A Propulsion Mechanism for a Ship Using the Weis-Fogh Mechanism*

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A model of a propulsion mechanism for a ship is proposed, using a novel and efficient lift generation mechanism found by Weis-Fogh. The model consists of a wing (or two wings) in a square channel, and the motion of the wing has three stages: opening, translating, and closing and their repeated cycles. Each stage of the motion of the wing is analyzed by assuming that the flow is a potential flow and using conformal mapping. The circulation around the wing and the dynamic properties, such as the thrust, drag, and moment on the wing in each stage are calculated. The conditions to make this propulsion mechanism work efficiently are established and the applicability of this mechanism to actual ships is predicted to be good.

Key Words : Unsteady Flow, Hydraulic Machine, Potential Flow, Wing, Analytical Study, Ship Propulsion, Propulsion Mechanism

1. Introduction

The Weis-Fogh mechanism⁽¹⁾ which was discovered by observing the hovering flight of a small insect has attracted great attention as a novel and efficient mechanism for lift generation. Lighthill⁽²⁾ has considered a two-dimensional inviscid-flow model and has shown that circulations in opposite directions to each other are generated respectively around two wings, and that is a characteristic of this mechanism. Maxworthy⁽³⁾ has performed an experimental simulation of this mechanism in a two-dimensional model and also in a three-dimensional model. He has found that the circulation in the two-dimensional case is even larger than that obtained by the inviscid model of Lighthill. Edwards and Cheng⁽⁴⁾ have obtained a result, applying a shedding vortex from the leading edge, which agrees well with the experiment by Maxworthy even though it is based on the assumption of an inviscid fluid. On the other hand, Haussling⁽⁵⁾ has solved the Navier-Stokes equation by the finite difference method in the two-dimensional model and has found solutions for the separating stage and the closing or clapping stage. A practical application of this mechanism has been tried by Furber and Ffowcs Williams⁽⁶⁾ and a possibility of raising the efficiency of turbo-machinery is suggested. Tsutahara and Kimura⁽⁷⁾ have studied the dynamic properties of the two-dimensional model for the fling stage and the separating stage and have shown that sufficient lift is generated on the assumption of an inviscid flow.

Here we propose a model of the propulsion mechanism so that we can apply the two-dimensional model of the Weis-Fogh mechanism to a ship's propulsion mechanism, and investigate analytically the properties of the above model.

2. A Model of Ship Propulsion

In a model shown in Fig.1, two wings move in a square channel as shown in (a) to (h) and repeat this motion. We call here the motion in (a) the opening stage, that in (b) the translating stage, and that in (c) and (d) the closing stage. The motions in (e) to (h) are the same respectively as those in (a) to (d) considering the images due to the wall of the channel. The uniform flow downward in Fig.1 corresponds to the advancing speed of the ship. In the opening stage in (a), when the angular velocity is large enough, a clockwise circulation is generated around the left-side wing and an anti-clockwise circulation which is the same in magnitude as the former around the right-side wing, and these produce the thrust (the force upward in this figure) when the wings translate in the opposite directions. As stated later, even in the opening stage sufficient thrust can be generated by applying an angular acceleration to the opening wings. In the closing stage in (c), the fluid between the wing and the wall is pushed downward as a jet and this produces a large thrust.

In the case where the channel is long enough, the flow fields on the left side and the right side of the center line shown in (a) and (b) in Fig.1 are symmetric to each other, and then the center line can be considered a solid wall. In the analysis described later, only the flow field on the left-hand side is considered. The flow is assumed

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Fig.1 A model of propulsion mechanism

to be a potential flow. In hovering flight, i.e., $U = 0$, a jet emanates from the gap between the two wings at the closing stage as shown by Haussling, and the flow may be completely different from the potential flow. But in this case, due to the uniform flow, the difference between the velocities of the jet and the ambient flow will be small, then even in the closing stage the assumption of the potential flow may be rather justifiable. The flow field is a singly connected region for the opening and closing stages, but it is a doubly connected region for the translating stage. So we shall analyze the opening stage and the closing stage in Section 4, and the translating stage in Section 5.

