Improvement of the Energy Method Using Finite Element

Division and Application to Flat-rolling*

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A combined method of the usual energy method with the rigid plastic finite element method (FEM) is proposed. In this method a velocity field is expressed by a linear combination of a number of simple velocity fields and the unknown coefficients of that expression are determined by minimizing the total energy dissipation. Improved points are as follows: (i) difficulty in introducing a suitable velocity field in the usual energy method is reduced to a great extent, (ii) computation time is shortened by applying a technique in FEM and limitation of degree of freedom in the velocity field in the energy method is removed, and (iii) main part of program of computation is made common to general plastic workings.

This method is applied to the analysis of flat-rolling of a bar, and it is found that rolling properties and deformation agree well with experimental ones.

Key Words: Plasticity, Forming, Energy Method, FEM, Rolling, Three Dimensional Deformation

1. Introduction

The energy method(1) and the rigid plastic finite element method (FEM)(2)(3) seem to be the most effective methods of analysis of a large, three-dimensional, steady, plastic deformation. However, some problems still remain in these methods. For example, in the analysis of a three dimensional deformation by the rigid plastic FEM, the number of elements is limited by the capacity of the memory of a computer and by the length of the computation time, and sufficient number of elements for precise calculation cannot be used for practical purpose.

On the other hand, recent analysis concerning flat rolling of a bar, etc.(4)-(8) proves that the energy method is effective for analysis of a rather, simple steady deformation. However, when deformation becomes complicated, the following problems arise: (i) a suitable velocity field must be assumed with respect to each plastic working, (ii) assumption of the velocity field satisfying the condition of volume constancy for a complicated deformation is not easy, (iii) computer programs for calculating strain rate, total energy dissipation, etc. must be made with respect to each problem and (iv) the direct search method must be applied to

minimization of the functional (total energy dissipation), because expressions of velocity fields are usually considerably complicated. Thus long computation time is needed and the degree of freedom in the velocity field is limited to within 5-6.

In this paper, a combined method of the energy method with the rigid plastic FEM is proposed. First, the velocity field is expressed by a linear combination of simple velocity fields(9), where coefficients in that expression are unknown constants. Then these constants are determined by the condition of minimizing the functional Φ . Here, some expressions in FEM are applied to calculation of strain-rate, integration of energy dissipation in plastic deformation, friction, etc.

2. Fundamental Equations and Method of Analysis

2.1 Fundamental equations

Flat rolling of a bar with rectangular cross section will be dealt with. It is shown in Fig.1, where one fourth of a billet is taken because of its symmetry. Cartesian coordinates x , y , z are set in the rolling direction, the width direction and the direction of reduction, respectively. Origin is

Fig.1 Coordinates and notations

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located in the cross section at the entry of roll bite.

The variational principle for rigid plastic solids(10) is expressed as follows:

$$\left. \begin{aligned} 2T\omega &\leq \Phi \\ \Phi &= \int_V \sqrt{3} k \dot{\epsilon}_{eq} dV + \int_{\Gamma^*} k \Delta v_k^* d\Gamma^* \\ &+ \int_{S_c} \tau_f \Delta v_k^* dS_c + Q_B v_B^* - Q_F v_F^* \end{aligned} \right\} \dots\dots(1)$$

where

T : true torque of upper and lower rolls

ω : angular velocity of upper and lower rolls

v_A^*, v_B^* : velocities of material at exit and entry of roll-bite

τ_f : frictional stress on roll surface

V, S_c, Γ^* : volume of material in plastic state, contact surface with rolls and interface on which discontinuity of tangential velocity arises, respectively

k : suffixes indicating kinematically admissible velocity fields

k : shearing yield stress

Δv_k^* : amount of discontinuity of velocity

$\dot{\epsilon}_{eq}$: equivalent plastic strain rate

Material is assumed to be a rigid-plastic, Mises' solid.

Let us divide plastic region into elements and represent a velocity field by values of velocity at nodal points as is done in FEM. Each velocity at a nodal point has three components. So the total number of velocity components is $3N$, where N is the number of nodes. These components are expressed by a vector v^* . Then the second expression in Eq.(1) may be written by $\Phi(v^*)$. Suffixes will be omitted in the following discussions.

2.2 Method of analysis

2.2.1 Rigid plastic FEM We will first explain the rigid plastic FEM briefly in order to make the difference between FEM and the authors' method clear.

