Study on Tooth Bearing of Bevel Gears*
(2nd Report, Relation between Tooth Bearing and
Assembly Error, Shaft Angle Error)

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The tooth bearing test of bevel gear is performed frequently, because it is necessary in this test to judge synthetically the precisions taking account of various gear errors. In the previous paper, the authors demonstrated a calculation method for the tooth bearing of bevel gear. In the present paper, the tooth bearing was measured with changing the assembly condition, using a tooth bearing tester specially produced for this study. It was found that the characteristic of tooth bearing of bevel gear can be made clearer by expressing the sizes and the positions of tooth bearing in the tooth trace and tooth profile directions by numerical values. Furthermore, the influence of the shaft angle error on the tooth bearing of bevel gear was also made clear.

Key Words : Gear, Bevel Gear, Straight Bevel Gear, Skew Bevel Gear, Tooth Bearing, Assembly Error, Shaft Angle Error

1. Introduction

With bevel gears, the measurement of the individual errors such as the tooth profile and tooth trace errors is more difficult than with cylindrical gears, such as spur and helical gears, so that the accuracy of the bevel gear is mainly judged not by examining the individual errors but by measuring the tooth bearing. Moreover, since the strength and the performance of bevel gear can be estimated from the tooth bearing, the tooth bearing test is often carried out before assembling the gears. Therefore, it is important to make clear quantitatively and theoretically the relation between the assembly conditions and the tooth bearings of bevel gear.

In the 1st report(1) of this study, the tooth bearing was calculated theoretically by considering the geometrical form of bevel gear, and the validity of this calculation method for the tooth bearing was confirmed by comparing the calculated result with the experimental result.

The condition of the tooth bearing of gears can be made clear from its size and position. And in every country the gear accuracy standards define the requisite length of tooth bearing according to the

accuracy grade. In the Japanese Industrial Standards (JIS)(2), the lengths of tooth bearing along the tooth trace and tooth profile directions are expressed as their ratios to the effective tooth trace and tooth profile lengths respectively, and the tooth bearings are classified according to these ratios. In the present paper, tooth bearing tests on three kinds of Gleason and Reinecker type straight bevel gears and Reinecker type skew bevel gear were carried out using a tooth bearing tester designed and produced for this study. The tooth bearing was measured

Table 1 Dimensions of test bevel gears

Tooth profile	Straight bevel gear		Skew bevel gear
	Gleason	Reinecker	Reinecker
Module m	6		6
Cutter pressure angle α_c	20°		20°
Number of teeth z	32		32
Shaft angle Σ	90°		90°
Reference diameter d_0	192 mm		192 mm
Face width b	30 mm		30 mm
Helix angle δ_m			20° (R.L)
Offset of tooth trace p			41.31 mm
Tooth depth h	13.5 mm		13.5 mm
Addendum h_k	6 mm		6 mm
Dedendum h_f	7.5 mm		7.5 mm
Cone distance (Heel) R_a	135.77 mm		135.77 mm
Pitch angle δ_0	45°		45°
Dedendum angle θ_f	3°10'		3°10'
Tip angle δ_k	47°32'		47°32'
Root angle δ_f	41°50'		41°50'
Material	SCM420		SCM420
Heat treatment	Normalized	Carburized	Carburized
Method of finishing teeth	Cut	Ground	Ground

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changing the assembly error of bevel gear, and the relation between the assembly error and the tooth bearing of bevel gear was investigated quantitatively by measuring the lengths and the positions of the tooth bearing along the tooth trace and tooth profile directions. Furthermore, the influence of the shaft angle error on the tooth bearing of bevel gear was also examined. By these investigations, the characteristics of the tooth bearing of each bevel gear under various assembly conditions were revealed. The details are reported below.

2. Experimental Apparatus and Experimental Procedure

2.1 Test gears

Table 1 shows the dimensions of Gleason and Reinecker type straight bevel gears and a Reinecker type skew bevel gear used in the experiment. All of these bevel gear pairs are miter bevel gears having the gear ratio 1, and their material is SCM420. The tooth flanks of Gleason type bevel gear are finished by only cutting with a Gleason gear cutting machine, and both Reinecker type straight and skew bevel gears are ground by using a Reinecker grinding machine after they are cut and carburized. Helix angle β_m of skew bevel gear in Table 1 is one at mid-facewidth. All of the bevel gears used in this experiment have no crowning on the tooth flank.