3. Nomenclature

α : opening angle or attack angle of the wing
 Ω : angular velocity of the wing rotation
 $\dot{\Omega}$: angular acceleration of the wing rotation
 c : chord length of the wing
 d : width of the channel
 U : velocity of the uniform flow or advancing speed of the ship
 V : velocity of translation of the wing
 $\dot{V}$: acceleration of translation of the wing
 z : physical plane $= x + iy$
 ζ, Z : mapped plane
 K : a constant of mapping
 $F(Z)$: complex potential on the Z -plane
 ϕ : velocity potential
 ψ : stream function

Fig.2 Physical Plane (z -plane)

$a_n, b_n, \alpha_n, \beta_n, \gamma_n, \delta_n$: Fourier coefficients

H_1 : distance between the center of the wing and the upper wall

H_2 : distance between the center of the wing and the lower wall

Γ : circulation about the wing (clockwise is positive)

$X+iY$: force on the wing

Q_x+iQ_y : suction force at the leading edge of the wing

T : thrust

D : drag

M : moment about the trailing edge

4. Analysis for the Opening and Closing Stages

4.1 Conformal mapping and the circulation about the wing

We consider here a flow field in which a plane of chord length c is rotating about the trailing edge with an angular velocity Ω in a channel of width d , upper wall DE , and lower wall EG . The above flow field is the left

half or the right half of (a) and (e), respectively, in Fig.1 and d is a half of the channel width in the same figure.

If the speed of the ship is denoted by U , the boundary conditions of the flow infinitely upstream and downstream are a uniform velocity U . If we put the variables Q and U as $-Q$ and $-U$, respectively, then this flow becomes the model of (c) or (g) in Fig.1. In the above case, the directions of the flow are reversed but the stream lines do not change. The actual flow, however, does not have this type of reversibility. We call the physical plane in Fig.2 the z -plane, and take the trailing edge A_1 (A_2) at the origin. The domain inside the channel in the z -plane is mapped onto the upper half of the ζ -plane shown in Fig.3 by the function

$$\frac{dz}{d\zeta} = K \frac{1}{\zeta-b} \frac{\zeta-a}{\zeta-1} \left(\frac{\zeta-1}{\zeta+1} \right)^{a/\pi} \quad (1)$$

The points denoted by the same letters in the z -plane and the ζ -plane correspond to each other.

Since points A_1 and A_2 represent the same point in the z -plane, we have

$$a = b + (1-b) \left(\frac{1+b}{1-b} \right)^{a/\pi} \quad (2)$$

$$K = d/\pi \quad (3)$$

$$\frac{c}{d} = \frac{1}{\pi} \int_{-1}^a \frac{1}{\lambda-b} \frac{a-\lambda}{1-\lambda} \left(\frac{1-\lambda}{1+\lambda} \right)^{a/\pi} d\lambda = \frac{h(a)}{\pi} \quad (4)$$

where c is the chord length of the wing and

$$h(\tau) = \int_{-1}^{\tau} \frac{1}{\lambda-b} \frac{a-\lambda}{1-\lambda} \left(\frac{1-\lambda}{1+\lambda} \right)^{a/\pi} d\lambda \quad (5)$$

The complex potential in the ζ -plane is expressed as

$$F(\zeta) = \frac{QK^2}{2\pi} \int_{-1}^{\zeta} \frac{h^2(\tau)}{\zeta-\tau} d\tau - \frac{Ud}{\pi} \log(\zeta-b) \quad (6)$$

The circulation about the wing is the difference of the velocity potential at A_1 and A_2 and is

$$\frac{\Gamma}{Qcd} = \frac{d}{\pi^2 c} \int_{-1}^1 (1-\tau^2)^{-1} h^2(\tau) d\tau - \frac{U}{Qc\pi} \log \frac{b-1}{b+1} \quad (7)$$

Fig.3 ζ -plane

Fig.4 Circulation about the wing

The circulation for $c/d=0.1$ is shown in Fig.4. In this figure the clockwise circulation is defined as positive, and the circulation is symmetrical with respect to the opening angle of 90° . In the case without a uniform flow ($U=0$), the circulation is always positive, but as the uniform flow velocity becomes large, the circulation becomes negative at a large opening angle (near 90°).