The condition of minimization of

$$\frac{\partial \Phi}{\partial v} = 0 \dots\dots\dots(2)$$

is a non-linear equation. This will be solved by Newton-Raphson method. Let us assume an approximate solution v_0 and express a true solution as follows:

$$v = v_0 + \delta v \dots\dots\dots(3)$$

Then

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial v} \Big|_v &= \frac{\partial \Phi}{\partial v} \Big|_{v_0} + \frac{\partial^2 \Phi}{\partial v^2} \Big|_{v_0} \cdot \delta v \\ &= -R + S \delta v \end{aligned} \right\} \dots\dots\dots(4)$$

where

$$\frac{\partial^2 \Phi}{\partial v^2} \Big|_{v_0} = S, \quad \frac{\partial \Phi}{\partial v} \Big|_{v_0} = -R \dots\dots\dots(4)'$$

S is a matrix which has $3N$ rows and $3N$ columns. R is a vector with $3N$ components.

From Eqs.(2) and (4), we have

$$S \delta v = R \dots\dots\dots(5)$$

In the rigid plastic FEM, Eq.(5) is solved for δv and an improved solution

$v_0 + \delta v$ is obtained. Such procedure is iterated. Here, velocity fields v_0 , δv are kinematically admissible ones. So we have

$$\left. \begin{aligned} \dot{\epsilon}_{ii} &= 0 && \text{in plastic region } V \\ \nu_r \cdot \delta v &= 0 && \text{on interface at which a} \\ &&& \text{discontinuity of tangential} \\ &&& \text{velocity arises} \\ \nu_s \cdot \delta v &= 0 && \text{on contact surface with} \\ &&& \text{rolls} \end{aligned} \right\}$$

where $\dots\dots\dots(6)$

$\dot{\epsilon}_{ii}$: volumetric strain rate

δv : amount of velocity discontinuity on Γ and S_c

ν_r, ν_s : normal vector to Γ and S_c

2.2.2 Proposal of new method of analysis Let us assume that a number of simple, kinematically admissible velocity fields $v^{(1)}, v^{(2)}, \dots, v^{(q)}$ are given. We call them fundamental velocity fields and express the velocity field of solution v as follows:

$$v = a_1 v^{(1)} + \dots + a_q v^{(q)}$$

$$= (v^{(1)} \dots v^{(q)}) \begin{Bmatrix} a_1 \\ \vdots \\ a_q \end{Bmatrix}$$

In another expression

$$\begin{Bmatrix} v_1 \\ \vdots \\ v_n \end{Bmatrix} = \begin{Bmatrix} v_{11} & \dots & v_{1q} \\ \vdots & & \vdots \\ v_{n1} & \dots & v_{nq} \end{Bmatrix} \begin{Bmatrix} a_1 \\ \vdots \\ a_q \end{Bmatrix} \dots\dots\dots(7)$$

$$v = Va \dots\dots\dots(7)'$$

where

V : a matrix with $n \times q$ components; $n = 3N$,

N : number of nodes

q : number of fundamental velocity fields

If we express using variables with suffixes, we get

$$v_j = V_{ji} a_i \dots\dots\dots(7)''$$

$$\frac{\partial v_j}{\partial a_i} = V_{ji} \quad (= v_{ji}) \dots\dots\dots(7)'''$$

Unknown constants $a_1, a_2, \dots, a_q$ will be obtained from Eq.(8):

$$\frac{\partial \Phi}{\partial a_i} = \frac{\partial \Phi}{\partial v_j} \cdot \frac{\partial v_j}{\partial a_i} = 0 \dots\dots\dots(8)$$

In solving Eq.(8), we will express a as follows:

$$a = a_0 + \delta a \dots\dots\dots(9)$$

a_0 : approximate solution

and, by substituting Eq.(9) into Eq.(7)', we obtain

$$v = v_0 + \delta v \dots\dots\dots(10)$$

where

$$v_0 = Va_0, \quad \delta v = V \delta a \dots\dots\dots(11)$$

V : Eq.(7)'''

The second of Eq.(11) is expressed as follows:

$$\delta v_k = V_{ki} \delta a_i \dots\dots\dots(11)'$$

From Eq.(4)

$$\frac{\partial \Phi}{\partial v_j} = -R_j + S_{jk} \delta v_k \dots\dots\dots(12)$$

By substituting Eqs.(11)' and (12) into Eq.(8)', we obtain

$$V_{ji}S_{jk}V_{kl}\delta a_l = V_{ji}R_j \dots\dots\dots(13)$$

or

$$V^T S V \delta a = V^T R \dots\dots\dots(13)'$$

where V^T is the transpose of V . This is the final equation to be solved in the authors' method.

