A Description of Bauschinger Curves Based
on the Random Barriers Theory*

by Yoichi OBATAYA** and Shinobu KOHNO***

A new approach to describing the Bauschinger curves for metals is presented on the concept of the field of the resisting force against the movement of dislocations. In a previous paper, it has been clarified that the cyclic plasticity model, proposed by the authors based on this concept, would be applicable to the quantitative descriptions of various cyclic deformation phenomena in the steady state, and the tensile stress-strain curves with a different type of yielding. In this paper, the Bauschinger curve can be expressed by applying this concept with some assumptions of the changes in the field of the resisting force caused by pre-loading. This has provided a satisfactory agreement with each experimental curve for a carbon steel and a brass with a wide range of strains.

Key Words: Plasticity, Bauschinger Effect, Dislocation, Field of Resisting Force

1. Introduction

Recently, the authors presented a cyclic plasticity model (1) based on the concept of the field of the resisting force in the random barriers theory (2) by Udoguchi and Okamura. In this model, the relation between the stress and the strain in any half cycle under the cyclic loading was described with three internal variables, that is, two variables related to the state of a force field and one variable related to the microscopic structure. And, it was experimentally clarified that the cyclic deformation behaviors of both carbon steel and brass having different characteristics of the slip system were able to be described in a unified form based on the change of the field of the resisting force. Moreover, it has been shown in the previous paper (3) that the descriptive method in this model would be applicable to describe the tensile stress-strain curves by considering the change of the internal structure variables depending on the strain history.

In this study, an equation to describe the Bauschinger curve in consideration of the change of the resisting force caused by pre-deformation is proposed and examined with the experimental curves of a carbon steel and a brass.

2. Description of Bauschinger Curve

In the random barriers theory, the field of the resisting force can be expressed by using the probability density function (hereinafter, simply referred to as density function) of the intensity of barriers. Therefore, in order to describe the Bauschinger curves by this theory, it is necessary to estimate the effect of pre-deformation on the characteristics of the change of the density function.

When the initial density function $h_0(y)$ of an annealed material is assumed to be expressed by the same function $w(|\tau|; \xi_0, \xi_b, \xi_d)^{1/y}$ on the tension side and the compression side as shown in Figure 1 (b)(i), the relation between the tensile stress $\sigma$ and the plastic strain $\varepsilon$ can be described by the following equation (3).

$$\int_{-\varepsilon}^{\varepsilon} A \text{d}\varepsilon = 2 \left(\frac{\sigma}{\xi_b}\right)^{1/y} - 1 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots About the NII-Electronic Library Service

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starting point of an arbitrary half cycle under the cyclic deformation was given by \( w(S;S_c;m) \), the plastic strain \( \varepsilon \) occurring in the cycle could be described by the following equation using the stress \( S_2 \) at the starting point (3).

\[
\int_{S_2}^{S} A d\varepsilon = \frac{(S/S_2)^{m_1}}{2} - \frac{(S/S_2)^{m_2}}{2}
\]  

(4)

Now, it is presumed that during the initial loading of the field of the resisting force changes to a field expressed by two density functions for different directions of movement of dislocations as shown in Figure 1(b) at the stress level \( \sigma_1 \). It is further assumed that the density function \( h_1(\sigma) \) for the fro-motion which regulates the Bauschinger curve is given by a composite Weibull distribution composed of \( h_2(\sigma) \) and \( h_2(\sigma) \) with use of the boundary stress \( \sigma_2 \) as follows.

\[
h(\sigma) = \begin{cases} h_1(\sigma) = w_1(\sigma) \; ; \; \sigma < \sigma_2 \\ h_2(\sigma) = w_2(\sigma;S_2;S) \; ; \; \sigma > \sigma_2 \end{cases}
\]  

(5)

If it is assumed that the ratio of the number of barriers exceeding the stress \( \sigma_2 \) to the total number of barriers estimated by the above two functions coincide with each other, the following relation is obtained.

\[
\left( \frac{\sigma_1}{\sigma_2} \right)^{m_1} = \left( \frac{\sigma_1}{\sigma_2} \right)^{m_2}
\]  

(6)

In this case, the relation between the stress \( S \) and the plastic strain \( \varepsilon \) in the region lower than the stress level \( \sigma_2 \) can be expressed as follows.

\[
\int_{S_2}^{S} A d\varepsilon = \left( \frac{S_2}{S} \right)^{m_1} - 1 \quad \cdots (7)
\]

If the strain \( \varepsilon \) in Equation (3) can be regarded as the accumulated strain, the Bauschinger curve in the region below the boundary stress \( (S \leq S_2; \varepsilon \geq \varepsilon_f) \) will be expressed by the following equation with the pre-strain \( \varepsilon_1 \).