4.2 Flow field

If we are to obtain the flow field by the potential on the ζ -plane, we must perform the integration to obtain the value of the potential at each point, and we must perform the singular integration to find the value on the wing surface. Accordingly we obtained the flow field by mapping again the upper half of the ζ -plane onto the inside of a circle of radius unity of the Z -plane shown in Fig.5 using the function

$$\zeta = -i \frac{Z-i}{Z+i} \quad (8)$$

The complex potential in the Z -plane is expressed in the general form

$$F(Z) = \sum_{n=0}^{\infty} c_n Z^n \quad (9)$$

$$c_n = a_n + ib_n$$

where a_n and b_n are real. Putting $Z = re^{i\theta}$ and substituting it into Eq.(9), the stream function (the imaginary part of $F(Z)$) is written

$$\psi = \sum_{n=0}^{\infty} r^n (a_n \sin n\theta + b_n \cos n\theta) \quad (10)$$

As Eq.(9) is considered a Fourier series expansion, the Fourier coefficients a_n and b_n are given by the stream function $\psi(\theta)$ at $r=1$, that is, at the surface of the circle as

Fig.5 Z-plane

$$\left. \begin{aligned} a_0 &\equiv 0 \\ b_0 &= \frac{1}{2\pi} \int_0^{2\pi} \Psi(\theta) d\theta \\ a_n &= \frac{1}{\pi} \int_0^{2\pi} \Psi(\theta) \sin n\theta d\theta \\ b_n &= \frac{1}{\pi} \int_0^{2\pi} \Psi(\theta) \cos n\theta d\theta \end{aligned} \right\} \dots\dots\dots (11)$$

($n=1, 2, \dots$)

In the stream function Ψ at the surface of the circle, the upper half corresponds to the surface of the wing, and it is expressed as

$$\Psi(z) = -\frac{1}{2} Q(z e^{-i\alpha})^2 \dots\dots\dots (12)$$

in the z -plane, so that $\Psi(\theta)$ can be obtained by using the functions of mapping Eq.(1) and Eq.(8). On the other hand, for the lower half, if $\Psi \equiv 0$ on the line of symmetry GE in the z -plane, then $\Psi = -Ud$ on the upper wall, so that in the Z -plane, we have

$$\Psi = 0 \quad \text{on } A_2G \text{ and on } A_1E$$

$$\Psi = -Ud \quad \text{on } DE$$

If the above boundary conditions are used, the convergence of the Fourier coefficients given in Eqs.(11) is not fast because of the stepwise change of the value of Ψ at D and E . Therefore, we put

$$\Psi \equiv 0 \quad \text{on } A_1EDA_2 \dots\dots\dots (13)$$

and we obtain the complex potential by putting a source and a sink

$$\frac{Ud}{\pi} \log(Z - Z_D) \quad \text{on } D \dots\dots\dots (14)$$

$$\frac{-dU}{\pi} \log(Z - Z_E) \quad \text{on } E \dots\dots\dots (15)$$

where Z_D and Z_E denote the complex variables representing the points D and E in the Z -plane.

4.3 Pressure, force, moment on the wing

The pressure will be given by the pressure equation

$$p = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} |v|^2 \right) + f(t) \dots\dots\dots (16)$$

where ρ is the density, $|v|$ is the absolute value of the fluid velocity, and $f(t)$ is a function of time, but we can put $f(t)=0$ because it cancels out when the integration of the pressure around the wing is performed. The term $\partial \phi / \partial t$ is the differentiation of ϕ with respect to time with $z=\text{const}$ in the z -plane, and is estimated in the Z -plane to be

$$\frac{\partial \phi}{\partial t} \Big|_z = \frac{\partial \phi}{\partial t} \Big|_Z + \text{Real} \left\{ \frac{dF}{dZ} \frac{dZ}{dt} \right\} \Big|_Z + \frac{Q}{Q} \phi \dots\dots\dots (17)$$

where "Real" indicates the real part. For the first term on the right hand side of Eq.(17), we obtain numerically the change of ϕ in the interval Δt at any point of $Z=\text{const}$, and calculate

$$\frac{\partial \phi}{\partial t} \approx \frac{\Delta \phi}{\Delta t} \dots\dots\dots (18)$$

for $\Delta t=0.0001$. The calculation of $\Delta \phi$ in this case is performed assuming Q as constant. The term dF/dZ is derived from Eq.(9). For dZ/dt , if we write

$$z = g(Z) \dots\dots\dots (19)$$

then differentiating both sides of Eq.(19) and equating them to zero, we have

$$\frac{dZ}{dt} = -\frac{\Delta g / \Delta t}{\partial g / \partial Z} \dots\dots\dots (20)$$

where $\Delta g / \Delta t$ is given numerically by calculating Δg , the change of $g(Z)$ during Δt , at any point where Z is constant. The third term, which is drawn out from $\partial \phi / \partial t$ because the effect of the angular acceleration can easily be estimated, is derived from the differentiation of Ψ in a_n and b_n in Eq.(11).