If velocity components at nodes are all chosen as fundamental velocity fields in the authors' method, V becomes a unit matrix ($q=3N$), and Eq.(13)' agrees with Eq.(5).

2.2.3 Comparison of characteristics of two methods Let us discuss about the number of data to be memorized in a computer. The matrix S has $(3N)^2$ components, whereas the matrix V has $3N \times q$ components. In the authors' method, $V^T S V$ must be memorized. But the number of components of it is only q^2 and it is much less than that of S , which must be memorized in FEM. It means that memory area in a computer is saved in the authors' method, though rigorous comparison is not so simple because saving of memory area is made by the use of a band matrix in FEM. From discussions above, it may be concluded that the authors' method becomes effective when the number of fundamental velocity fields (q) is not large.

2.2.4 Condition of steady deformation

In a steady deformation we are dealing with, the shape of a deforming material (shape of stress-free, side surface) is not known and it is presumed.

Then δv is obtained from Eqs.(13)' and (11). If the shape is deformed by the velocity field $v_0 + \delta v$, the shape is updated and calculation is iterated. This procedure yields a solution of the steady deformation (refer to Reference (5)).

2.3 Procedure of introducing admissible velocity fields

We must provide many, kinematically admissible velocity fields as fundamental velocity fields in Eq.(7), but this is not so easy. So two components of velocity (x and z components) are assumed and the third component (y component) is calculated numerically by the equation of volume constancy.

Let us explain with reference to an example of a flat rolling. We will consider elements with marks $\circ$ in Fig.2 at first and will express the velocity at nodes 1-5 by a polynomial of z :

$$v_y = A_1 + A_2 z^2 + A_3 z^3 + A_4 z^4 \quad (z \geq 0) \dots\dots\dots(14)$$

Coefficients $A_1, \dots, A_4$ are determined by the condition of volume constancy at the centers of those four elements. Then, velocity at nodes 6-10 is determined by the condition of volume constancy at elements in the neighboring column.

Velocities at nodes on all sections in a billet are obtained from the exit section to the entry one successively. When the number of elements in z direction is j , polynomials of j degree are used, instead of Eq.(14).

In this calculation the number of

Fig.2 Determination of v_y by the condition of volume constancy
(a) Longitudinal section, (b) Cross section, (c) Distribution of v_y

data to be memorized in a computer is quite small.

2.4 Elimination of singularity in functional

Chen and Kobayashi(11) pointed out the singularity of functional due to the discontinuity of frictional stress at the neutral point in the analysis of ring compression by the rigid plastic FEM and suggested a solution to the problem of the singularity. A similar problem arises concerning the velocity discontinuity at the entry section in the flat-rolling in this paper. For this problem, another simple method will be proposed.

Let the amount of contribution of the velocity discontinuity at an integration point to the functional Φ be designated by Φ_1 . It is expressed as follows:

$$\Phi_1 = B \sqrt{(\Delta v_{yP})^2 + (\Delta v_{zP})^2} \dots\dots\dots(15)$$

where

$\Delta v_{yP}, \Delta v_{zP}$: y and z components of velocity discontinuity at an integration point P

B : constant

Derivatives of Φ_1 with respect to Δv_{yP} and Δv_{zP} vary steeply in the neighborhood of the point $\Delta v_{yP} = \Delta v_{zP} = 0$. So, we cannot get convergence of the solution by Newton-Raphson method, because smoothness of derivatives up to the second order is needed.

Then we will introduce Δv^* :

$$\Delta v^* = \sqrt{(\Delta v_{yP})^2 + (\Delta v_{zP})^2} / U \dots\dots\dots(16)$$

U : circumferential velocity of roll surface

and modify Φ_1 as follows:

$$\left. \begin{aligned} \Phi_1 &= B' \Delta v^* && \text{when } \Delta v^* \geq \delta \\ \Phi_1 &= B' (\Delta v^{*2} / 2\delta + \delta / 2) && \text{when } \Delta v^* < \delta \end{aligned} \right\} \dots\dots(17)$$

B' : constant ($=BU$)

Values of Φ_1 before and after modification are indicated by solid and broken lines in Fig.3, respectively, where Δv_{yP} is set at zero for the sake of simplicity.