\[
A_{\infty} + A_{e} - \frac{\varepsilon - \varepsilon_{1}}{\varepsilon_{1}} - 1 \quad \cdots (8)
\]

On the other hand, when the strain \( \varepsilon \) at the time of \( S = S_2; \varepsilon = \varepsilon_f \) is determined by the above equation and denoted by \( \varepsilon_f \), the stress-strain relation in the region exceeding the stress \( S_2 \) can be given by

\[
\int_{S_2}^{S} A d\varepsilon = \left( \frac{S}{S_2} \right)^{m_2} - 2 \quad \cdots (9)
\]

By using Equations (6) and (8), the above equation is rewritten as follows.

\[
A_{\infty} + A_{e} - \frac{\varepsilon - \varepsilon_{1}}{\varepsilon_{1}} - 1 \quad \cdots (10)
\]

The stress-strain relations given by Equations (8) and (10) will describe the Bauschinger curve. Here, we introduce an equivalent strain \( \varepsilon \) given by the following

--- STATIC TENSION CURVE
--- BAUSCHINGER CURVE

(a) Stress-strain curve  (b) Distribution of strength of barriers

Fig. 1 Change of the field of resisting force accompanying pre-deformation (schematic diagram)

*1 It is assumed that the density function \( h(\sigma) \) taking part in the deformation at an arbitrary stress level is given by a Weibull distribution \( w(S;Sc,m) \) having the stress \( S \) as a variable measured from the maximum or minimum stress experienced in the past, the median value \( Sc \), the shape parameter \( m \) and the location parameter \( \gamma \) which is always zero. That is

\[
h(\sigma) = w(S;Sc,m) = \min \left( \frac{S}{Sc} \right)^{m-1} \frac{2^{1-(S/Sc)^m}}{(S/Sc)^m}
\]

In this case, the following equation holds.

\[
\int_{S}^{S} w(S;Sc,m)dS = 2 \\min \left( \frac{S}{Sc} \right)^{m-1} \frac{2^{1-(S/Sc)^m}}{(S/Sc)^m}
\]
equation reflecting the pre-strain $\epsilon_1$.

$$\tilde{\epsilon} = \epsilon_1 \left(1 + \frac{A_2 - A_{\infty}}{A_{\infty}} \epsilon^{\alpha} \left(1 - \frac{\epsilon}{\epsilon_1}\right)\right)$$

(11)

Then, Equations (8) and (10) can be simply expressed as follows, respectively.

$$\left(\frac{S}{S_2}\right) = \left(\frac{\log_2(A_{\infty} \epsilon + 1)}{\epsilon_0}\right)$$

(12)

$$\left(\frac{\sigma_1}{\sigma_{01}}\right) = \left(\frac{\log_2(A_{\infty} \epsilon + 1)}{\epsilon_1}\right)$$

(13)

Here, Equation (6) is assumed to be applicable to all the cases. The value of the second term in braces of right side in Equation (11) calculated with the experimental values of $A_2$, $A_{\infty}$ and $\alpha$ for the carbon steel and the brass can be considered to be sufficiently small in comparison with unity in the case when $\epsilon_1$ is not very small. Therefore, in such a case, $\tilde{\epsilon}$ may be considered to be nearly equal to $\epsilon$.

The characteristics of Equation (12) and Equation (13) are examined. When these equations are expressed on the Inlog$_2(A_{\infty} \epsilon + 1)$ - ln S or the Inlog$_2(A_{\infty} \epsilon + 1)$ - ln $\sigma$ coordinate systems as shown in Figure 3, they become two straight lines. The values of $m_1$ and $m_2$ can be decided by the slopes of these lines and the values of $S$ and $\sigma$ at log$_2(A_{\infty} \epsilon + 1) = 1$ correspond to $S_2$ and $\sigma_0$, respectively. On the other hand, Equation (13) represented in the Inlog$_2(A_{\infty} \epsilon + 1)$ - ln S coordinates becomes the curve schematically shown in Figure 3 as the broken line. Accordingly, the experimental Bauschinger curve with a wide range of strains plotted on the Inlog$_2(A_{\infty} \epsilon + 1)$ - ln S coordinates will be expressed by a curve having a turning point such as a deep solid line shown in Figure 3. The stress at this turning point corresponds to the boundary stress $\sigma_2$.

3. Examination by Experimental Curves

In order to examine the propriety of the relations presented here and the characteristics of the change of the field of the resisting force caused by pre-deformation, axial load tests on a carbon steel and a brass were carried out using hour-glass-shaped test pieces (minimum diameter 8mm). The main mechanical properties of these materials are shown in Table 1. The strain range in the Bauschinger curves after the initial tensile or compressive loading was regulated to be about $|\epsilon_1| = 0.2$.