The forces and moment on the wing are obtained by integrating the pressure over the wing surface. The integration is performed on the Z -plane. Putting X and Y as the forces in the x -direction and the y -direction, respectively, and M as the moment about the trailing edge of the wing, we have

$$X + iY = i\rho \int_0^\pi \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} \left| \frac{dF}{dZ} \frac{dz}{dZ} \right|^2 \right\} \frac{dz}{dZ} i e^{i\theta} d\theta \dots\dots\dots (21)$$

$$M = -\rho \text{Real} \int_0^\pi \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} \left| \frac{dF}{dZ} \frac{dz}{dZ} \right|^2 \right\} \times \bar{z}(\bar{Z}) \frac{dz}{dZ} i e^{i\theta} d\theta \dots\dots\dots (22)$$

The pressure becomes negative infinity at the leading edge of the wing and a suction force acts on this point. This suction force can be given by the theorem of residue as

$$Q_x - iQ_y = i\pi \operatorname{Res} \left\{ \left(\frac{dF}{dZ} \right)^2 \frac{dz}{dZ} \right\}_{Z=e^{i\theta}} \quad (23)$$

where Q_x and Q_y are, respectively, the components in the x-direction and in the y-direction of the suction force, and Res denotes the residue at the leading edge $Z=e^{i\theta}$ and is obtained by

$$\operatorname{Res} \left\{ \left(\frac{dF}{dZ} \right)^2 \frac{dz}{dZ} \right\}_{Z=e^{i\theta}} = \left\{ \left(\frac{dF}{dZ} \right)^2 \frac{d^2 z}{dZ^2} \right\}_{Z=e^{i\theta}} \quad (24)$$

The thrust T on the wing, which is the x-component of the force, is represented

Fig.6 Relationships between thrust and opening angle ($\dot{\Omega} = 0$)

Fig.7 Moment on the wing ($\dot{\Omega} = 0$)

Fig.8 Thrust ($\dot{\Omega} \neq 0$)

Fig.9 Moment ($\dot{\Omega} \neq 0$)

by

$$T = X + Q_x \quad (25)$$

4.4 Results

Although we show here the results only for $c/d=0.1$, the tendency is almost the same for $c/d=0.2$. As the accuracy of the calculation is not good for small opening angles, the results are given for angles larger than 20° . The thrust for the case of constant rotating velocity is shown in Fig.6 as a function of the opening angle. The curve is symmetrical with respect to the point at which the thrust is zero and the angle is 90° , and we can consider 0° to 90° as the opening stage and 90° to 180° as the closing stage. In the opening stage, the thrust becomes negative, and this means that the wing produces a drag as the ship sails. But in the closing stage, sufficient thrust is given. The moment about the trailing edge of the wing is shown in Fig.7. The moment is defined as positive when it is anti-clockwise, and it is also symmetrical at 90° and is shown only for 0° to 90° . This figure also shows that the wing has a moment in the direction of its opening, and then the wing receives energy from the flow when it is opening at constant angular velocities.

When angular acceleration is applied to the rotation of the wing, the thrust and moment for $U/Qc = 0.5$ become as shown in Fig.8 and Fig.9 and the wing generates positive thrust and does work to the flow when the angular velocity is adequately large. Here we note that the curves in Figs.8 and 9 do not represent the history of the wing motion.