Results of numerical calculation at the standard rolling condition (refer to Table 1) are plotted in Fig.4, where δ is changed. We understand that the rolling properties change little and that convergence is rather well, if we set $\delta = 0.01$. So we choose this value in the following calculation.

A similar modification is made concerning the neutral point.

Fig.3 Modification of singularity in functional ϕ_1 (in the situation of $\Delta v_{yP}=0$)

Table 1 Standard rolling condition

Roll radius	$R/h_B = 20$
Reduction	$r = 0.4$
Width/high ratio of material	$b_B/h_B = 1$
Friction ratio	$m = 1$

Fig.4 Influence of range of modification δ

3. Analysis of Flat-rolling of Bar

3.1 Definition of rolling property variables

We will define non-dimensional variables. Notations shown in Fig.1 will be adopted.

Width spread: $w = (b_F - b_B)/b_B$

$2b_F$: mean width at exit

Barrel of side surface: $\Delta b/h_F$

Warping of cross section: u_h/h_F

u_h : amount of warping difference of x coordinates of points 1 and 5 after deformation (see Fig.5)

Neutral point: x_N/l

x_N : x coordinate at which x component of velocity of material, relative to roll surface, vanishes

Forward slip: $\xi = (v_F - U)/U$

v_F : velocity of material at exit

U : circumferential velocity of roll surface

Roll torque: $T^* = \Phi / (R\omega \cdot 4b_B h_B \cdot \sqrt{3}k)$

Reduction: $r = (h_B - h_F)/h_B$

Friction factor: $m = \tau_r/k$

3.2 Preliminary discussions

Some numerical calculations will be done and appropriate numbers of elements,

Fig.5 Various divisions into elements at entry of roll-bite (M_s : number of singular elements)

Fig.6 Effect of division into elements at entry of roll-bite ($b_B/h_B=1, r=0.4, m=1$)

integral points, etc. will be determined. Here, velocity field (iv), which will be discussed in 3.3 is used. Rolling condition is shown in Table 1.

3.2.1 Element at entry of roll-bite

We will use a linear hexahedral, isoparametric element. At the entry of roll-bite, the velocity changes rapidly. Then the element will be modified so that one plane may be reduced to a line and that the element may become a triangular cylinder as is shown in Figs.5(b),(c). Numerical calculations of rolling will be done in the following three situations: $M_s = 0, 1$ and 2 , which correspond to Figs.5(a), (b) and (c), respectively. Here, discontinuity of tangential velocity at the cross section at the entry of roll-bite is admitted.

Results are shown in Fig.6. Abscissas are roll radii R in Figs.6(a)-(f). Dotted lines, broken lines and dot-dash lines correspond to cases of $M_s = 0$,

Fig.7 Effect of number of integration points

1 and 2, respectively. From these figures we see that difference between those lines is little. However, amounts of velocity discontinuity in the cases of $M_s = 1$ and 2 are smaller than that in the case of $M_s = 0$ and the values of functional (or roll torque) in the former cases are also smaller, exactly speaking. Then we will set $M_s = 1$ in the following analyses.

3.2.2 Number of integration points The following three cases will be examined: number of integration points in an element $\bar{N} = 1, 8$ and an intermediate value, respectively. The intermediate value means that $\bar{N} = 8$ in elements in the first and the second columns at the entrance (elements in left-hand side of interface 11-12-... 15 in Fig.5(b)) and $\bar{N} = 1$ in other elements. Results of numerical calculation of the three cases are shown in Figs.7(a) and (b), where those in the intermediate case coincide with those of $\bar{N} = 8$. From Fig.7, we know there is only small difference between them. Computation time is 15, 30 and 20 sec. by the computer M280H in cases of $\bar{N} = 1, 8$ and the intermediate value, respectively. Consequently, we use $\bar{N} = 1$ in the following analyses.