2.2 Bevel gear flank tester

Figure 1 shows the appearance of a base cone type bevel gear flank tester produced for this study. This bevel gear flank tester is composed of the base cone (2) having a shaft and an apex which are common to the measured bevel gear (3), the motor (6) with reduction gears (number of revolutions 0.294 rpm) and the driving arm

Fig.1 Base cone type bevel gear flank tester

Fig.2 Tooth profile curves of Gleason type straight bevel gear

Fig.3 Tooth profile curves of Reinecker type straight bevel gear

Fig.4 Tooth profile curves of Reinecker type skew bevel gear

⑦ to roll the base cone on the circular surface plate ①, and the electric micrometer ④ for the tooth profile measurement. The tooth profile errors of bevel gear are measured as deviations from the spherical involute. In the profile measurement, the base cone is given a rolling motion in which the apex of the base cone ② fitted with the measured gear ③ always coincides with the center of the circular surface plate ①. The electric micrometer ④ is fixed to the circular surface plate ① through the slider ⑤ and the attachment arm ⑧. The position of the contact needle of electric micrometer is set up at the height of the circular surface plate. By rotating the slider screw, the measuring position of the electric micrometer can be moved to any position between the toe and the heel in the tooth trace direction. A deflection of the contact needle of electric micrometer is recorded by the recorder. The base cone angle of this bevel gear flank tester is $\delta_g = 41^\circ 38'$, and the tooth profile errors of bevel gear are measured by using the same base cone for all of Gleason and Reinecker type bevel gears.

Figures 2~4 show the tooth profile error curves (these are represented as the deviation from the spherical involute) of each bevel gear measured by using the bevel gear flank tester shown in Fig.1. The chained lines in Figs.2~4 express the calculated deviation of the tooth profile of each bevel gear from the spherical involute. It is evident that the measured tooth profile curves (solid lines) of Gleason type straight bevel gear and Reinecker type skew bevel gear are considerably different from the calculated results (chained lines). The difference can be attributed to the discrepancy between the design apex of bevel gear and the apex after cutting (grinding) or the apex of base cone fitted with the bevel gear, and detailed consideration on this point is made as follows.

Figure 5 shows an example of the relation among the three positions of the design apex O_0 of bevel gear, the apex O_c after cutting (grinding) and the apex O_b

of base cone, and f_a is the deviation of the apex O_c after cutting (grinding) from the design apex O_0 , which is called the apex error of bevel gear, and f_b is the deviation of the apex O_c after cutting (grinding) from the apex O_b of base cone, which is called the apex error in measuring the tooth profile. Here, the sign of f_a is plus when O_c is outside of O_0 and the sign of f_b is plus when O_c is outside of O_b as shown in Fig.5. The position of the apex O_b of base cone changes with the distance from the reference surface of bevel gear, namely, with the length L of the boss of bevel gear. Therefore, the measured lengths of the boss of each bevel gear are also shown in a table in Fig.5. The design length of the boss in manufacturing the gear blank is 47 mm for all of the bevel gears, and it is evident from the table in Fig.5 that L of Reinecker type straight and skew bevel gears is identical to the design value L_0 (thus, O_0 and O_b coincide with each other), but that L of Gleason type straight bevel gear is 46.1 mm for the driving gear and 46.5 mm for the driven gear. Therefore in this case, the differences of $L_0 - L = 0.9$ mm and 0.5 mm appear between O_0 and O_b on the driving and driven gears respectively. The relation among f_a , f_b and $L_0 - L$ is expressed by the following equation referring to Fig.5:

$$f_b = f_a + (L_0 - L) \quad (1)$$

$f_a = 0$ combined with $f_b = 0$ means that both O_c and O_b coincide with the design apex O_0 and that there is not any apex error on the bevel gear, and $f_a = f_b \neq 0$ means that though O_0 and O_b coincide with each other, an apex error exists between O_c and O_0 .

