -0.1$ mm	0 mm	-0.55 mm
Driven gear	$f_a = 0.1$ mm	0.2 mm	-0.35 mm

tance equal to the apex error.

When the tooth bearings of Gleason and Reinecker type straight bevel gears in Figs.11 and 12 are compared with each other, it is recognized that the tendencies of the changing of the tooth bearing of the two with changing assembly errors f_x and f_y are similar to each other regardless of the tooth profiles. Namely, it is found that both straight bevel gears have a tendency that the tooth bearing in the tooth trace direction moves from the toe to the heel through the whole tooth trace with a decreasing of f_x or an increasing of f_y , and then the tooth bearing in the tooth profile direction moves from the tip to the root with a decreasing of f_x and from the root to the tip with an increasing of f_y . With Reinecker type skew bevel gear in Fig. 13, in the same manner as with the straight bevel gear, the tooth bearing in the tooth profile direction moves from the root to the tip with an increasing of f_y and also the tooth bearing in the tooth trace direction moves from the toe to the heel through the whole tooth trace with a decreasing of f_x . However, the whole tooth trace bearing is obtained only at $f_x=0.55$ mm (the value of f_x for the complete tooth bearing). Therefore, it is clear that the decreasing or the increasing of f_y has little influence on the tooth bearing in the tooth trace direction.

5. Conclusions

The results obtained in the present paper are summarized as follows:

(1) A method to calculate theoretically the tooth bearing of bevel gear having an assembly error, a shaft angle error and an apex error considering the geometric tooth shape and the meshing condition is presented. The tooth bearing calculated by this method is in good agreement with the measured result. Therefore, it can be concluded that this method for calculation of tooth bearing is valid.

(2) If the bevel gear has apex error (the deviation of the apex at cutting or grinding from that at designing), a satisfactory tooth bearing can be obtained by moving the bevel gear in its axial direction from the regular position by a distance equal to the apex error.

(3) The tendencies in the changing of the tooth bearing of both Gleason and Reinecker type straight bevel gears with changing assembly errors are similar to each other regardless of the difference in the tooth profile.

(4) The tooth bearing of the straight bevel gear in the tooth trace direction is influenced by both assembly errors f_x and f_y (the assembly errors in each axial direction of the driving and driven gears), but that of the skew bevel gear is influenced only by f_x and hardly by f_y .

References

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- (4) p.2492 in Reference (3).