3.2.3 Number of elements Three cases ①, ② and ③ are calculated, where number of elements in one fourth of a billet $M = 4 \times 1 \times 2, 8 \times 2 \times 4$ and $12 \times 3 \times 6$. Results are shown in Fig.8. Consequently, we will adopt the element division of $8 \times 2 \times 4$ in the following.

3.3 Fundamental velocity fields

Several cases of different combinations of fundamental velocity fields will be examined. Number of fundamental velocity fields means degree of freedom of the velocity field representing the deformation under consideration and it corresponds to q in Eq.(7).

(i) Velocity field of $q=3$ Let us consider the velocity fields of plane strain rolling without width spread and the velocity field in which only width spread occurs and no elongation in rolling direction does, as in the previous paper. Then x and z components of velocity are expressed as follows, where those velocities are identified by suffixes I and II.

$$\left. \begin{aligned} v_{xI} &= v_B \cdot h_B/h, & v_{xII} &= v_B \cdot h_B h'/h^2 \cdot z \\ v_{zI} &= v_B, & v_{zII} &= v_B \cdot h'/h \cdot z \end{aligned} \right\} \dots (18)$$

Fig.8 Effect of number of elements

where

$2h$: thickness of material at x ;
 $h' = dh/dx$

U : velocity at entry of roll-bite

We will combine them:

$$\left. \begin{aligned} v_x &= \alpha v_{xI} + (1-\alpha) v_{xII} \\ v_z &= \alpha v_{zI} + (1-\alpha) v_{zII} \end{aligned} \right\} \dots (19)$$

$$\alpha = m_0 - m_1(1-x/l)^2$$

where

l : projected contact length with roll

m_0, m_1 : unknown constants

From the condition of volume constancy

$$\partial v_x / \partial x + \partial v_y / \partial y + \partial v_z / \partial z = 0 \dots (20)$$

we obtain y component of a velocity

$$v_y = -(v_x h') / h \cdot y \dots (19)'$$

where $()' = d()/dx$. From Eqs.(19) and (19)', the velocity field can be expressed as follows:

$$v = a_1 v^{(1)} + a_2 v^{(2)} + a_3 v^{(3)} \dots (21)$$

where

$$a_1 = m_0 \frac{v_B}{U}, \quad a_2 = m_1 \frac{v_B}{U}, \quad a_3 = \frac{v_B}{U} \dots (22)$$

$$v_x^{(1)} = \left(\frac{h_B}{h} - 1 \right) U, \quad v_x^{(2)} = \left(1 - \frac{x}{l} \right)^2 \left(\frac{h_B}{h} - 1 \right) U,$$

$$v_x^{(3)} = U$$

$$\dots (23)$$

$$v_z^{(i)} = v_x^{(i)} \frac{h'}{h} z, \quad v_y^{(i)} = - \frac{(v_x^{(i)} h')}{h} y, \quad i=1, 2, 3$$

$$\dots (23)'$$

U : circumferential velocity of roll surface

We will choose $v^{(1)}, v^{(2)}$ and $v^{(3)}$ in Eq.(21) as fundamental velocity fields.

(ii) Velocity field of $q=5$ In addition to the three velocity fields, mentioned above, two more velocity fields will be introduced. One is a field representing warping of cross section of a billet:

$$v_x^{(4)} = g(x) \left(\frac{z^2}{h_B^2} - \frac{h^2}{3h_B^2} \right) U, \quad v_z^{(4)} = v_x^{(4)} \frac{h'}{h} z \dots (24)$$

$$g(x) = - \frac{x - x_N}{l} \cdot \frac{x}{l} \cdot \left(1 - \frac{x}{l} \right)^{G_2} \dots (25)$$

G_2 : constant, depending on rolling condition ($=1.9-6.5$)(5)

and the other is a field representing barreling of side surface of a billet:

Fig.9 Effect of number of fundamental velocity fields

$$v_x^{(5)}=0, \quad v_z^{(5)}=\left(\frac{z}{h}-\frac{z^3}{h^3}\right)h'U \quad (26)$$

Thus we obtain

$$v = a_1 v^{(1)} + a_2 v^{(2)} + a_3 v^{(3)} + a_4 v^{(4)} + a_5 v^{(5)} \quad (27)$$

(ii)' Velocity field of $q=5$ (another field) Eq.(28) will be adopted, instead of Eq.(24).

$$\left. \begin{aligned} v_x^{(4)} &= g(x) \left(\frac{z^2}{h^2} - \frac{h^2}{3h^2} \right) U \\ v_y^{(4)} &= 0, \quad v_z^{(4)} = - \int_0^z \frac{\partial v_x^{(4)}}{\partial x} dz \end{aligned} \right\} \quad (28)$$

Other basic velocity fields are the same as those in (ii).

