Application of the Boundary Element Method to Threaded Connection*

(2nd Report, In case of Threaded Connection with Free Flat Boundaries)

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This paper describes the application of the boundary element method to the stress analysis of a threaded connection where there is a large flat unstressed surface, by using the Mindlin solution that enables us to solve the problem involving free flat boundaries without having to discretize those boundaries. The advantage of this method is in its simplicity of giving data as input for concrete problem undertaken by fastener designer in practice. This method make it possible to analyze the axi-symmetric and the three-dimensional problems of a bolted fastener with a contact surface and under service loading, by using the unit partitioning technique.

Key Words : Threaded Connection, B.E.M, Mindlin Solution, Free Boundary

1. Intriduction

As a new numerical method, the boundary element method has come to be widely used in the field of structural analysis. Unlike in the finite element method that is based on the discretization of the internal domain of the problem, the discretization process in the boundary element method involves only the boundary of the problem. The boundary element method not only saves excessive labour in the preparation of input data but also enables good prediction of displacement and stress. It is especially useful for the stress analysis of machine elements whose geometry and size are prescribed by standard.

The authors have presented a fundamental method for the boundary element analysis of threaded connections, and applied this method to the stress analyses of gears. Although the finite element method and the boundary element ones are difficult from each other in the formulation of equations, the system equations for the system have the same form of expressing the relationship between the nodal forces and displacements. It is quite easy to couple these equations. The advantage of the boundary element method is in its simplicity for users. It gives accurate solutions in spite of small data preparation. We can decrease the amount of data preparation in solving a specific problem if we choose the fundamental solutions appropriate for the problem. This paper describes an application of the boundary element method to the stress analysis of a threaded connection with free flat boundaries.

2. Analytical Method

Threaded connections are the most wide-

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ly used machine elements to fasten together various parts of an assembly. The fastened plate has a very simple contour and a flat unstressed surface in most cases. The boundary integration equations used in the boundary element method can be derived from the reciprocal theorem in the theories of elasticity. If we choose the fundamental solutions that automatically satisfy the boundary conditions of the problem, the data preparation necessary to solve the problem will be greatly decreased. In this paper the authors investigated a fundamental method for the boundary element analysis of threaded connections with free flat surface by using the Mindlin solutions.

The solution for a unit point load in a semi-infinite elastic medium was deduced by Mindlin and has widely used in the field of solid mechanics, because the ground is equivalent to a large flat unstressed surface. Although the Mindlin solution does not satisfy all free surfaces of a fastened plate automatically, the amount of data preparation will be considerably reduced comparing to the Kelvin solution.

The boundary integration equations used in the boundary element method are

Semi-infinite elastic medium

Fig.1 Semi-infinite elastic medium

$$c_{ki}u_i + \int_S p_{ki}^* u_i dS = \int_S u_i^* p_{ki} dS + \int_V u_i^* b_i dV \quad \dots\dots\dots(1)$$

where p_i and u_i are traction and displacement respectively. $\int_S dS$ is surface integral and $\int_V dV$ domain integral. u_i^* is the displacement at load point l , b_i is body force. In a general case the effect of body force is so small that we can ignore the term of b_i in Eq.(1). p_{ki}^* and u_{ki}^* are the traction and the displacement at the field point in the i direction due to a point load applied at l (the load point) in the k direction in a semi-infinite elastic medium, and they are identical with the Mindlin solutions. As the Mindlin solutions are expressed in a very complicated form, we write here the special case using $h=0$ (Fig.1) of the first solution that is written in a polar coordinate.

$$\left. \begin{aligned} U_r &= \frac{P}{4\pi G} \left\{ (1-2\nu) \frac{1}{r} \left(\frac{z}{R} - 1 \right) + \frac{r^2 z}{R^3} \right\} \\ U_z &= \frac{P}{4\pi G} \left\{ 2(1-\nu) \frac{1}{R} + \frac{z^2}{R^3} \right\} \\ \tau_{rr} &= \frac{P}{2\pi} \left\{ (1-2\nu) \frac{1}{r^2} \left(1 - \frac{z}{R} \right) - 3 \frac{r^2 z}{R^3} \right\} \\ \tau_{zz} &= -\frac{3P}{2\pi} \frac{z^2}{R^3} \\ \tau_{rz} &= (1-2\nu) \frac{P}{2\pi} \left(-\frac{1}{r^2} + \frac{z}{Rr^2} + \frac{z}{R^3} \right) \\ \tau_{rz} &= -\frac{3P}{2\pi} \frac{rz^2}{R^3} \end{aligned} \right\} \dots\dots(2)$$