5. Analysis of the Translating Stage

5.1 Mapping onto the circular ring

A flow shown in Fig.10 will be

considered as a model in which the two wings are separated and translating in the direction vertical to a uniform flow after opening to some angle. In this figure, the origin is taken at the intersection of the lower wall and the vertical line passing through the center of the wing. The lower wall can be considered the line of symmetry as in Fig.2. The domain inside the channel in the z -plane is mapped onto the inside of the circular ring in the Z -plane by the function⁽⁸⁾

$$z = K + i \frac{2d}{\pi} \left\{ \frac{1}{2i} \log \frac{Z - qe^{i\theta_2}}{Z - qe^{-i\theta_2}} + \sum_{n=1}^{\infty} \frac{q^{2n} \sin n\theta_2}{n(1 - 2q^{2n} \cos 2\alpha + q^{4n})} \times \left[\left(\frac{Z}{q} \right)^n (e^{2i\alpha} - q^{2n}) - \left(\frac{q}{Z} \right)^n (e^{-2i\alpha} - q^{2n}) \right] \right\} \quad (26)$$

The radius of the outer circle of the ring is always taken to be unity, and the radius of the inner circle q ($q < 1$) depends on the position, angle, or the chord length of the wing. If we put the points in the Z -plane corresponding to the leading edge as A , the trailing edge as A' , the point of infinity upstream and the one downstream as

$$\left. \begin{aligned} A &= e^{i\theta_1}, & A' &= e^{i\theta_2} \\ H &= qe^{i\theta_2}, & H' &= qe^{-i\theta_2} \end{aligned} \right\} \quad (27)$$

then we have a relation between θ_1 and θ_2 as

$$\theta_1 + \theta_2 = -2\alpha \quad (28)$$

We also have expressions from the relation between the z -plane and the Z -plane as

$$K = \frac{4d}{\pi} e^{i\alpha} \sum_{n=1}^{\infty} \frac{q^n \sin n\theta_2 \cos n(\theta_1 + \alpha)}{n(1 - 2q^{2n} \cos 2\alpha + q^{4n})} \times \{-\sin(n-1)\alpha + q^{2n} \sin(n+1)\alpha\} \quad (29)$$

$$C = \frac{8d}{\pi} \sum_{n=1}^{\infty} \frac{q^n \sin n\theta_2 \sin n(\theta_1 + \alpha)}{n(1 - 2q^{2n} \cos 2\alpha + q^{4n})} \times \{\cos(n-1)\alpha - q^{2n} \cos(n+1)\alpha\} \quad (30)$$

$$H_1 = \frac{d\theta_2}{\pi} - \text{Im}(K) \quad (31)$$

Fig.10 z -plane

$$H_2 = d \left(1 - \frac{\theta_2}{\pi} \right) + \text{Im}(K) \quad (32)$$

For determining the mapping we need another equation, for example at the trailing edge of the wing as follows

$$iH_2 - \frac{c}{2} e^{i\alpha} = K + \frac{d}{\pi} \log \frac{e^{i\theta_2} - qe^{i\theta_1}}{e^{i\theta_2} - qe^{-i\theta_2}} + \frac{i2d}{\pi} \left\{ \sum_{n=1}^{\infty} \frac{q^{2n} \sin n\theta_2}{n(1 - 2q^{2n} \cos 2\alpha + q^{4n})} \times \left[\left(\frac{e^{i\theta_2}}{q} \right)^n (e^{2i\alpha} - q^{2n}) - \left(\frac{q}{e^{i\theta_2}} \right)^n (e^{-2i\alpha} - q^{2n}) \right] \right\} \quad (33)$$

Substituting Eq.(29) into Eqs.(32) and (33), these two equations and Eq.(30) with given d , c , and H_2 determine the variables θ_1 , θ_2 , and q .

5.2 The complex potential inside the ring

A complex potential $F_1(Z)$ inside the ring is generally expressed by

$$F_1(Z) = \alpha_0 + i\beta_0 + \sum_{n=1}^{\infty} (\alpha_n + i\beta_n) Z^n + \sum_{n=1}^{\infty} (\gamma_n + i\delta_n) Z^{-n} + \frac{i\Gamma}{2\pi} \log Z \quad (34)$$

where α_n , β_n , γ_n , δ_n , and Γ are real. The outer circle of the ring corresponds to the wing surface and the inner circle to the wall; then the stream functions on the circles are written as

$$\left. \begin{aligned} \Psi_1(\theta) & \quad \text{on the inner circle} \\ \Psi_H(\theta) &= \Psi_A' + \Psi_2(\theta) \quad \text{on the outer circle} \end{aligned} \right\} \quad (35)$$