(iii) Velocity field of $q=18$ We will use the three velocity fields $v^{(1)}$, $v^{(2)}$ and $v^{(3)}$ in Eq.(21) as fundamental velocity fields first.

Now let us consider a velocity field representing warping of a "cross section", which means a curved section, such as the section 11-12-...15 in Fig.5(b). Here, x and z components on the "cross section" under consideration are expressed by

$$v_x^{(i)} = \left(\frac{z^2}{h^2} - \frac{h^2}{3h^2} \right) U, \quad v_z^{(i)} = v_x^{(i)} \frac{h'}{h} z \quad (29)$$

and those on other "cross section" vanish.

We introduce velocity fields of this type with respect to all the "cross sections" except those at the entry and the exit of roll-bite. Then we get seven fundamental velocity fields $v^{(4)}, \dots, v^{(10)}$.

In addition, we will introduce eight velocity fields $v^{(11)}, \dots, v^{(18)}$ representing

barreling, or non-uniform flow in y direction. Here, x and z components of velocity on a "cross section" under consideration are expressed by

$$v_x^{(i)}=0, \quad v_z^{(i)} = \left(\frac{z}{h} - \frac{z^3}{h^3} \right) h' U \quad (30)$$

and the other components vanish.

Thus, we get a velocity field from Eq.(7), where $q=18$. In this velocity field no special assumption is made, whereas the function $g(x)$ must be assumed in the velocity field (ii).

We get another velocity field (iii)' by modifying the field (iii) in a similar way to that explained in (ii)'.

(iv) Velocity field of $q=27$ In addition to $v^{(1)}$, $v^{(2)}$ and $v^{(3)}$ in Eq.(21), velocity fields, which are represented by v_x and v_z at nodes, respectively, are used as fundamental velocity fields. Those nodes are chosen on the first and the second "cross sections" (1-2-...5 and 6-7-...10 in Fig.5(b)). Details are discussed in the foot-note*.

We get another velocity field (iv)' in a similar way to that explained in (ii)'.

(v) Velocity field of $q=33$ The velocity field (iv) is modified, so that velocity fields represented by v_x and v_z at nodes on three "cross sections" at the entry, respectively, may be used as fundamental velocity fields.

(vi) Velocity field of $q=39$ In the explanation in (v), four "cross sections" are considered.

3.4 Discussions about results of analyses by the use of various velocity fields

Results of numerical calculation of a flat-rolling are shown in Fig.9, where those by the use of velocity fields (i)-(vi) are plotted by solid lines. It is found that they do not change so much, when degree of freedom in a velocity field becomes greater than that in the field (iv).

Results of using velocity fields

* Only one component of v_x or v_z at one node is set at non-zero value, for example U , and all other components are set at zero, where y component v_y is calculated from the condition of volume constancy. This type of velocity field is considered concerning each component of v_x and v_z at nodes 1, ... 10 in Fig.5(b). Then we get 20 velocity fields. Among them only 12 velocity fields are valid because some of them are fixed by boundary conditions, v_x at nodes 1, ... 5 and v_z at nodes 5, 6, 10.

Further, we use velocity fields in (iii), among which three fields are removed — two fields representing distortion of the first and the second "cross section" and one field representing a barreling in the second "cross section". Then 15 velocity fields remain.

Consequently, 27 velocity fields are used as fundamental velocity fields.

Fig.10 Distribution of equivalent strain rates in longitudinal section ($\dot{\epsilon}_{eq} \cdot h_B / U \times 10^2$) ($R/h_B=20, b_B/h_B=1, m=1, y=0$)

Fig.11 Distribution of equivalent strain rates in cross section (hatched part: high strain rate) (a) Standard velocity field (iv) (b) high degree of freedom in v_v

(i)-(iv)' are plotted by broken lines. Here, $G_2 = 2.4$ in Eq.(25) in these cases(5). When degree of freedom q increases, the results tend to agree with the former ones.