where G and ν are the shear modulus and Poissons' ratio. After transforming Eq.(2) into an expression in terms of the Cartesian coordinates, we obtain U_i and τ_{ij} . The fundamental solutions for the semi-infinite case are given by $u_{ki}^* = U_i$ and $p_{ki}^* = n_j \tau_{ij}$. n_j is a component of the outward normal unit vector on the boundary surface. c_{ki}^* is the evaluation of the singular integration $\int_S p_{ki}^* dS$ at the load point.

In this study the magnitude of the point load is determined so as to give c_{ki}^* a constant value of 1.0 on a smooth surface ($P=1$ at $h=0$, and $P=2$ at $h=0$). We can solve a three-dimensional problem by using Eq(1). For an axi-symmetric problem, Eq(1) is transformed into an expression in cylindrical coordinates. Performing coordinate transformation, and neglecting the components associated with circumferential direction about its axis of symmetry gives.

$$\begin{aligned} & \begin{bmatrix} C_{11} & C_{13} \\ C_{31} & C_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} + \int_S \begin{bmatrix} p_{11}^* & p_{13}^* \\ p_{31}^* & p_{33}^* \end{bmatrix} \\ & \times \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_r \\ u_z \end{bmatrix} dS \\ & = \int_S \begin{bmatrix} u_{11}^* & u_{12}^* & u_{13}^* \\ u_{31}^* & u_{32}^* & u_{33}^* \end{bmatrix} \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_r \\ p_z \end{bmatrix} dS \quad \dots\dots\dots(3) \end{aligned}$$

The integration in Eq(3) can be evaluated by the line integral $\int_S dS = \int_r \int_\phi r / |J| d\phi dr$. $|J|$ is a Jacobian determinant corresponding to the transformation. ϕ is the angular position of a field point about x_2 axis from x_1 axis. As the load point lies on x_1 axis, $[u_1 \ u_2]^T$ can be written as $[u_r \ u_z]^T$. Eqs(1) and (3) are numerically

evaluated by using the Gauss quadrature or the process devised by Hammer.

Generally several types of fundamental solutions are available to solve a given problem. The procedure using the Mindlin solution involves long computational time because of its complexity. However, in the boundary element analysis, it is necessary to evaluate numerically the boundary integral for each load point in order to set up a system of linear equations. This means that Mindlin solution has great advantage in the analysis of threaded connection with free flat surfaces that need no discretization. It is one of the main advantages of the boundary element method that the time and the labour of data preparation can be saved.

This study presents a three-dimensional axi-symmetric procedure which makes it possible to analyze the threaded connections by using Eqs(1) and (3). This procedure is designed such that not only any favourite type of element among ones that are constant linear and quadratic can be used but also in the case of a three-dimensional problem, triangular and rectangular elements can be coupled. It is necessary to deal with the singularities at a corner of the boundary in a specific problem. However, the threaded connections with free surface do not involve any special procedure such as double nodes treatment about its boundary discontinuities because of a suitable mixture of tractions and displacement boundary conditions. In this paper, the unit partitioning method is applied to the threaded connections which consists of more than two bodies regarding it as a contact problem. The contact zone in the mating surfaces is found by an iterative process. The reason for applying the unit partitioning method to the contact problem is that the boundary conditions on the contact surface agree with the special conditions of inter-unit continuity, and this method provides the most efficient solution to the problem. If friction is taken into account the incremental and the iterative technique must be used for finding a contact area. However, in the three-dimensional axi-symmetric case, the procedure becomes more complicated since it has to be two-dimensional. For this reason and for the purpose of saving the long computational time, analyses are made here by using frictionless models of threaded connections.

3. Boundary Element Analyses of Threaded Connections

In the previous report, the fundamental method for the boundary element analysis of threaded connections is introduced in consideration of the threaded parts of a bolt and a nut. However, the practical and engineering approach often needs a simple model neglecting a locally disturbed flow of forces in threaded connection. Especially it is very important to investigate the effects of various types of external loads upon the sealing performance and the strength of threaded connection.