The uniform flow is estimated by the source and the sink on the boundary, and therefore we can put

$$\Psi_1(\theta) \equiv 0 \quad (36)$$

For $\Psi_H(\theta)$ on the outer circle, Ψ_A' represents the stream function at the

Fig.11 Z -plane

trailing edge of the wing and it is determined if the circulation around the wing is given. The function $\Psi_2(\theta)$ is expressed as

$$\Psi_2(\theta) = -V|z - z_A|\cos\alpha \quad (37)$$

where V denotes the speed of translation of the wing. We have

$$\left. \begin{aligned} \frac{\Gamma}{2\pi} \log q + \Psi_A &= -\frac{1}{2\pi} \int_0^{2\pi} \Psi_2(\theta) d\theta \\ \alpha_n &= \frac{1}{1-q^{2n}} \frac{1}{\pi} \int_0^{2\pi} \Psi_2(\theta) \sin n\theta d\theta \\ \beta_n &= \frac{1}{1-q^{2n}} \frac{1}{\pi} \int_0^{2\pi} \Psi_2(\theta) \cos n\theta d\theta \\ \gamma_n &= \frac{-1}{1-q^{-2n}} \frac{1}{\pi} \int_0^{2\pi} \Psi_2(\theta) \sin n\theta d\theta \\ \delta_n &= \frac{1}{1-q^{-2n}} \frac{1}{\pi} \int_0^{2\pi} \Psi_2(\theta) \cos n\theta d\theta \end{aligned} \right\} \quad \dots (38)$$

in a similar manner, when the Fourier coefficients are obtained in § 4.2. In the case where a uniform flow exists, we obtain the complex potential by adding the sink and the source of strength Ud/π and $-Ud/\pi$ at the points H and H' to the function in Eq.(34). Considering the images of these source and sink, we finally have

$$\begin{aligned} F(Z) &= \alpha_0 + i\beta_0 + \sum_{n=1}^{\infty} (\alpha_n + i\beta_n) Z^n \\ &+ \sum_{n=1}^{\infty} (\gamma_n + i\delta_n) Z^{-n} + \frac{i\Gamma}{2\pi} \log Z \\ &+ \frac{Ud}{\pi} \sum_{m=1}^{\infty} \left\{ \log \frac{Z - q^m e^{-i\theta_2}}{Z - q^m e^{i\theta_2}} + \log \frac{q^m Z - e^{-i\theta_2}}{q^m Z - e^{i\theta_2}} \right\} \end{aligned} \quad (39)$$

If we assume that the Kutta condition is satisfied at the trailing edge of the wing, then the circulation is given by the condition that the point of $A' = e^{i\theta_2}$ becomes a stagnation point

$$\left. \frac{dF}{dZ} \right|_{Z=e^{i\theta_2}} = 0 \quad (40)$$

5.3 Calculation of the thrust, drag, and moment on the wing

The pressure is obtained from the pressure equation (16). When the acceleration of the translating speed V is to be considered, the term $\partial\phi/\partial t$ is expressed by

$$\left. \frac{\partial\phi}{\partial t} \right|_z = \left. \frac{\partial\phi}{\partial t} \right|_z + \text{Real} \left\{ \frac{dF}{dZ} \cdot \frac{dZ}{dt} \right\} \Big|_z + \frac{\dot{V}}{V} \phi \quad (41)$$

instead of Eq.(17). Each term on the right hand side of the above equation can be calculated in the same manner as in § 4.3. If the x-component and the y-component of the force in the z-plane are denoted by X and Y , respectively, and the moment about the center of the wing by M , then we have

$$\begin{aligned} X + iY &= i\rho \int_0^{2\pi} \left\{ \frac{\partial\phi}{\partial t} + \frac{1}{2} \left| \frac{dF}{dZ} \right| \frac{dz}{dZ} \right\} \\ &\times \frac{dz}{dZ} i e^{i\theta} d\theta \quad (42) \end{aligned}$$

$$\begin{aligned} M &= -\rho \text{Real} \int_0^{2\pi} \left\{ \frac{\partial\phi}{\partial t} + \frac{1}{2} \left| \frac{dF}{dZ} \right| \frac{dz}{dZ} \right\} \\ &\times \overline{(z - iH_2)} \frac{dz}{dZ} i e^{i\theta} d\theta \quad (43) \end{aligned}$$