The broken lines in Figs.2 and 4 show the calculated tooth profile curves, which are expressed as the deviation from the spherical involute, for the bevel gear having the apex error f_b in measuring the tooth profile. It is found from Fig.2 that the tooth profile curve calculated under the condition of $f_b = 1.0$ mm for the driving gear of Gleason type straight bevel gear is in good agreement with the measured tooth profile curve, and that for the driven gear the former under $f_b = 0.6$ mm is in good agreement with the latter. As described above, the driving and driven gears of Gleason type bevel gear have manufacturing errors of the boss of $L_0 - L = 0.9$ mm and 0.5 mm respectively, and also $f_b = 1.0$ mm and 0.6 mm are nearly equivalent to $L_0 - L = 0.9$ mm and 0.5 mm respectively. Therefore, it can be concluded that the discrepancy between the measured tooth profile curve and the calculated tooth profile curve (chained line) of Gleason type bevel gear shown in Fig.2 is caused by the manufacturing error of its boss, and that such a seeming result comes from this. If the apex error f_a of bevel gear is calculated substituting the above values of f_b and $L_0 - L$ into Eq.(1), it seems an apex error of about 0.1 mm will occur in both the driving and driven gears of Gleason type straight bevel gear. It is seen by considering in the same way that

	Straight bevel gear		Skew bevel gear
	Gleason	Reinecker	Reinecker
Driving gear	$L = 46.1$ mm	47.0 mm	47.0 mm
Driven gear	$L = 46.5$ mm	47.0 mm	47.0 mm

Fig.5 Apex error and measured value of boss length L

the Reinecker type skew bevel gear shown in Fig.4 has apex errors of $f_a = -0.55$ mm and -0.35 mm in the driving and driven gears respectively. Furthermore, Reinecker type straight bevel gear seems to have hardly any apex error as seen from Fig.3.

2.3 Tooth bearing tester

Figure 6 shows the bevel gear tooth bearing tester designed and produced for this experiment. This tester is composed of two bearing stands ②, ④ and two gear shafts ①, ③ fixed to the bearing stand. In this tester, the test gears (the driving gear ⑬ and the driven gear ⑫) rotate slipping around these shafts. In order to measure the change of the tooth bearing pattern under different assembly conditions, an assembly error and a shaft angle error are given to the test gear pair by moving the bearing stand ④ of the driving gear in two horizontal directions under the condition of fixing the bearing stand ② of the driven gear and by inserting shims having various thicknesses under each bearing stand. The bearing stand ④, namely, the driving gear can be moved to any position by turning the nut on the screw bars ⑤ and ⑥. The movement is measured by the dial gauges ⑪ attached on two sides of the bearing stand ④. The bearing stand is fixed by tightening the stopper ⑧ with a bolt after it is moved. The knock pin ⑦ is used in order to determine the normal assembly position of the bevel gear, at which the design apexes of the driving and driven gears coincide with each other and at which the shaft angle is 90° . The

Fig.6 Tooth bearing tester of bevel gear

scale of dial gauge is set on the basis of this position.

2.4 Experimental procedure and arrangement method for tooth bearing data

The tooth bearing is measured by the following method using red lead: The red lead applied on the right tooth flank of driven gear is transferred to the tooth flank of driving gear by meshing both tooth flanks with each other under a light load (nearly no load), and the transferred red lead is further transferred to a transparent adhesive tape. The red lead is applied uniformly with utmost care to about $20\mu\text{m}$ in thickness in this experiment.

In order to measure the change of the tooth bearing under different assembly conditions of bevel gear, the direction of the movement and the assembly error of bevel gear are defined as shown in Fig.7. In this figure, the design apex of driving gear is at the origin O , the driving gear shaft is on the X -axis, the direction of the driven gear shaft perpendicular to the X -axis is set parallel to the Y -axis, and the Z -axis is perpendicular to the X - and Y -axes. The moved lengths of the design apex O_2 of driven gear in the X , Y and Z directions from the design apex O of driving gear are defined as the assembly errors f_X , f_Y and f_Z . The signs of f_X , f_Y and f_Z are set plus respectively when the design apex O_2 of driven gear lies in the position at which X , Y and Z are plus as shown in Fig.7. The positive sign of f_X represents the state that the driven gear approaches the driving gear from the normal assembly position in the direction of the driving gear shaft (X direction), and the positive sign of f_Y represents the state that the driven gear recedes from the driving gear in the direction of the driven gear shaft (Y direction). Thus, even if the signs of f_X and f_Y are the same, their meanings are different from each other. The shaft angle error is set plus when the shaft angle of bevel gear is wider than the normal shaft angle (design value) and set minus when the shaft angle is narrower than the latter, and the re-

Fig.7 Moving direction and assembly error of bevel gear

lation between the shaft angle error and the tooth bearing also is investigated.