In Figure 4(a) and (b), the Bauschinger curves for the carbon steel are expressed in the Inlog$_2(A_{\infty} \epsilon + 1)$ - ln S coordinates. The turning point mentioned in the preceding.

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### Table 1: Main mechanical properties of the materials tested

<table>
<thead>
<tr>
<th>Material</th>
<th>S45C</th>
<th>SM41M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield point $\sigma_y$ (MPa)</td>
<td>328</td>
<td>—</td>
</tr>
<tr>
<td>Proof stress $\sigma_{ps}$ (MPa)</td>
<td>182</td>
<td>—</td>
</tr>
<tr>
<td>Tensile strength $\sigma_t$ (MPa)</td>
<td>615</td>
<td>419</td>
</tr>
<tr>
<td>Reduction of area $\psi$ (%)</td>
<td>40.5</td>
<td>40.4</td>
</tr>
<tr>
<td>Young's modulus $E$ (GPa)</td>
<td>220</td>
<td>87.4</td>
</tr>
<tr>
<td>Poisson's ratio $\nu$</td>
<td>0.29</td>
<td>0.37</td>
</tr>
</tbody>
</table>

---

Fig. 3 Diagram for explaining the relation between Eq. (12) and Eq. (13)

Fig. 4 Bauschinger curves (carbon steel)
ing paragraph can be found in each curve regardless of the direction of pre-deformation. Similar behaviors are observed for the brass as shown in Figure 5. The boundary stress $\sigma_2$, the distribution median $S_{c2}$ and the shape parameter $m_2$ for each curve can be decided based on the curve shown in these figures. The relation between $S_{c2}$ and $|\sigma_1|$ and the relation between $|\sigma_2|$ and $|\sigma_1|$ are shown in Figure 6 and Figure 7, respectively. It is clearly confirmed from these figures that both $|\sigma_2|$ and $S_{c2}$ increase linearly as the pre-stress $|\sigma_1|$ increases. In Figure 8(a) (for the carbon steel) and Figure 8(b) (for the brass), the experimental curves exceeding the boundary stress are expressed in the $\ln \log_{2} (A_{m} \bar{e} + 1)$ vs. $\ln |\sigma|$ coordinates. It may be seen that all curves have the same slope nearly equal to unity regardless of the materials. Figure 9 shows the relation between $\sigma_{yb}$ decided by the experimental curve and the pre-stress $|\sigma_1|$. The value of $\sigma_{yb}$ for the brass increases as the pre-stress increases but in the carbon steel $\sigma_{yb}$ does hardly depend on the pre-stress. This difference is considered to be mainly caused by the difference of the slip system in these materials.

Fig. 5 Bauschinger curves (brass)

Fig. 6 Relation between $S_{c2}$ and $\sigma_1$

Fig. 7 Relation between $|\sigma_2|$ and $\sigma_1$

Fig. 8 Bauschinger curves in the region exceeding $|\sigma_2|$
A similar effect of the pre-stress on the boundary stress has been observed in Figure 7.

Figure 10 shows the relation between the pre-stress \( \sigma_1 \) and the value of \( B \) calculated on the basis of the experimental curves, where \( B \) is the ratio of the both sides of Equation (6). That is,

\[ B = \left( \frac{\sigma_1 + \sigma_2}{\sigma_2} \right) \frac{\left( \frac{\sigma_2}{\sigma_{y_2}} \right) m_2}{1} \]

Since the experimental data are close to \( B = 1 \) regardless of the materials and the pre-stress, the validity of Equation (6) is confirmed. Accordingly, the value of \( m_2 \) can be determined by Equation (6) with three values of \( Sc \), \( \sigma_2 \) and \( \sigma_{yb} \) depending on the pre-stress.

From the above results, the characteristics of the change of the resisting force caused by the pre-deformation can be described as follows.

If \( \Delta \sigma_{y_2} \), the change of the median of the initial density distribution \( \sigma_{y_2} \) during the initial loading is in proportion to the pre-stress \( \sigma_1 \), the following equation holds.

\[ \Delta \sigma_{y_2} = (Sc_2 - \sigma_2) \sigma_{y_2} = \mu_1 \sigma_1 \]

where \( \mu_1 \) is a proportional constant. Accordingly,

\[ Sc_2 = \sigma_{y_2} + (1 + \mu_1) \sigma_1 \]

Moreover, if the change in the stress level from \( \sigma_{y_2} \) to \( \sigma_{y_2} \) which is the median of density function \( B_0 \) is in proportion to the stress increment \( \Delta \sigma_{y_2} \), that is, if

\[ \sigma_{yb} = \sigma_{y_2} = \mu_2 \Delta \sigma_{y_2} \]

then, from Equation (14) the following equation is obtained.