Although there have been many studies on threaded connections subjected to an external load using the finite element method or the theory of elasticity, their methods are not so convenient for designers of threaded connection to use who are not experts in numerical analyses. The fundamental method on the strength of materials is also very easy to use but the range of its applications is very narrow, and it is very difficult to simulate the actual behaviour of threaded connection. In this paper, the authors introduce a

Fig.2 Threaded Connection

a) rectangular plate

b) Circular plate

Fig.3 Discretization of the fastened plate

fundamental method for a concrete practical problem of a threaded connection subjected to an external load.

3.1 Three-dimensional analysis of threaded connection. Figure 2 shows the simplest model of threaded connection. The part to be connected is a thick rectangular plate and the bolt hole is made in the middle of the plate. Although the boundary element method is applied to the analysis of threaded connection, even when taking the free boundaries into account, the involved computational time is very long. For finding the fundamental behaviour of threaded connection in a simple way, the fastened plate is assumed to be subjected to a uniformly distributed load from the rigid bolt. Figure 3-(a) shows one fourth of a threaded connection modelled by quadratic elements with 15 elements and 64 nodes. The planes of symmetry are not discretized for reducing the amount of data preparation. Figure 3-(b) shows the case of a circular fastened plate modelled by quadratic elements with 12 element and 53 nodes. It should be noted that the upper surface of the fastened plate is not discretized. The process for applying the external load

Thickness : SH = 25 mm

a) Rectangular plate

Thickness : SH = 25 mm

b) Circular plate

Fig.4 Contact pressure distribution

necessitates a discretization of the area contacting with the bolt bearing surface because the load term in Eq.(1) has to be evaluated numerically. The sides of the fastened plate need discretization as the Mindlin solution does not satisfy all free boundaries of the plate. The analyses were made by mounting the fastened plate on a rigid foundation (or symmetric with respect to the mating surface) and fixing the bottom of the plate. In the process of finding a succession of solutions the iteration continues until the convergence is achieved by releasing the nodes when a negative traction is presented and coupling the nodes that overlap the foundation. Figure 4 shows the contact area attained in the model shown in Fig.3 when the load applied by bolt is 98 MPa. It is found from this result that the contact area remains in the vicinity of the bolt hole because of the deformation of the plate.

The geometries of the fastened plate do not have great effect upon the extent of the contact area. The extent of the contact area and the magnitude of the contact pressure agree with those obtained from an axis-symmetric analysis that will be discussed in the following section. Figure 3 shows the fastened plate on a rigid

foundation. However, as might be suspected, the analysis of a thick plate on an elastic foundation is suitable for some cases of threaded connections. As elastic foundation is equivalent to a semi-infinite medium, it is advantageous to use the Mindlin solution for the boundary element analysis of a threaded connection on an elastic foundation. Figure 5 shows a threaded connection on an elastic foundation modelled by quadratic elements with 17 elements and 78 nodes. As the region to be discretized on the elastic foundation is the mating surface and the discretization pattern is the same as that of interface of the plate, the surface of elastic foundation does not explicitly appear in Fig.5. Establishing and coupling two sets of linear equations for both of the fastened plate and the elastic foundation result in the unit partitioning procedure. The mating faces should be modelled with corresponding contact nodes and elements and they have the same length as the counterparts contacting with them. In this paper, the discretization of elastic foundation is schemed such that it can automatically done from the discretization pattern of the fastened plate.

For saving the required storage and computation effort that are greatly increased in the three dimensional analysis using the unit partitioning method, the bolt hole is neglected in the model and the boundaries are discretized by using the rectangular and/or the triangular elements.

Fig.5 Fastened plate on elastic foundation

Thickness : SH = 25 mm

Fig.6 Deformation of the fastened plate

Fig.7 Contact zone

Figure 6 shows a deformed state of the fastened plate when the load applied by bolt is 98 MPa. The region mating with the bolt bearing surface is largely deformed and the interface is tightly pressed in the vicinity of the bolt axis against the elastic foundation.

Figure 7 shows the effect of plate thickness upon the contact pressure distribution and the extent of contact area. Figure 7-(d) shows a comparison with the case of rigid foundation. In Fig.3-(a) it is clear that the contact area increases together with the thickness of the plate and the extent of contact area when the case with elastic foundation is considerably different from the case with rigid foundation. This arises from the fact that the mutual displacement due to the local deformation of a warped plate and the elastic foundation diminishes and makes the contact area increase whereas the plate's warp makes it separate from the rigid foundation.