In order to investigate the relation between the tooth bearing and the assembly error of bevel gear quantitatively, the measured tooth bearing data are arranged as follows. Figure 8 shows the symbols used for expressing the tooth bearing with the numerical value. In this figure, the tooth bearing of bevel gear is represented by the hatched portion, l_o is the working length of the tooth trace of bevel gear, l_a is the maximum length of tooth bearing in the tooth trace direction, h_a is the maximum width of tooth bearing along the tooth profile direction, and h_{ao} is the working tooth depth at the position of h_a . The point O, which is called the center of the tooth bearing, is determined as the middle point of the width of tooth bearing at the middle point of the maximum length l_a of tooth bearing in the tooth trace direction. The position of the center of tooth bearing is denoted with the symbols l_c , h_c and h_{co} , where l_c , h_c and h_{co} are the distance along the tooth trace from the heel of bevel gear to the center O, the distance from the tooth tip to the center O, and the working tooth depth at the position of the center O respectively. When these symbols are used, the ratio of the maximum length of tooth bearing in the tooth trace direction to the working length of the tooth trace is denoted by l_a/l_o (%), and the ratio of the maximum width of tooth bearing in the tooth profile direction to the working tooth depth is denoted by h_a/h_{ao} (%). Moreover, l_c/l_o and h_c/h_{ao} are used for defining the positions of the center of tooth bearing in the tooth trace direction and in the tooth profile direction, where they are expressed in percentage for convenience dividing l_c and h_c by the working length of the tooth trace and by the working tooth depth respectively. Thus, the tooth bearing can

Fig. 8 Symbols for expressing tooth bearing with numerical value

Table 2 Example of tooth bearing pattern and its numerical value

Assembly error	Tooth bearing	l_a/l_o	l_c/l_o	h_a/h_{ao}	h_c/h_{ao}
$f_x=0.55$ mm $f_y=-0.05$ $f_z=0.0$	Tip Heel Toe Root	30%	82%	60%	29%
$f_x=0.25$ $f_y=0.55$ $f_z=0.0$	Tip Heel Toe Root	97	48	46	29
$f_x=-0.35$ $f_y=0.55$ $f_z=0.0$	Tip Heel Toe Root	27	13	64	29

be expressed with these numerical values. Here, l_a/l_o being 100% means that there is a portion of the complete tooth bearing along the tooth trace, and h_a/h_{ao} being 100% means that there is a portion of the complete tooth bearing along the tooth profile. Furthermore, l_c/l_o being 100% and 0% mean the tooth bearings at the toe and at the heel of bevel gear respectively, and h_c/h_{ao} being 100% and 0% mean the tooth bearings at the tooth root and at the tooth tip respectively.

Table 2 shows an example of the tooth bearing pattern measured for Reinecker type straight bevel gear and its numerical values. It is found from this table that the numerical values of l_a/l_o etc. express the condition of the tooth bearing well.

3. Experimental and Calculated Results and Consideration

3.1 Relation between assembly error and tooth bearing

Fig. 9 Change of tooth bearing of Gleason type straight bevel gear ($f_z=0^\circ$)

Fig. 10 Relation between assembly error and l_a/l_o , l_c/l_o , h_a/h_{ao} , h_c/h_{ao} (Gleason type straight bevel gear, $f_z=0^\circ$)

3.1.1 Tooth bearing of straight bevel gear

Figure 9 shows the change of the tooth bearing of Gleason type straight bevel gear, which is measured on the right tooth flank of driving gear, with the change of the assembly errors f_X , f_Y and f_Z . The tooth bearing pattern at $f_Z=0$ mm is already shown in the 1st report(1). Figure 10 shows the numerical values of l_a/l_o , l_c/l_o , h_a/h_{ao} and h_c/h_{co} for the tooth bearing shown in Fig.9, which are plotted against the assembly error f_Y in abscissa together with the assembly error f_X . It is found from Fig.10 that the length l_a/l_o of tooth bearing in the tooth trace direction at $f_Z=0.4$ mm increases with a decrease of f_X or with an increase of f_Y and the tooth bearing in the tooth trace direction is better, but then l_a/l_o at $f_Z=-0.4$ mm shows a tendency contrary to this. Namely, when the sign of f_Z (offset between two gear shaft centers) differs, such tooth bearing as obtained by exchanging the driving and driven gears with each other occurs along the tooth trace. In the case of $f_Z=0$ mm, as f_Y increases from a negative value, l_a/l_o also increases gradually from a small value (the incomplete tooth bearing along the tooth trace), and l_a/l_o becomes nearly 100% (the complete tooth bearing along the tooth trace) when f_Y reaches a certain value regardless of f_X . When f_Y increases over this value, l_a/l_o decreases to the contrary, and the tooth bearing along the tooth trace becomes incomplete again. And the value of f_Y at which l_a/l_o is 100% decreases with f_X . Therefore, if the tooth bearing in the tooth trace direction is incomplete, the complete tooth bearing in the tooth trace direction can be obtained by changing f_X or f_Y suitably.