As the Kutta condition is applicable at the trailing edge for obtaining the circulation, the pressure at this point is finite, but at the leading edge the pressure becomes infinite and a suction force acts as in § 4.3. This can be given in a similar manner in the form of Eq.(23) by

$$Q_x - iQ_y = i\pi \text{Res} \left\{ \left(\frac{dF_1}{dZ} \right)^2 \frac{dz}{dZ} \right\} \Big|_{Z=e^{i\theta_2}} \quad (44)$$

Here we define the thrust and the drag on the wing as

$$T = X + Q_x \quad (45)$$

$$D = -(Y + Q_y) \quad (46)$$

5.4 Results

The results are shown for the cases of the wing separated from the walls at some distance because otherwise the accuracy will not be good. The circulation about the wing for the opening angles of $\alpha = 30^\circ$ and $\alpha = 45^\circ$ is shown in Fig.12 as a function of the position of the wing in the channel. For $\alpha = 30^\circ$ the circulation does not vary, but for 45° it varies rapidly as the wing approaches the upper wall, especially for $U/V=2$. This means that some vortices are shed from the wing and it contradicts the assumption of a vortex free flow, and it is doubtful that the Kutta condition is satisfied at

Fig.12 Relationships between circulation and position of the wing

the trailing edge when the wing is near the wall. In any case, the approximation of the analysis of this situation becomes poor.

The thrusts for $U/V=1$ and $U/V=2$ are shown, respectively, in Figs.13 and 14, and the drags in Figs.15 and 16. For $U/V=1$, when $\alpha=30^\circ$ the thrust is generated even for the acceleration of the translation $V=0$, but when $\alpha=45^\circ$ the wing becomes resistant to the flow if $V=0$. As the force due to the acceleration acts in the direction perpendicular to the wing, it is advantageous to have a large opening angle when the acceleration is given. For $U/V=2$, the wing is resistant for both $\alpha=30^\circ$ and 45° when $V=0$, and so it is necessary to apply the acceleration to generate the thrust. As seen from the curves of the drag, when the flow velocity is larger than the translating speed the wing receives energy from the flow for the case where the wing translates at a constant speed.

6. Discussions

In this analysis, we treated the opening stage and the translating stage of the wing separately and did not refer to the connection between the two. It may be possible to connect them by some

limiting operations, but the analysis described so far gives us much information as it considers a model of propulsion mechanism.

The circulation for the opening stage shown in Fig.4 becomes negative as the opening angle becomes large because of the uniform flow. As seen in Figs 12 and 16, for the translating stage, it can be roughly estimated that the circulation must be positive to generate a thrust without the acceleration. When the stage shifts from opening to translating, it is more effective to keep the variation of the circulation as small as possible because if so, the vortices shed diminish and the energy loss is small. Therefore in the opening stage, it will be desirable to keep the circulation positive by sufficient angular velocity and let it shift into the translating stage.

We did not calculate the propulsive efficiency of this mechanism because the energy loss could not be estimated from the assumption of the vortex free flow. For this purpose it will be necessary to employ another model of the flow or calculation method.

Fig.13 Relationships between thrust and position of the wing ($U/V = 1$)

Fig.15 Relationships between drag and position of the wing ($U/V = 1$)

Fig.14 Relationships between thrust and position of the wing ($U/V = 2$)

Fig.16 Relationships between drag and position of the wing ($U/V = 2$)

7. Conclusions

We have proposed a model to apply the Weis-Fogh mechanism to a ship's propulsion mechanism and analytically obtained the dynamic properties of the model on the assumption of a potential flow. We have the following conclusions.

- (1) In the opening stage, we get a thrust by applying adequate angular acceleration to the wing.
- (2) In the closing stage, we get sufficient thrust by a constant angular velocity of the wing.
- (3) In the translating stage, when the wing translates at a constant velocity, the thrust is large when the opening angle is small. On the other hand, when acceleration is applied, the thrust is large when the opening angle is large.
- (4) In the ship's propulsion mechanism, it is desirable to give sufficient angular acceleration to the wing in the opening stage and let it shift to the translating stage at a small opening angle.

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