Consequently, the velocity field (iv) is found to be the most effective, because the result is reliable and computation time is not so long. So we will use this velocity field in the following.

In the examples, mentioned above, deformation in the cross section is rather simple. So an example of using a more elaborate velocity field is supplemented. In (iv), one fundamental velocity field (Eq.(29)) is adopted concerning one "cross section", whereas two velocity fields are adopted this time. It means that a fundamental velocity field is adopted concerning each of columns of nodes, 1-2-... 5 and 6-7-...10 in Fig.2(b), respectively. Results are shown by the mark $\square$ in Fig.9. They agree with those mentioned above.

3.5 Distribution of strain rates

3.5.1 Strain rates in longitudinal section In the velocity field in 3.3(iv), the distribution of strain rates

Fig.12 Effect of friction factor m ($R/h_B=20, b_B/h_B=1, r=0.4$)

Fig.13 Effect of reduction r ($R/h_B=20, b_B/h_B=1, m=1$)

Fig.14 Effect of width of billet ($R/h_B=20, r=0.4, m=1$)

in longitudinal section is expressed rather well, because a considerable degree of freedom is included, especially at nodes in the neighborhood of the entry. Distributions of equivalent plastic strain rates are shown in Figs.10(a) and (b), which correspond to $r=0.4$ and 0.3 , respectively (r : reduction). Other working conditions are the same as those in Table 1. They prove that the region of small strain rates at the entry extends as far as the part inside of roll-bite and that shearing strain at the line of velocity discontinuity γr is small ($\gamma r \lesssim 0.01$). Further, regions of small strain rates exist near contact surfaces with rolls. Such features agree with experimental ones well.

3.5.2 Distribution of strain rates in "cross section" The distribution of strain rates in "cross section" can be expressed only roughly, because rather small degree of freedom is included in the deformation in "cross section". So we will divide elements in cross sections into two groups depending on the magnitude of strain rate. Results are shown in Figs.11(a) and (b), which correspond to the velocity field (iv) and the supplemented one (the mark $\square$ in

Fig.9) in 3.3, respectively. We understand that the result in Fig.11(b) agrees with experimental one better, because the degree of freedom in the velocity field in "cross section" is higher in that case.

3.6 Rolling properties

Fundamental rolling properties will be obtained, with use of the velocity field (iv) in 3.3. Effect of roll radius is shown by broken lines in Fig.6, already. Results in the previous paper(5) are plotted by two dots and dash lines for reference. The velocity field includes the one in the previous paper approximately. Results in this paper are not so different from those in the previous paper, but the former is more accurate than the latter, because degree of freedom in the velocity field in the former is much greater and the velocity field in the former includes the one in the latter approximately. Experimental results by Yanagimoto(14) are plotted by two dots-dash lines in Fig.6 (parameters $\beta=1.02$ and $\mu=0.38$ are used). The authors' analyses agree with them well.

Rolling properties are shown by solid lines in Figs.12, 13 and 14, where friction factor m , reduction r and width of a billet $2b_s$ are changed. Results in the previous paper are shown by two dots-dash lines in them. Both results agree with each other.

4. Conclusions

A new method of analysis of three dimensional steady plastic deformation was proposed and some applications of it to flat-rolling were shown. Results are as follows:

- (i) A combined method of the energy method with the rigid plastic FEM is proposed. In this method the velocity field is expressed by a linear combination of simple, known velocity fields and some expressions in FEM are applied to integration of functional (Φ) and to minimization of it. Consequently, the limitation of degree of freedom in the velocity field in the usual energy method is removed and the analysis of a rather complicated three dimensional deformation in rolling becomes possible.
- (ii) A method of constructing the velocity field in the analysis is shown. In short, rather simple velocity fields,

similar to those in the usual energy method are used in the region of monotonous distribution of strain rates and, in the region of steep variation of strain rate, many fundamental velocity fields are introduced and degree of freedom is increased in the velocity field.

(iii) The flat rolling of a bar was analysed by the use of the velocity fields, mentioned above, and it was found that rolling properties and distribution of strain rates agree with experimental ones well.

(iv) Rolling properties with respect to fundamental rolling conditions were obtained. Results approximately agree with those in the previous paper(5), but the former is superior to the latter in that no special assumption is needed in constructing a velocity field and that main part of the computer program can be used commonly in other problems.

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