3.2 Axi-symmetric analysis of threaded connection. In solving three dimensional problems by the boundary element method, even if the free boundaries are taken into account, we are faced with the task of solving a large set of linear equations. One of the simplest methods to investigate the threaded connection is to analyze the system using an axi-symmetric model that will be adequate and more economical. Figure 8 shows a fastened plate on a rigid foundation modelled by quadratic elements with 14 ele-

ment with 14 elements and 29 nodes. The fastened plate is assumed to be subjected to a uniformly distributed load from the rigid bolt. The iteration technique is used to find a succession of solutions.

The contact pressure distribution between the plate and the foundation plotted for three different thicknesses of the plate ($h=10, 25$ and 40 mm) is shown in Fig.9. The relationships between the thickness and the maximum pressure, and between the thickness and the contact length are shown in Fig.10. The contact length increases with thickness of the plate and the contact pressure distribution becomes uniform. For designing threaded connections, it is presumed that the force is transmitted from the bolt head and the nut to the fastened plates along cones of influence whose generatrices are inclined to the mating surface at 45° . However, the results obtained from this analysis show the smaller base of a cone inclined at 40° . It is also found that the diameter of the fastened plate does not have great effect on the contact length. When the stiffness of the bolt is taken into account, the elastic behaviour of a threaded connection can be analyzed with high accuracy.

Figure 12 shows a threaded connection modelled with quadratic elements that consists of a bolt and a clamped plate. It is not necessary to discretize the top of the bolt and the free surface of the plate. For the boundary condition the interface of the fastened plates is axially fixed and the

Fig.8 Axi-symmetric model

Fig.10 Effect of the plate thickness upon the contact length

Fig.9 Contact pressure distribution

Fig.11 Effect of the plate thickness upon the contact pressure

axial tension is caused by applying a forced displacement to the bottom of the bolt. A point load is considered by putting the body force equal to the Dirac delta function. The contact pressure distribution for $F = 0$ is shown in Fig.13.

In this paper the unit partitioning method is applied to the boundary element method for the calculation of the change in the axial tension and the contact pressure in the axi-symmetric model of threaded connection subjected to an external load. Figure 13 shows the change in the contact pressure caused by the external load in both cases of the boundary element analysis and the finite element one. The position of load point is at $r = 50$ mm on the top surface of the plate.

The contact zone when no load acts has approximately the same length as the case with $SH = 25$ mm. It is found from these results that the form of the load distribution on the bolt bearing surface and the diameter of the plate do not have great effect upon the contact length. As can be seen the finite element method and the boundary element method are in good agreement.

It is very important to investigate the effects of various types of external loads upon the strength of a threaded connection. So far there have been many studies on threaded connections subjected to an external load using the strength of materials. Although the fundamental method based on the strength of materials is very easy to use, the range of its applications is very narrow. In the previous paper, the authors have studied the application of the finite element method to a threaded connection subjected to an external load. Figure 14 compares the results of the finite element

method with those of the boundary element one. These results agree with those of the experiment performed by Yoshimoto et al. It is found from this figure that the axial tension does not show a remarkable increase against the external load. Although the deformation of the threaded part may also have an effect upon the elastic behaviour of the threaded connection under an arbitrary type of loading, the change in the axial tension under axi-symmetric loading is well simulated by a model without a threaded part.

Conclusions

This paper introduces the boundary element analysis of a threaded connection characterized by economic advantage and small volume of data required. The results obtained may be summarized as follows:

- 1) This study presents a new simple method for the analysis of threaded connection using the boundary element technique.
- 2) This method using Mindlin's solution makes it easy to save the labour in the

Fig.13 Contact pressure distribution

Fig.12 Boundary element model of the threaded connection.

Effects of subsequent service loading on bolted connection

Fig.14 Effects of subsequent service loading on bolted connection.

data preparation.

- 3) This paper describes a fundamental method for the three-dimensional axisymmetric analyses of threaded connections.
- 4) This method makes it possible to analyze the threaded connections subjected to an external load.
- 5) By the unit partitioning method, the stress analyses of threaded connections are reduced to a simple problem of multiple elastic bodies with contact surfaces.

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