The tendency of the tooth bearing position l_c/l_o in the tooth trace direction is explained using Fig.10 as follows: l_c/l_o at $f_Z=0.4$ mm does not change so largely as a whole with the changing of f_X and f_Y and it is always 75~90%, which means the tooth bearing at the toe, but l_c/l_o decreases with an increase of f_Y as f_X decreases further, while the tooth bearing at $f_Z=-0.4$ mm always occurs at the heel. In the case of $f_Z=0$ mm, l_c/l_o shows a tendency to decrease with an increase of f_Y regardless of f_X . This decreasing of l_c/l_o is more remarkable as f_X becomes smaller, and it is seen that the position of the tooth bearing changes from the heel to the toe according to the values of f_X and f_Y .

The tooth bearing width h_a/h_{ao} in the tooth profile direction shows about the same tendency regardless of the change of f_Z , and h_a/h_{ao} is within the range of 30~70% for all of f_X and f_Y . The tooth bearing position h_c/h_{co} in the tooth profile direction has a tendency to decrease with an increase of f_Y or f_X , which means that the tooth bearing moves from the tooth root to the tooth tip. This tendency and the value of h_c/h_{co} are about the same for all of f_Z . Thus, it is found

that the tooth bearing in the tooth profile direction is hardly influenced by f_Z .

Figure 11 shows the relation between the assembly error and l_a/l_o , l_c/l_o , h_a/h_{ao} , h_c/h_{co} for Reinecker type straight bevel gear. It is evident from this figure that all of them are nearly similar to those for Gleason type straight bevel gear. Figure 12 shows the calculated results (solid lines) of l_a/l_o , l_c/l_o , h_a/h_{ao} and h_c/h_{co} at $f_Z=0$ mm for Reinecker type straight bevel gear in comparison with their experimental results (broken lines). The calculated results of l_a/l_o , l_c/l_o and h_c/h_{co} are in good agreement with the experimental results. The calculated result of h_a/h_{ao} shows a peak value near 100% at each f_X , but the peak value

(a) $f_Z=-0.4$ mm (b) $f_Z=0.0$ mm (c) $f_Z=0.4$ mm

Fig.11 Relation between assembly error and l_a/l_o , l_c/l_o , h_a/h_{ao} , h_c/h_{co} (Reinecker type straight bevel gear, $f_Z=0^\circ$)

(c) h_a/h_{ao} (d) h_c/h_{co}

Fig.12 Comparison between calculated and experimental results (Reinecker type straight bevel gear, $f_Z=0$ mm, $f_Z=0^\circ$)

of h_a/h_{ao} obtained from the experiment is as small as 60~70%. This may be explained from the fact that the tooth profile error of bevel gear is not taken into consideration in calculation, though a real gear has a concave error at the tooth tip or tooth root as shown in Fig.3.

3.1.2 Tooth bearing of skew bevel gear

Figure 13 shows the tooth bearing pattern for Reinecker type skew bevel gear, and Fig.14 shows the relation between the numerical values of l_a/l_o , l_c/l_o , h_a/h_{ao} , h_c/h_{co} and the assembly error. It is seen from Fig.14 that the tooth bearing length l_a/l_o in the tooth trace direction at $f_z = 0.4$ mm increases initially and approaches 100% as f_x decreases from a positive value, but l_a/l_o decreases again with a further decrease of f_x . When f_x is 0.55 mm, l_a/l_o at $f_z = 0$ mm is nearly 100% regardless of the value of f_y and shows a complete tooth bearing along the tooth trace. And as f_x increases or decreases more than 0.55 mm, l_a/l_o decreases rapidly to almost 20% and

Fig.13 Change of tooth bearing of Reinecker type skew bevel gear ($f_z = 0^\circ$)