\[ \sigma_{yb} = \sigma_{y_2} + \mu_1 \mu_2 \sigma_1 \]

where \( \mu_2 \) is a proportional constant. Comparing Equation (15) and Figure 9, the values of \( \sigma_{yb} \), \( \sigma_{y_2} \), \( \mu_1 \mu_2 \) can be evaluated for each material. The value of \( \sigma_{yb} \) for the carbon steel and the brass is 94.8 (MPa) and 36.7 (MPa), respectively. On the other hand, the relationships of Equation (14) using these values of \( \sigma_{yb} \) seem to agree with the experimental results shown in Figure 6 for both the materials. Therefore, \( \mu_1 = 0.038 \) for the carbon steel and \( \mu_1 = 0.32 \) for the brass are obtained. From these values of \( \mu_1 \) and the values of \( \mu_2 \) obtained above, the value of \( \mu_2 \) is decided as \( \mu_2 = 0.231 \) for the carbon steel and \( \mu_2 = 0.0341 \) for the brass. The relation between the boundary stress \( \sigma_{y_2} \) and the pre-stress can be formulated by the following equation based on the experimental results shown in Figure 7.

\[ \sigma_{y_2} = \mu_2 \sigma_1 \]

Here, \( \mu_2 \) is a material constant. It is 0.966 for the carbon steel and 1.19 for the brass.

The value of \( \sigma_{yb} \) for the brass evaluated in this study, however, is much smaller than that estimated in the previous study based on the hysteresis curves in the steady state with an assumption that the initial field of the resisting force might be preserved in the stress region not experienced in the past. This is not the case in the carbon steel. This suggests that the characteristics of the change of the force field under the cyclic loading up to the steady state depends on the materials. Nevertheless, it has been confirmed that the experimental tensile stress-strain curve for the brass can be fairly expressed by the descriptive method presented in the previous paper if the value of \( \sigma_{yb} \) obtained here is used. The material constants obtained in this study and those revalued are shown in Table 2.

From the above examination, it is clear that the Bauschinger curves can be described by the two density functions determined by the pre-stress. In Figure 11 the experimental curves for the carbon

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**Table 2 Material constants related to the field of resisting force**

<table>
<thead>
<tr>
<th></th>
<th>S45C</th>
<th>BsBM1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{y2} ) (MPa)</td>
<td>94.8</td>
<td>36.7</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>0.94</td>
<td>0.74</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>12650</td>
<td>12000</td>
</tr>
<tr>
<td>( A_m )</td>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td>( a )</td>
<td>1000</td>
<td>364</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>-0.038</td>
<td>0.32</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>-0.231</td>
<td>0.341</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>0.966</td>
<td>1.19</td>
</tr>
</tbody>
</table>
steel are compared with the stress-strain curves as obtained by using the formulae presented here. As a whole, they are in good agreement. Figure 12(a) and (b) show examples of the calculated Bauschinger curves in the region exceeding the stress $\sigma_1$ in comparison with the static tension curves. The calculated curves are obtained by the approximation of $\varepsilon_0$. In these figures, the effects of the pre-stress on the Bauschinger curve are remarkable in the lower strain region for both the carbon steel and the brass. In the higher strain region, however, the Bauschinger curves for the carbon steel are close to each other but those for the brass keep the effects of the pre-stress. Such a difference between the carbon steel and the brass in Bauschinger curves at large strain seems to be closely related to the characteristics of the force field shown in Figure 9 as the relation between $\alpha_0$ and $\alpha_1$.

Takahashi et al. showed that each curve obtained by moving the Bauschinger curves through the magnitude of pre-strain along the strain axis coincides with the loading curve in large strain for an aluminum(4). Such a material may be considered to have a force field characterized by $\alpha_0 = \varepsilon_0 + \alpha_1 = \alpha_1 + \varepsilon_0$.

4. Conclusions

A new approach to describing the Bauschinger curves for metals is presented based on the concept of the field of resisting force in the random barriers theory. It is assumed that the field of the resisting force after the initial loading can be expressed by a composite Weibull distribution composed of two density functions divided at a boundary stress. By making a comparison of the experimental Bauschinger curves with the stress-strain relationships formulated with the force field mentioned above, the effects of the pre-deformation on these density functions are discussed. As the result, it has been revealed that the characteristics of the change in these density functions might be estimated quantitatively to depend on the pre-stress only. The descriptive formulae presented here have provided a satisfactory agreement with each experimental Bauschinger curve for a carbon steel and a brass with a wide range of strains.

References


![Fig.11 Comparison of calculated curves and experimental results (carbon steel)](image)

![Fig.12 Effect of pre-stress $\sigma_1$ on the Bauschinger curves)](image)