Fig.14 Relation between assembly error and l_a/l_o , l_c/l_o , h_a/h_{ao} , h_c/h_{co} (Reinecker type skew bevel gear, $f_z = 0^\circ$)

the tooth bearing in the tooth trace direction becomes incomplete. In the case of $f_z = -0.4$ mm, a complete tooth bearing along the tooth trace is obtained at $f_x = 0.85$ mm, but when f_x changes from this value, l_a/l_o becomes nearly 10% for all of f_y and the tooth bearing in the tooth trace direction becomes incomplete. As above-mentioned, the value of f_x at which the tooth bearing in the tooth trace direction for Reinecker type skew bevel gear becomes complete is limited to a certain value, and though the tooth bearing length along the tooth trace is influenced by f_y at a transition period from the incomplete condition to the complete condition of the tooth bearing in the tooth trace direction, f_y has little effect on it except in this period. This is one of the remarkable characteristics of the tooth bearing in the skew bevel gear, which is different from the characteristic of the tooth bearing in the tooth trace direction for the straight bevel gear as shown in Fig.10 or 11.

In this experiment, the tooth bearing of the skew bevel gear is measured on the right tooth flank of driving gear by pairing the driving gear of left-hand teeth with the driven gear of right-hand teeth. As inferred from the above-mentioned experimental result, however, if the tooth bearing on the right tooth flank is measured with a combination of the helix directions different from this experiment, namely, by pairing the driving gear of right-hand teeth with the driven gear of left-hand teeth, the tooth bearing in the tooth trace direction may be influenced by f_y and f_z , but f_x may have little effect on it, contrary to the above experimental result.

The influences of f_x , f_y and f_z on h_a/h_{ao} and h_c/h_{co} of the tooth bearing in the tooth profile direction for Reinecker type skew bevel gear show a similar tendency to those for the straight bevel gear shown in Fig.10 or 11. Thus, the difference in the shapes of the skew and straight bevel gears has little effect on the tooth bearing in the tooth profile di-

Fig.15 Comparison between calculated and experimental results (Reinecker type skew bevel gear, $f_z = 0$ mm, $f_z = 0^\circ$)

rection. Figure 15 shows comparison between the calculated result and the experimental result of l_a/l_o , l_c/l_o , h_a/h_{ao} and h_c/h_{co} for Reinecker type skew bevel gear. It is found from this figure that the two are in good agreement with each other for all of l_a/l_o , l_c/l_o , h_a/h_{ao} and h_c/h_{co} .

3.2 Relation between shaft angle error and tooth bearing

Since the tooth bearing tester used in this experiment has two dial gauges each on the two sides of the bearing stand (4) of the driving gear as shown in Fig. 6, it is possible to calculate the shaft angle error of the tooth bearing tester, after moving the bearing stand, from the readings of these dial gauges, but it is very difficult due to the structure of the tooth bearing tester to give the tester only the shaft angle error f_Z without giving the assembly errors f_X and f_Y . Therefore, in this section the tooth bearing influenced by only the shaft angle error is calculated using the theoretical calculation method for the tooth bearing described already in the 1st report(1), and the relation between the shaft angle error and the calculated tooth bearing is investigated.

Figure 16 shows the calculated results of the tooth bearing change with the shaft angle errors for Gleason and Reinecker type straight bevel gears and Reinecker type skew bevel gear. These were calculated on the assumption that all of bevel gears are ideal ones having no apex errors ($f_{a1}=f_{a2}=0$ mm, suffixes 1 and 2 stand for the driving and driven gears respectively). The positive and negative signs of the shaft angle error f_Z in Fig. 16 mean that the shaft angle of bevel gear is larger and smaller than the normal shaft angle (design value) respectively. It is found from Fig. 16 that the tooth bearings in the tooth trace direction for all of the bevel gears are always complete and do not change even if the shaft angle deviates from the normal value, namely, even if a shaft angle error exists. As shown in Fig. 16(a), the tooth bearing of Gleason type straight bevel gear occurs at the tooth root when the shaft angle is smaller than

Fig. 16 Relation between gear shaft angle error and tooth bearing ($f_{a1}=f_{a2}=0$ mm, $f_X=f_Y=f_Z=0$ mm)

the normal value (f_Z is minus), and the tooth bearing moves toward the tooth tip with an increasing shaft angle. In the case of Reinecker type straight bevel gear, however, the condition that no tooth bearing occurs at the tooth tip under minus f_Z , which is observed with Gleason type bevel gear, is not observed, though the area of tooth bearing decreases with an increasing shaft angle. This is attributed to the fact that the tooth profile of Gleason type bevel gear has a concave deviation from the spherical involute at the tooth tip and the tooth root(3), and the tooth bearing of Gleason type bevel gear is still incomplete even at the normal shaft angle ($f_Z=0^\circ$). It is clear from Fig. 16(c) that the tooth bearing of Reinecker type skew bevel gear occurs at the tooth tip if f_Z is minus and it does at the tooth root if f_Z is plus, and a complete tooth bearing appears only under the normal shaft angle. Thus, the tooth bearing of Reinecker type skew bevel gear shows a characteristic different from that of the straight bevel gear.

Figures 17 and 18 show the measured and calculated results of the tooth bearing which changes with both the assembly error and the shaft angle error for Gleason type and Reinecker type straight bevel gears. When the tooth bearing of Gleason type bevel gear for the assembly error $f_X=f_Y=0.1$ mm and the shaft angle error $f_Z=0^\circ$ is compared with that for $f_X=0.09$ mm, $f_Y=0.17$ mm and $f_Z=1.0^\circ$, and when the tooth bearing of Reinecker type bevel gear for $f_X=0$ mm, $f_Y=0.2$ mm and $f_Z=0^\circ$ is compared with that for $f_X=0.05$ mm, $f_Y=0.26$ mm and $f_Z=2.0^\circ$, it is found that the tooth bearing in the tooth trace direction is satisfactory in all the cases, and that there is little difference in the tooth bearing in the tooth trace direction between the two for each gear. In this case, the assembly errors f_X and f_Y of one for each gear are about the same as f_X and f_Y of the other, and only the shaft angle errors differ from each other. Therefore, it is

f_X mm	f_Y mm	f_Z mm	f_Z deg	Experimental	Theoretical
0.1	0.1	0	0	Tip Heel Toe Root	Heel Toe
0.09	0.17	0	1.0	Tip Heel Toe Root	Heel Toe
-0.02	0.63	0	1.9	Tip Heel Toe Root	Heel Toe

Fig. 17 Tooth bearing of Gleason type straight bevel gear

f_X mm	f_Y mm	f_Z mm	f_Z deg	Experimental	Theoretical
0	0.2	0	0	Tip Heel Toe Root	Heel Toe
0.04	0.07	0	1.0	Tip Heel Toe Root	Heel Toe
0.05	0.26	0	2.0	Tip Heel Toe Root	Heel Toe

Fig. 18 Tooth bearing of Reinecker type straight bevel gear

confirmed also experimentally that the tooth bearing in the tooth trace direction does not change even if a shaft angle error exists, which is clear already from the calculated result shown in Fig.16.

4. Conclusions

The results obtained in the present paper are summarized as follows:

(1) The method of measuring the tooth profile error of bevel gear which is represented as a deviation from the spherical involute is demonstrated using a base cone type bevel gear flank tester produced for this study. From the comparison of the tooth profile curve measured by this tester with the calculated curve, it is confirmed that the tooth profile error measured by the base cone type bevel gear flank tester shows an almost satisfactory result. In the case of using the base cone type bevel gear flank tester, however, the tooth profile should be measured carefully because a seeming tooth profile curve different from the real curve will be obtained if there is a manufacturing error in the length of the boss of bevel gear to which the base cone is attached.

(2) The apex error of bevel gear which is defined as a deviation of the apex after cutting or grinding from the design apex can be estimated from the measured tooth profile curve.

(3) When the size and the position of the measured tooth bearing of bevel gear are expressed with numerical values in each

direction of the tooth trace and the tooth profile, the characteristics of the tooth bearing can be known easily.

(4) The tooth bearing in the tooth trace direction for the skew bevel gear is influenced by the assembly errors f_X and f_Z , but f_Y has little effect on it. This can be attributed to only the condition of the helix direction of the bevel gear used in this experiment. And in the case of the helix direction different from this experiment, it can be inferred that the tooth bearing in the tooth trace direction will be influenced by f_Y and f_Z , but hardly by f_X .

(5) Even if the shaft angle error exists in the bevel gear, no change of the tooth bearing in the tooth trace direction appears on either the straight or skew bevel gears.

(6) In the case of Gleason type straight bevel gear, even if the assembly is normal, a complete tooth bearing can not be obtained with a concave deviation of its tooth profile from the spherical involute at the tooth tip and tooth root.

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